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Requirements and detailed content

Name of module: Strength of materials 2

Module code: 18503

a. Number of credits: 02 credits **ASSIGNMENT** ☐ **PROJECT** ☐

b. Department: **Strength of materials**

c. Time distribution:

- Total: 30 lessons. - Theory: 18 lessons.
- Experiment: 0 lesson. - Exercise: 10 lessons.
- Assignment/Project instruction: 0 lesson. - Test: 2 lessons.

d. Prerequisite to register the module: After studying Strength of materials 1.

e. Purpose and requirement of the module:

Knowledge:

On the basic of the fundamental knowledge taught in Strength of materials 1, Strength of materials 2 supplies students with necessary knowledge and calculating methods to solve complicated load-resistant cases, the most popular cases of dynamic load in technics, the ways to compute the stability of the column subjected to axial compressive load as well as curved bars.

Skills:

- Be able to correctly think, analyse, evaluate the load-resistant state of construction parts, machine parts.
- Be capable of applying the knowledge of the subject to solve practical problems.
- Be able to solve the basic problems of the subject proficiently.

Job attitude:

- Obviously understand the important role of the subject in technical fields. As a result, students have serious, active attitude and try their best in study.

f. Describe the content of the module:

Strength of materials 2 module consists of content below:

- Chapter 7: Combined stresses.
- Chapter 8: Buckling of columns.
- Chapter 9: Dynamic load.
- Chapter 10: Curved bar.

g. Compiler: **MSc Nguyen Hong Mai, Strength of materials Department – Basic Science Faculty**

h. Detailed content of the module:

CHAPTER	LESSON DISTRIBUTION				
	SUM	THEORY	EXERCISE	EXPERIMENT	TEST
Chapter 6: Combined stresses	**9**	**6**	**3**		

6.1. Concept - Principle of superposition		0.5			
6.2. Oblique bending (Bending in two directions)		1,5			
6.3. Bending and tension or compression		1.5			
6.4. Simultaneous bending and torsion in round shaft		1.5			
6.5. Round shaft is subjected to general loadings		1			
Exercises			3		
Self-taught contents (18 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 6.5 in reference materials [1] in section l by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes).*					
Chapter 7: Buckling of columns	**7**	**4**	**2**		1
7.1. Concept		0,5			
7.2. The Euler's formula to determine critical load		0.5			
7.3. The Euler's formula to determine critical stress. Scope to use this formula		0.5			
7.4. The formula to determine the critical stress of column as material works outside elastic region		0.5			
7.5. Calculate the stability of the column subjected to axial compressive load thanks to factor of safety about stability (K_{buck})		0.5			
7.6. Calculate the stability of the column subjected to axial compressive load thanks to standard code (Use coefficient φ)		1			
7.7. The suitable shape of cross-section and the way to choose material		0,5			
Exercises			2		
Periodic test					1
Self-taught contents (14 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 7.6, 7.7, 7.8 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes).*					
Chapter 8: Dynamic load	**9**	**6**	**3**		

8.1. Concept, research direction		0.5			
8.2. The problem of translational motion with constant acceleration		1			
8.3. The problem of rotational motion with constant angular velocity		1			
8.4. The problem of oscillation		2			
8.5. The problem of impact		1			
8.6. The critical speed of shafts		0.5			
Exercises			3		
Self-taught contents (18 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 8.6, 8.7 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes.*					
Chapter 9: Curved bar	**5**	**2**	**2**		1
9.1. Concept – Internal force diagram		0.5			
9.2. Calculate curved bar subjected to pure flexure		0.5			
9.3. Determine the radius of curvature of neutral layer		0.5			
9.4. Calculate the curved bar subjected to complicated loads		0.5			
Exercises			**2**		
Periodic test					1
Self-taught contents (18 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 9.3 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes.*					

i. Describe manner to assess the module:

- To take the final exam, students have to ensure all two conditions:
  + Attend class 75% more than total lessons of the module.
  + $X \geq 4$
- The ways to calculate X : $X = X_2$

- X_2 is average mark of two tests at the middle of term (the mark of each test includes incentive mark of attitude at class, self-taught ability of students).

- Manner of final test (calculate Y):
  Written test in 90 miniutes.

- Mark for assessing module: **Z = 0,5X + 0,5Y**

In case students aren't enough conditions to take final test, please write **X = 0** and **Z = 0.**

In case **Y < 2**, **Z = 0**.

X, Y, Z are calculated by marking scheme of 10 and round up one numberal after comma.

After calculated by marking scheme of 10, Z is converted into marking scheme of 4 and letter-marking scheme A+, A, B+, B, C+, C, D+, D, F.

k. Textbooks:

[1]. Nguyen Ba Duong, *Strength of materials*, Construction Publishing House, 2002.

l. Reference materials:

[1]. Le Ngoc Hong, *Strength of materials*, Science and Technique Publishing House, 1998

[2]. Pham Ngoc Khanh, *Strength of materials*, Construction Publishing House; 2002

[3]. Bui Trong Luu, Nguyen Van Vuong, *Strength of materials exercises*, Education Publishing House, 1999.

[4]. I.N. Miroliubop, XA. Engaluutrep, N.D. Xerghiepxki, Ph. D Almametop, N.A Kuristrin, KG Xmironop - Vaxiliep, L.V iasina, *Strength of materials exercises*, Construction Publishing House; 2002.

m. Approved day: 30/5/2015
n. Approval level:

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CHAPTER 6: COMBINED STRESSES

6.1. Concept - Principle of superposition

6.1.1. Concept

In previous chapters, we researched the simply load-resistant manners of bars, including axial tension or compression, pure torsion, planely horizontal bending. In this chapter, we will research complicatedly load-resistant cases which are combined by the simply load-resistant manners as shown above. In case of complicatedly load-resistant bars, on their cross-sections, many different components of internal forces will appear. Complication is shown by the number of internal forces appearing on cross-section. We will research from less complicated case to general case.

6.1.2. Principle of superposition

We use Principle of superposition to research the complicatedly load-resistant cases. Its content is expressed below:

When we research the bar subjected to action of many loads causing many types of internal forces on cross-sections, stress and displacement at a point will equal the sum of stress and displacement caused by each separate component of loads.

To use this principle, problems have to satisfy the following conditions:

- Materials work in elastic region and relationship between stress and deformation is linear.
- Deformation of bar is small and displacement of points on which loads are put is insignificant.
- When we consider the problems of complicated load-resistance, because the influence of shear force on the strength of bar is insignificant, we can ignore it.

6.2. Oblique bending (Bending in two directions)

6.2.1. Concept

A bar is called oblique bending if on its each cross-section, there are two internal forces being bending moment M_x and M_y in two centroidally principal planes of inertia of the bar.

We can combine two vectors $\vec{M}_x$ and $\vec{M}_y$ to form a totalvector $\vec{M}_u$:

$$\vec{M}_u = \vec{M}_x + \vec{M}_y$$

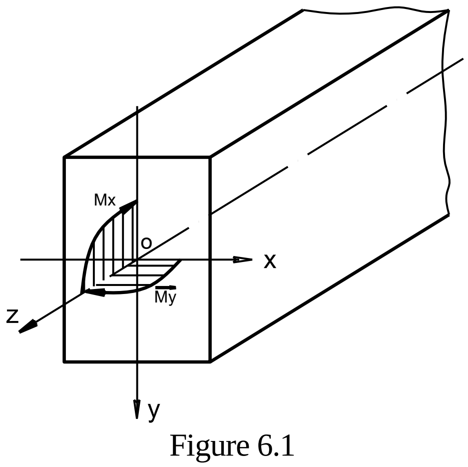


Figure 6.1

Therefore, we have an other concept: a bar is subjected to oblique bending if on each its cross-section, there is one bending moment M_u which is not in centroidally principal planes of inertia. The plane containing bending moment M_u is called loading plane. In the figure 6.2, loading plane is the plane π. Line of intersection between loading plane and cross-section is loading line. We realise that loading line goes through the centroid of cross-section and does not concide with centroidally principal axes of inertia.

Call α the angle formed by loading line and centroidally principal axis of inertia Ox, $\alpha$is considered to be positive if it rotates clockwise from axis x to loading line. (figure 6.2)

According to the figure, we have:

$M_x = M_u \sin\alpha$ (a)

$M_y = M_u \cos\alpha$

$$tg\alpha = \frac{M_x}{M_y}$$

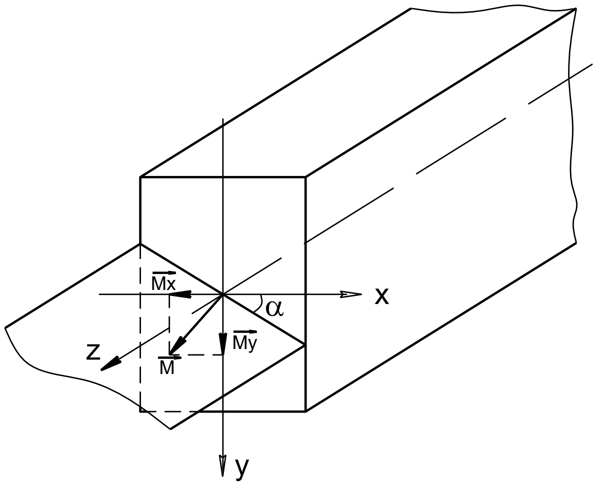


Figure 6.2

6.2.2. Stress on cross-section

Use Principle of superposition, stress at a point having co-ordinates (x, y) will equal the sum of normal stresses caused by each bending moment:

$$\sigma_z = \sigma_z^{M_x} + \sigma_z^{M_y} \quad \text{(b)}$$

However $\sigma_z^{M_x} = \dfrac{M_x}{J_x} y$ (c)

Similarly $\sigma_z^{M_y} = \dfrac{M_y}{J_y} x$ (d)

Hence $\sigma_z = \dfrac{M_x}{J_x} y + \dfrac{M_y}{J_y} x$ (6-1)

The sign of each term in (6-1) depends on the sign of M_x, M_y, x and y.

To avoid mistaken about sign, we can use the following formula:

$$\sigma_z = \pm\frac{|M_x|}{J_x}|y| \pm \frac{|M_y|}{J_y}|x| \quad \text{(6-2)}$$

In this formula, M_x, M_y, x, y are in absolute values and the signs are considered to be positive or negative in front of each term. This depends on the action of M_x and M_y causing tension or compression at researched point.

6.2.3. Neutral axis

Neutral axis is the line consisting of all the points on cross-sections which have normal stress equaling zero. Hence, equation of neutral axis is inferred from the equation $\sigma_z = 0$ as below:

$$y = -\frac{M_y}{M_x} . \frac{J_x}{J_y} . x \quad \text{(6-3)}$$

Therefore, neutral axis is a line going through the centroid of cross-section.

If we call β the angle formed by neutral axis and axis x: $tg\beta = -\frac{M_y}{M_x}.\frac{J_x}{J_y} = -\frac{1}{tg\alpha}.\frac{J_x}{J_y}$ (6-4)

Hence, we have some comments about neutral axis.

- Loading line and neutral axis are not in the same quadrant of cross-section.
- Neutral axis and loading line are not perpendicular each other.

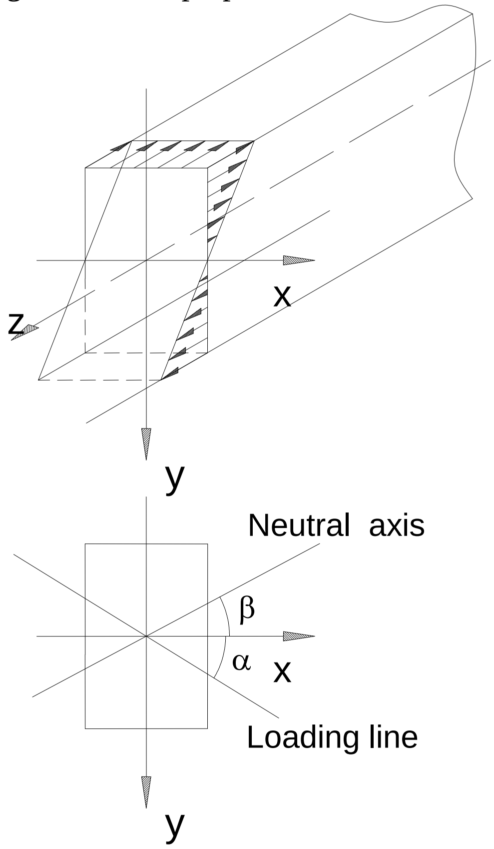


Figure 6.3

6.2.4. Normal stress diagram on cross-section

To draw normal stress diagram on cross-section, we have some following comments:

- All the points which are in the same line parallel to neutral axis have the same values of normal stress.

We can prove the above comment as shown below:

Assume that we have two points (1) and (2) which are in the same line parallel to neutral axis and have the co-ordinates: $1(x_1, y_1)$, $2(x_2,y_2)$.

Because the line 1-2 is parallel to neutral axis, its equation is:

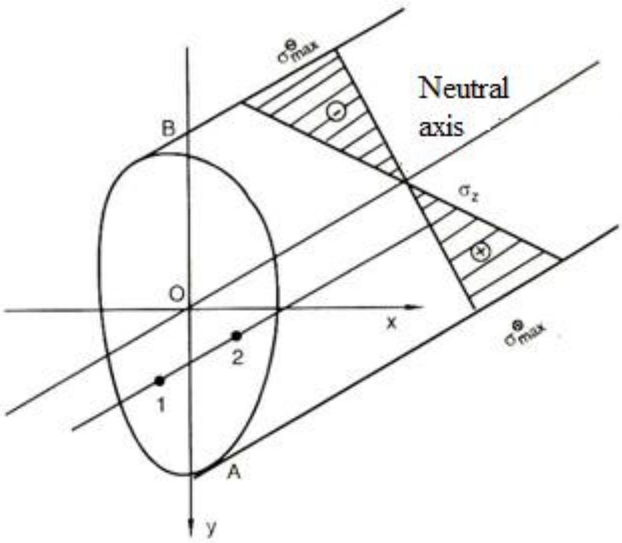


Figure 6.4

$$\frac{M_x}{J_x}y + \frac{M_y}{J_y}x + C = 0 \tag{e}$$

Here, C is a determined constant.
Subsitute the co-ordinates of the point (1) and (2) in the equation (e) and turn the term C into the right of equal sign, we get:

$$\begin{cases} \sigma_z^{(1)} = \dfrac{M_x}{J_x}y_1 + \dfrac{M_y}{J_y}x_1 = -C \\ \sigma_z^{(2)} = \dfrac{M_x}{J_x}y_2 + \dfrac{M_y}{J_y}x_2 = -C \end{cases} \tag{f}$$

Hence, stresses at two points (1) and (2) are equal.
- The law of the change of normal stress over distance of neutral axis is linear.

Thanks to two comments, we can draw normal stress diagram through the following procedure:
- Determine the position of neutral axis and stretch across cross-section.
- Draw a line perpendicular to neutral axis to be the directrix and determine the limit of cross-section.
- Determine two points:
+ Point 1 is the intersection between the directrix and the neutral axis.
+ Point 2 is the point expressing stress at an arbitrary point: $\sigma_z^{(2)} = \dfrac{M_x}{J_x}y^{(2)} + \dfrac{M_y}{J_y}x^{(2)}$
- Join two points, mark and rule diagram.

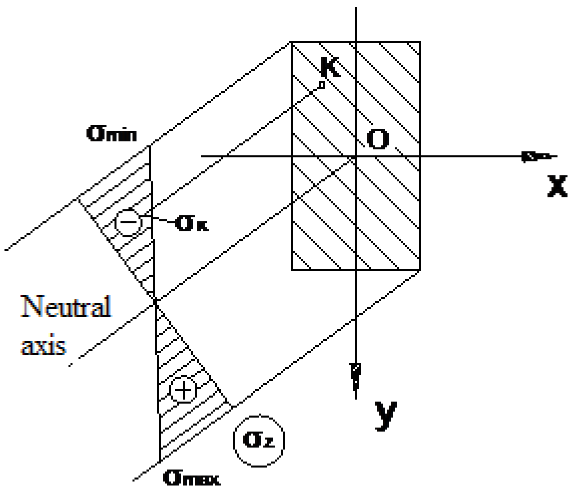


Figure 6.5

The diagram is shown as in the figure 6.4. Thanks to the diagram, the points having the maximum normal stresses are the furthest from neutral axis to two sides of tension and compression.

$$\begin{cases} \sigma_{\max}^{k} = \dfrac{M_x}{J_x} y_A + \dfrac{M_y}{J_y} x_A \\ \sigma_{\max}^{n} = \dfrac{M_x}{J_x} y_B + \dfrac{M_y}{J_y} x_B \end{cases} \qquad (6\text{-}5)$$

In case of the bars having rectangular section, I-section, [- section, the points which are the furthest from neutral axis are always in the corner of section and have the maximum co-ordinates (x_{max}, y_{max}). Therefore, the maximum normal stress is:

$$\begin{cases} \sigma_{\max}^{k} = +\dfrac{|M_x|}{W_x} + \dfrac{|M_y|}{W_y} \\ \sigma_{\max}^{n} = -\dfrac{|M_x|}{W_x} - \dfrac{|M_y|}{W_y} \end{cases} \qquad (6\text{-}6)$$

The normal stress diagram is shown as in the figure 6.5.

6.2.5.The condition of strength and three basic problems

a. The condition of strength

In case of the bar subjected to oblique bending, dangerous points are the furthest from neutral axis at the dangerous cross-sections. The stress state of dangerous points is single stress state. Hence, the condition of strength is:

- Brittle materials:

$$\begin{cases} \sigma_{z\max}^{k} \le [\sigma]_k \\ |\sigma_{z\max}^{n}| \le [\sigma]_n \end{cases} \qquad (6\text{-}7)$$

- Ductile materials:

$\max|\sigma_z| \le [\sigma]$ in which $\max|\sigma_z| = \max(\sigma_{z\max}^{k}, \sigma_{z\max}^{n})$ (6-8)

And $\sigma_{z\max}^{k}, \sigma_{z\max}^{n}$ are determined by the formula (6.5) or (6.6), which depends on the shape of cross-sections.

According to the condition of strength (6-7) and (6-8), we have:

It is noted that in case of the problem of determining the dimensions of cross-section, we have to use the gradually correct method. For example, in case of the problem which beam is made from ductile material and cross-section is symmetric, the condition of strength will be:

- $\frac{|M_x|}{W_x} + \frac{|M_y|}{W_y} \le [\sigma]$

It is evident that this inequality has two unknowns W_x và W_y. To be comfortable, we can write in the following form:

- $\frac{1}{W_x}\left[|M_x| + \frac{W_x}{W_y}|M_y|\right] \le [\sigma]$

We can solve the problem of determining the dimensions of cross-section as below:

Select the ratio $\frac{W_x}{W_y}$, subsitute it in the condition of strength, we can infer W_x

Through W_x, we can choose the dimensions or sign number of cross-section.

Thanks to the determined cross-section, check the condition of strength again and try beam to choose the minimum section satisfying the condition of strength.

We choose the ratio $\frac{W_x}{W_y}$ in accordance with the shape of cross-section.

- The rectangular section:

$$\frac{W_x}{W_y} = \frac{h}{b}$$

- The I-section:

$$\frac{W_x}{W_y} = 8 \div 10$$

- The [-section:

$$\frac{W_x}{W_y} = 5 \div 7$$

b. Three basic problems

According to the condition of strength (6-7) and (6-8), we also have three basic problems, including the test problem, the problem of determining allowable load and the problem of determining the dimensions or sign number of cross-section. The content and solution of these problems are similar to the basic problems in the previous chapters.

Example 1: Check the strength of the beam subjected to oblique bending as shown in the figure 6.6. Know that q = 6kN/m; l = 4m; angleφ = 30^0, [σ] = 160MN/m^2; E = 2.10^5 MN/m^2, $IN^0$20-section.

Solution:

- Analyse q into two components $\vec{q}_x$ và $\vec{q}_y$ with $q_x = q.\sin\varphi$ and $q_y = q.\cos\varphi$.

- Draw diagrams M_x and M_y as in the figure.

- Through the diagram, we realise that dangerous section is in the middle of the beam and has $|M_x|_{\max} = \frac{q_y.l^2}{8}; |M_y|_{\max} = \frac{q_x.l^2}{8}$.

The condition of strength of ductile material:

$$\max\sigma_z^{\max} = \frac{|M_x|_{\max}}{W_x} + \frac{|M_y|_{\max}}{W_y} \le [\sigma]$$

Consult the index:IN^o20 has W_x =152 cm^3 ; W_y = 20,5 cm^3.

Max $|\sigma_z|$ = $\frac{q_y.l^2}{8Wx} + \frac{q_x.l^2}{8Wy} \le [\sigma]$

Substitute the values, we have Max $|\sigma_z|$ = 36,45 kN/cm^2.

Compare and realise that Max $|\sigma_z|$ =36,45 kN/cm^2> [σ] =16 KN/cm^2.

Hence, the beam does not satisfy the condition of strength.

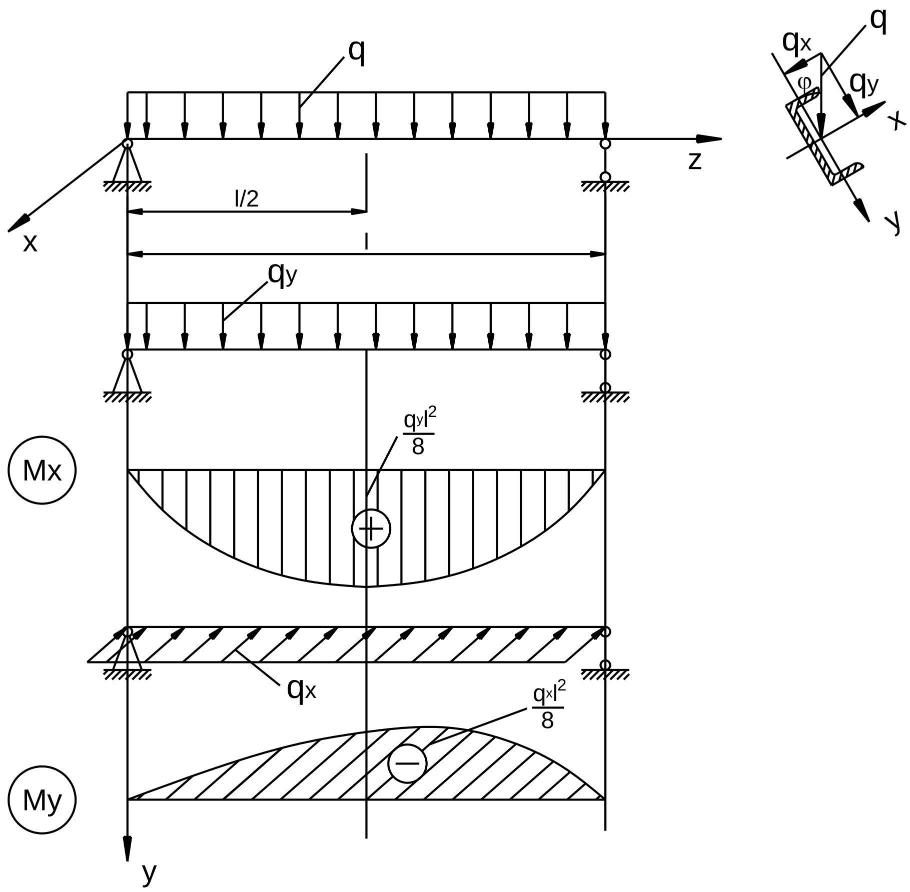


Figure 6.6

<u>Example 2</u>: The beam is shown in the figure 6.6. Assume that we do not know the magnitude of load q. Determine the allowable value of load q thanks to the condition of strength. The other values are given as in the example 1.

<u>Solution</u>:

- Analyse q into two components $\vec{q}_x$ và $\vec{q}_y$ with $q_x = q.\sin\varphi$ and $q_y = q.\cos\varphi$.
- Draw diagrams M_x and M_y as in the figure.
- Through the diagram, we realise that dangerous section is in the middle of the beam and has $\left|M_x\right|_{\max} = \dfrac{q_y.l^2}{8}; \left|M_y\right|_{\max} = \dfrac{q_x.l^2}{8}$.

The condition of strength of ductile material:

$$\max\sigma_z = \frac{\left|M_x\right|_{\max}}{\mathrm{W_x}} + \frac{\left|M_y\right|_{\max}}{\mathrm{W_y}} \le [\sigma]$$

Consult the index: IN°20 has $W_x = 152\ cm^3$; $W_y = 20,5\ cm^3$

$$\text{Max } |\sigma_z| = \frac{q_y.l^2}{8\text{Wx}} + \frac{q_x.l^2}{8\text{Wy}} = q\left(\frac{\sqrt{3}.l^2}{16\text{Wx}} + \frac{l^2}{16\text{Wy}}\right)$$

According to the condition of strength: $\max\sigma_z \le [\sigma]$, we infer:

$$[q] = \frac{[\sigma]}{\left(\dfrac{\sqrt{3}.l^2}{16\text{Wx}} + \dfrac{l^2}{16\text{Wy}}\right)}$$

Substitute the values, we have

$$[q] = \frac{16}{\dfrac{\sqrt{3}.(400)^2}{16.152} + \dfrac{(400)^2}{20{,}5.16}} = 265{,}8.10^{-4}\,\mathrm{kN/cm}$$

Example 3: A I-beam is subjected to oblique bending as in the figure 6.7. Determine the sign number of section thanks to the condition of strength. Know that: P = 10kN; l = 4m; $\varphi = 30^0$; $[\sigma] = 16\mathrm{kN/cm}^2$.

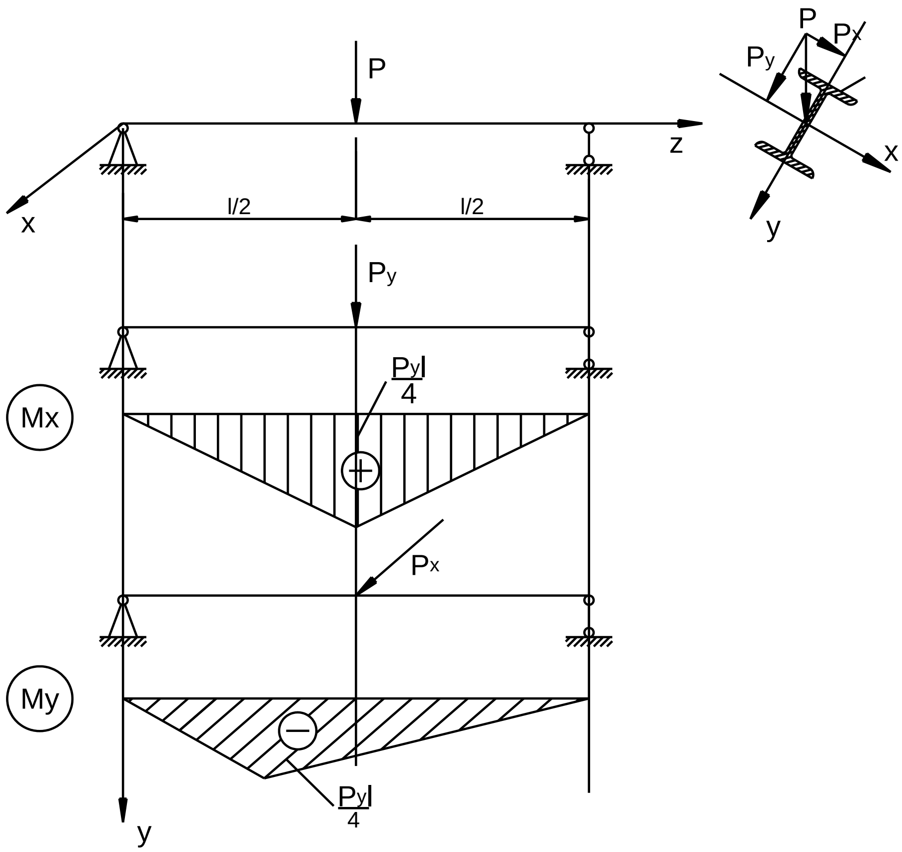


Figure 6.7

Solution:

- Analyse P into two components P_x and $P_y$with $P_x = P.\sin\varphi$; $P_y = P.\cos\varphi$.
- Draw diagrams M_x and M_y as in the figure.
- Through the diagram, we realise that dangerous section is in the middle of the beam

and has $|M_x|_{\max} = \dfrac{P_y.l}{4}; |M_y|_{\max} = \dfrac{P_x.l}{4}$

The condition of strength of ductile material:

$$\sigma_z^{\max} = \frac{1}{W_x}\left[|M_x|_{\max} + \frac{W_x}{W_y}|M_y|_{\max}\right] \le [\sigma]$$

In case of I-section, we choose $\dfrac{Wx}{Wy}$ = 9. Therefore, we have:

$$W_x \ge \frac{\left[|M_x|_{\max} + \dfrac{W_x}{W_y}|M_y|_{\max}\right]}{[\sigma]} = 335{,}4cm^3$$

Consult the index and choose IN°27-steel which has Wx = 371 cm^3, $W_y = 41{,}5\ cm^3$.

Check the condition of strength of the beam when it is made from IN°27-steel, we get:

$$\max \sigma_z = \frac{|M_x|_{\max}}{W_x} + \frac{|M_y|_{\max}}{W_y} = 14{,}4kN/cm^2 \le [\sigma] = 16kN/cm^2$$

We realise that max$\sigma_z$is much smaller than [σ].

We choose steel which has smaller sign number. It is N^{o}24a which has W_x = 317 cm^3, W_y = 41,6 cm^3. Check the condition of strength again.

We realise that max | σ_z | =14,7 kN/cm^2< [σ]

We continue to choose smaller sign number. It is N^{o}24 which has W_x = 289 cm^3, W_y = 34,5 cm^3.

We realise that max | σ_z | =17,5 kN/cm^2> [σ] =16 KN/cm^2 with a percentage of 8,6 %. This does not satisfy the condition of strength.

Hence, the sign number of cross-section is IN^{0}24a.

6.2.6. Deflection in oblique bending

We call the deflection of the cross-section of beam f. According to Principle of superposition, we have:

$$\vec{f} = \vec{f_x} + \vec{f_y}$$

In terms of magnitude:

$$f = \sqrt{f_x^2 + f_y^2}$$

In which: $f_x$is deflection which follows the direction x and is caused by M_y; f_y is deflection which follows the direction y and is caused by M_x. We can independently determine them by the methods researched in the previous chapters.

The condition of stiffness: $f_{\max} \le [f]$

Or: $\frac{f_{\max}}{l} \le \left[\frac{f}{l}\right]$

6.3. Bending and tension or compression

6.3.1. Concept

A bar is called simultaneous bending and tension (compression) if on its each cross-section, there are both bending moment M_u and longitudinal force N_z.

In general case: $\vec{M}_u = \vec{M}_x + \vec{M}_y$

At that moment, internal forces on cross-sections consist of three components: M_x, M_y, N_z.

In particular case, there are only N_z, $M_x$or N_z, M_y.

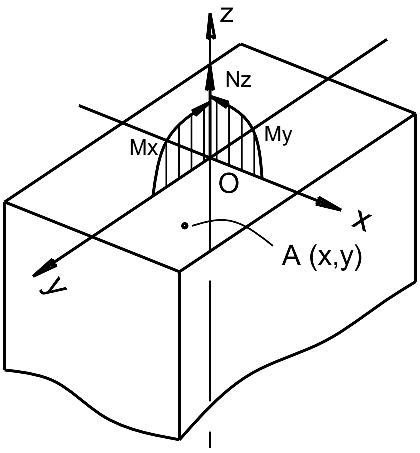


Figure 6.8

6.3.2. Stress on cross-section

According to Principle of superposition, stress at a point on cross-section equals the sum of stresses caused by three internal forces N_z, M_x, M_y:

$$\sigma_z = \frac{N_z}{F} + \frac{M_x}{J_x} y + \frac{M_y}{J_y} x \qquad (6\text{-}9)$$

To aviod mistake about sign, we can use the technical formual below:

$$\sigma_z = \pm\frac{|N_z|}{F} \pm \frac{|M_x|}{J_x}|y| \pm \frac{|M_y|}{J_y}|x| \qquad (6\text{-}10)$$

6.3.3. Neutral axis and normal stress diagram on cross-section

According to the concept of neutral axis, we have its equation:

$$\frac{N_z}{F} + \frac{M_x}{J_x} y + \frac{M_y}{J_y} x = 0$$

or $y = -\frac{M_y}{M_x} . \frac{J_x}{J_y} x - \frac{N_z J_x}{M_x F}$ (6-11)

According to the equation (6-11), we find that neutral axis does not go through the centroid of section.

To draw normal stress diagram on cross-section, we also have two comments as the previous part:

- The normal stresses of the points having the same distance of neutral axis are equal.
- The law of the change of normal stress over distance of neutral axis is linear.

Normal stress diagram is drawn as in the figure (6.10).

Because the free term $\frac{Nz}{F}$ has arbitrary magnitude, it is likely that neutral axis will pass beyond the area of cross-section. At that moment, normal stress diagram has only either tensile region or compressive one as shown in the figure (6.11).

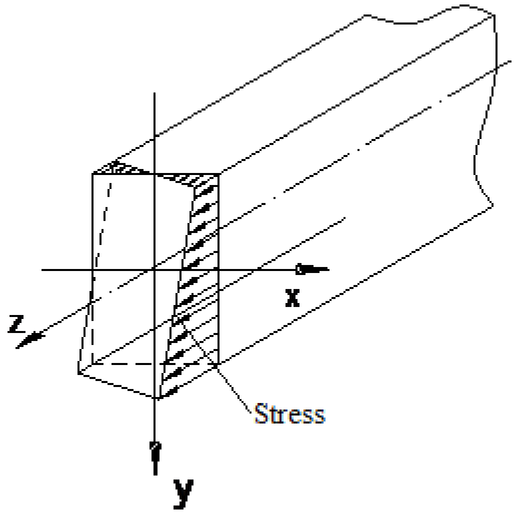


Figure 6.9

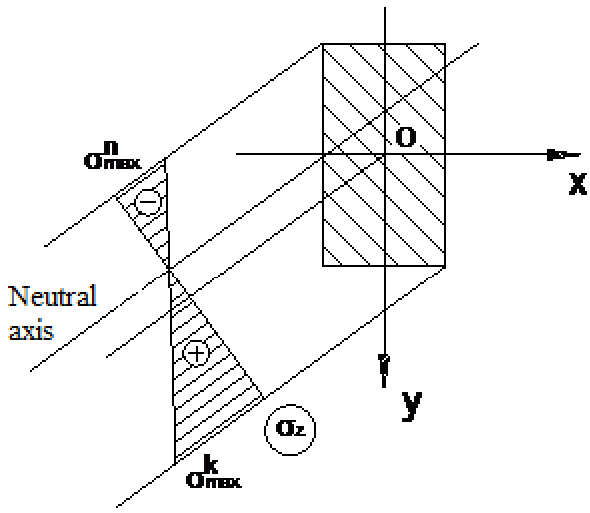


Figure 6.10

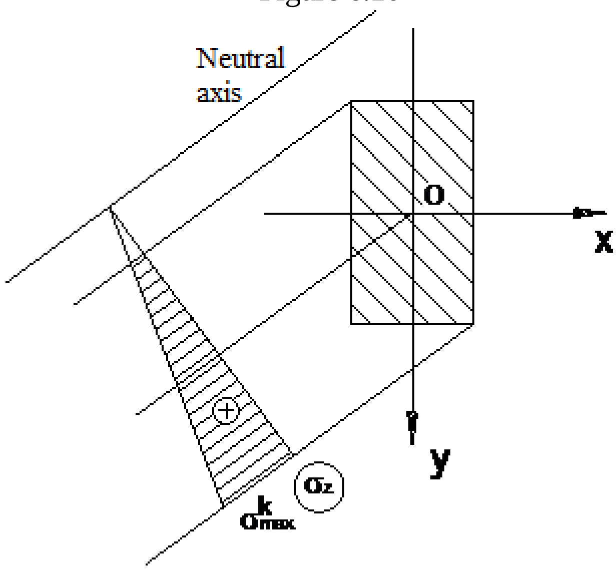


Figure 6.11

The maximum normal stress is at the points which are the furthest from neutral axis. The maginute of this maximum normal stress can be determined by the formula below:

$$\begin{cases} \sigma_{\max}^{k} = \dfrac{N_z}{F} + \dfrac{M_x}{J_x} y_A + \dfrac{M_y}{J_y} x_A \\ \sigma_{\max}^{n} = \dfrac{N_z}{F} + \dfrac{M_x}{J_x} y_B + \dfrac{M_y}{J_y} x_B \end{cases} \qquad (6\text{-}12)$$

6.3.4. The condition of strength

$$\begin{cases} \sigma_{\max}^{k} \le [\sigma]_k \\ \sigma_{\max}^{n} \le [\sigma]_n \end{cases} \qquad (6\text{-}13)$$

Thanks to this condition of strength, we also have three basic problems as in the previous part.

6.3.5. Eccentric loading

a. Concept

A bar is subjected to eccentric tension (compression) if external forces acting on it can gather up into the forces which are parallel to the axis of bar and does not concide with the axis of bar.

Assume that we have a force-setting point K (x, y) at a distance e of centroid O. The distance e is called eccentric distance. Consider internal forces on cross-section.

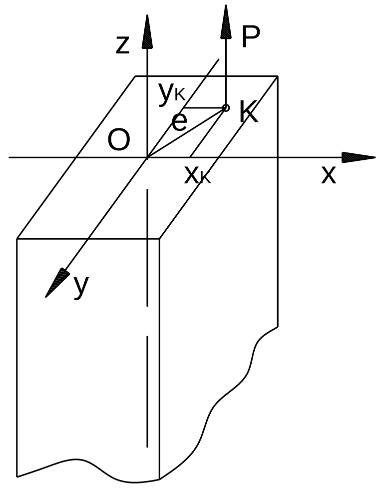


Figure 6.12

$N_z = P$

$M_u = P.e$

Analyse M_u into:

$M_x = P.y_k$

$M_y = P.x_k$

Hence, the bar subjected to eccentric tension (compression) will suffer from tensile (compressive) and bending deformation simultaneously.

b. Stress on cross-section

Thanks to the formula (6-9), we have:

$$\sigma_z = \frac{P}{F}\left(1 + \frac{y_k}{i_x^2} y + \frac{x_k}{i_y^2} x\right) \tag{6-14}$$

In which:

$$i_x^2 = \frac{J_x}{F} ;\; i_y^2 = \frac{J_y}{F}$$

c. Neutral axis

According to the concept of neutral axis, we find that its equation is :

$$1 + \frac{y_k}{i_x^2} y + \frac{x_k}{i_y^2} x = 0 \tag{6-15}$$

If we replace: $a = -\frac{i_y^2}{x_k}$ (6-16)

$$b = -\frac{i_x^2}{y_k}$$

We get the equation of neutral axis:

$$\frac{x}{a} + \frac{y}{b} = 1 \tag{6-17}$$

Hence, we find some following properties of neutral axis:

- Neutral axis is the line which does not go through the centroid of cross-section and cuts axis x at a and cuts axis y at b.
- Thanks to (6-17), we realise that a and b are always opposite in sign with x_k and y_k, so neutral axis never goes through the quadrant containing force-setting point.
- If force-setting point is on an axis, neutral axis will be parallel to another axis.
- The position of neutral axis only depends on the position of force-setting point and the shape and dimension of the cross-section of bar. It does not depend on the magnitude of force P.

- When force-setting point moves on a line which does not going through the origin O (the centroid of cross-section), neutral axis will correspondingly rotate around an arbitrarily fixed point.
- If force-setting point moves on the line going through the origin O, neutral axis will move in parallel to itself. If force-setting point moves near centroid, neutral axis will move far centroid. By constrast, if force-setting point moves far centroid, neutral axis will move near centroid.

Example 4: A steel beam is made from two [$N^0$12 - sections joined together and has load-resistant layout as in the figure 6.13a. Determine allowable load [q], know that: $[\sigma] = 16kN/cm^2$; l = 80cm.

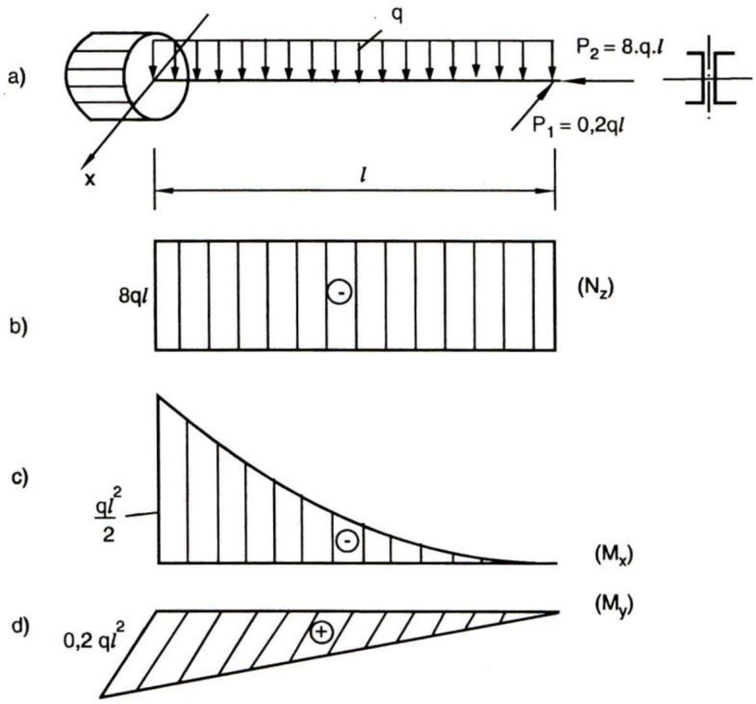


Figure 6.13

Solution: Longitudinal force diagram N_z, bending moment diagram M_x and M_y are shown as in the figure 6.13b, c, d. Dangerous section is at the cantilever and its internal forces are:

$$N_z = -8ql$$

$$M_{x\max} = -\frac{ql^2}{2}$$

$$M_{y\max} = 0{,}2ql^2$$

Consult the index of shaped-steel [$N^0$12, we get:
h = 12cm; b = 5,2cm; $J_{x1} = 304cm^4$; $J_{y1} = 31{,}2cm^4$; $z_0 = 1{,}54cm$; $F_1 = 13{,}3m^2$.
We determine flexure-resistant moments W_x and W_y:

$$J_x = 2J_{x1}; W_x = \frac{J_x}{\frac{h}{2}} = \frac{2.304}{6} = 101{,}3cm^3$$

$$J_y = 2\left(J_{y1} + z_0^2F_1\right); W_y = \frac{J_y}{b} = \frac{2\left[31{,}2 + (1{,}54)^2.13{,}3\right]}{5{,}2} = 24{,}1cm^3$$

We determine the maximum stresses as below:

$$\sigma^k_{\max} = \frac{N_z}{F} + \frac{|M_x|}{W_x} + \frac{|M_y|}{W_y}$$

$$\sigma^n_{\max} = \frac{N_z}{F} - \frac{|M_x|}{W_x} - \frac{|M_y|}{W_y}$$

Because $N_z < 0 \rightarrow |\sigma_z|_{max} = |\sigma^n_{z\max}|$

$$\sigma^n_{\max} = -q\left(\frac{8l}{2F_1} + \frac{l^2}{2W_x} + \frac{0,2l^2}{W_y}\right) = -107,06qkN/cm^2$$

Because the material of beam is ductile, the condition of strength will be: $|\sigma|_{\max} \leq [\sigma]$ or $107,06q \leq [\sigma]$

$$q \leq \frac{[\sigma]}{107,06} = \frac{16}{107,06} = 14,9.10^{-2}\,kN/cm$$

Hence $[q] = 14,10^{-2}$ kN/cm = 14,9kN/m.

Example 5: A wooden column is subjected to a compressive force set at point K (3,-6)cm. Ignore the gravity of coulumn. Check the strength of column. Know that: P = 30kN, $[\sigma]_k = 0,8kN/cm^2$ $[\sigma]_n = 1kN/cm^2$.

Solution: The properties of the cross-sections of column:

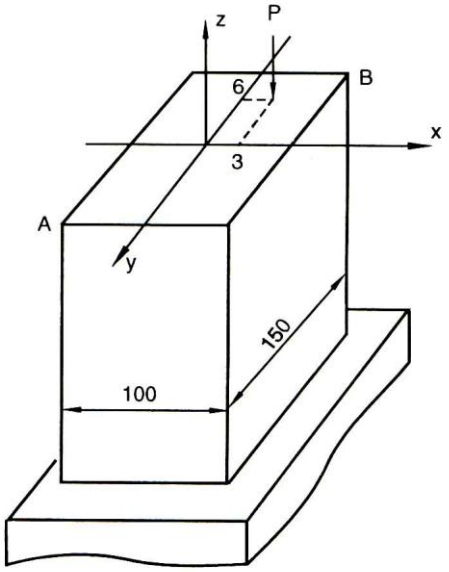


Figure 6.14

$F = 15 \times 10 = 150cm^2$

$$W_x = \frac{bh^2}{6} = \frac{10.15^2}{6} = 375cm^3$$

$$W_y = \frac{b^2h}{6} = \frac{10^2.15}{6} = 250cm^3$$

Internal forces on the cross-section of column:

$N_z = -P = -30kN$

$M_x = P.y_k = 30.6 = 180kNcm$

$M_y = P.x_k = -30.3 = -90kNcm$

$$\sigma_{\max}^{k} = \sigma_A = -\frac{|N_z|}{F} + \frac{|M_x|}{W_x} + \frac{|M_y|}{W_y} = 0,64 kN/cm^2$$

$$\sigma_{\max}^{n} = \sigma_B = -\frac{|N_z|}{F} - \frac{|M_x|}{W_x} - \frac{|M_y|}{W_y} = -1,04 kN/cm^2$$

Compare with the allowable stresses, we realize that:

$\left|\sigma_{\max}^{k}\right| < [\sigma]_k$

$\left|\sigma_{\max}^{k}\right| > [\sigma]_n$. However, the discrepancy is about 4%.

Therefore, the column has enough strength.

6.4. Simultaneous bending and torsion in round shaft

6.4.1. Concept

A bar is subjected to bending and torsion simultaneously when on its cross-sections, there are bending moment M_u and torque M_z.

If there are both bending moments M_x and M_y, we always have $\vec{M}_u = \vec{M}_x + \vec{M}_y$ and axes x, y, u, v…are centroidally principal axes of inertia. The flexure of round shaft is always single bending. It is not oblique bending.

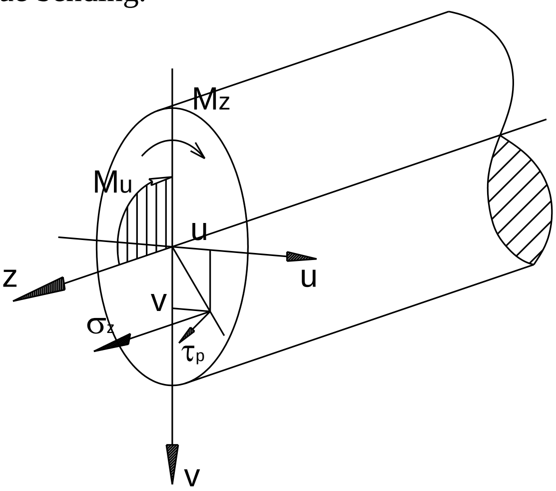


Figure 6.15

6.4.2. Stress on cross-section

In case of round shaft, flexure caused by M_u is pure bending because loading plane containing M_u is centroidally principal plane of inertia and loading line is a centroidally principal axis of inertia.

Because it is pure bending, neutral axis is perpendicular to loading line.

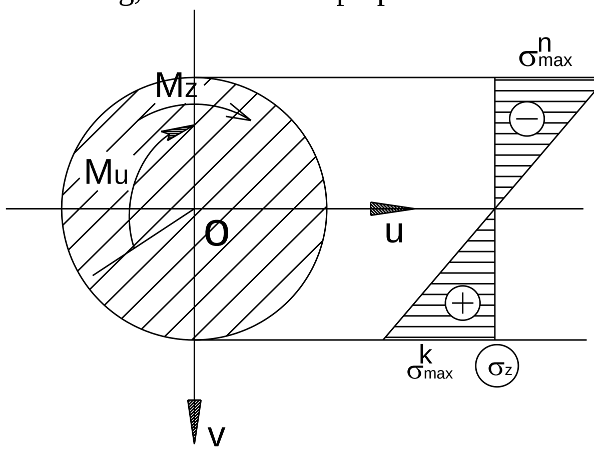


Figure 6.16

Normal stress at a point on cross-section will be:

$$\sigma_z = \frac{M_u}{J_u} v \tag{6-18}$$

Normal stress diagram on cross-section is shown as in the figure 6.16. The maximum normal stresses are at the points which are the furthest from neutral axis, such as point A and point B in the figure 6.17.Their magnitudes are:

$$\sigma_{max}^{(+)} = -\sigma_{max}^{(-)} = \frac{M_u}{W_u} \tag{6-19}$$

Because $M_u = \sqrt{M_x^2 + M_y^2}$ and in case of circular section $W_u = W_x = W_y$:

$$\sigma_{max}^{(+)} = -\sigma_{max}^{(-)} = \frac{\sqrt{M_x^2 + M_y^2}}{W_x} \tag{6-20}$$

Besides normal stress on cross-section, there is also shear stress caused by torque M_z:

$$\tau_\rho = \frac{M_z}{J_P}.\rho \tag{6-21}$$

The diagram of this stress is shown in the figure 6.16. The maximum shear stresses are at the points on the perimeter of section and their magnitudes are:

$$\tau_{max} = \frac{M_z}{W_\rho} = \frac{M_z}{2W_x} \tag{6-22}$$

6.4.3. The condition of strength

Thanks to normal stress diagram and shear stress diagram on the cross-section of the bar subjected to bending and torsion simultaneously, we realize that there are two dangerous points which are the points A and B because at these points, there are both maximum normal stress and maximum shear stress.

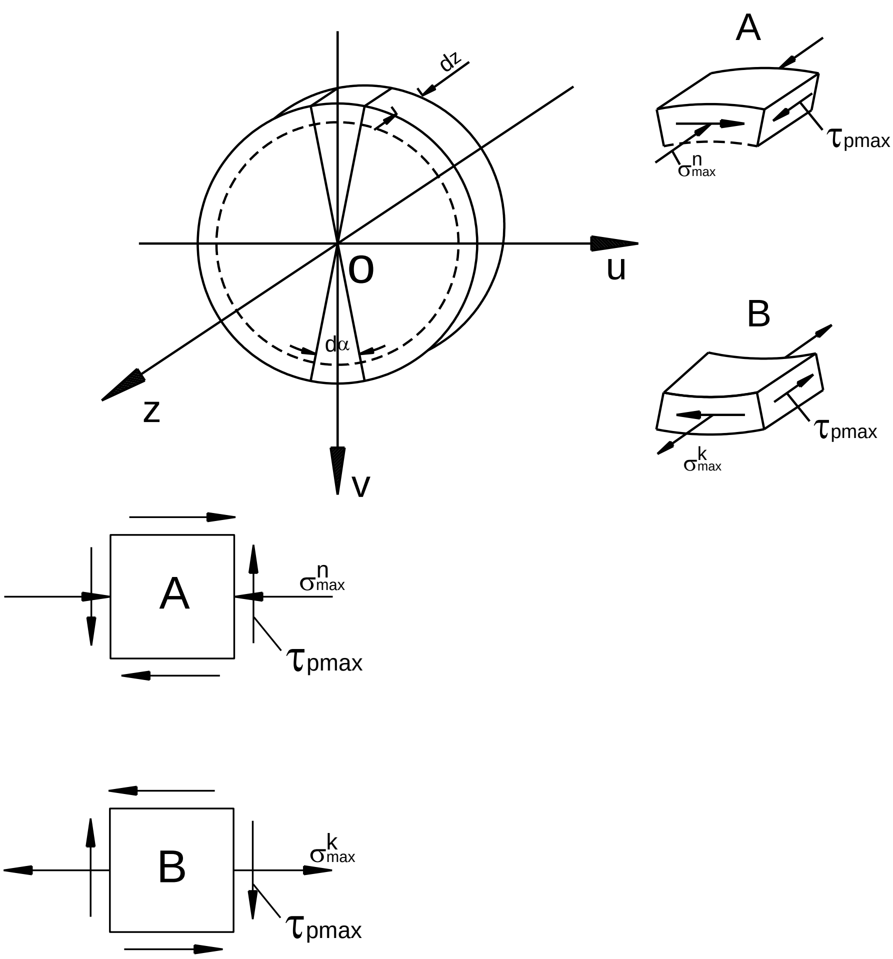


Figure 6.17

The stress state of these elements is single stress state. Therefore, the condition of strength has to conform to reliability theories.

According to the third reliability theory, we have:

$$\sigma_{t3} = \sqrt{\sigma_{max}^2 + 4\tau_{max}^2} \le [\sigma]$$

Substitute the expression σ_{max} and τ_{max} in the formula (6-20) và (6-22), we get:

$$\sigma_{t3} = \frac{1}{W_x}\sqrt{M_x^2 + M_y^2 + M_z^2} \le [\sigma] \qquad (6\text{-}23)$$

According to the forth reliability theory: $\sigma_{t4} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} \le [\sigma]$

Substitute the formula (6-20) and (6-22), we get:

$$\sigma_{t4} = \frac{1}{W_x}\sqrt{M_x^2 + M_y^2 + \frac{3}{4}M_z^2} \le [\sigma] \qquad (6\text{-}24)$$

According to the reliability theory Morh: $\sigma_{tMo} = \frac{1-\alpha}{2}\sigma_{max} + \frac{1+\alpha}{2}\sqrt{\sigma_{max}^2 + 4\tau_{max}^2} \le [\sigma]$

Substitute σ_{max} and τ_{max} in the formula (6-20) và (6-22), we get:

$$\sigma_{tMo} = \frac{1}{W_x}\left[\frac{1-\alpha}{2}\sqrt{M_x^2 + M_y^2} + \frac{1+\alpha}{2}\sqrt{M_x^2 + M_y^2 + M_z^2}\right] \le [\sigma]$$

$$\text{with}\ \alpha = \frac{[\sigma]_k}{[\sigma]_n} \qquad (6\text{-}25)$$

Thanks to the condition of strength above, we also have three basic problems. We will illustrate one of three basic problems as below:

Example 6: A shaft is subjected to loads as in the figure 6.18a. Check the strength of shaft thanks to the third reliability theory. Know that: the diameter of shaft d = 10cm, $[\sigma]$ = 16kN/cm^2. The other parameters are given as in the figure 6.18a.

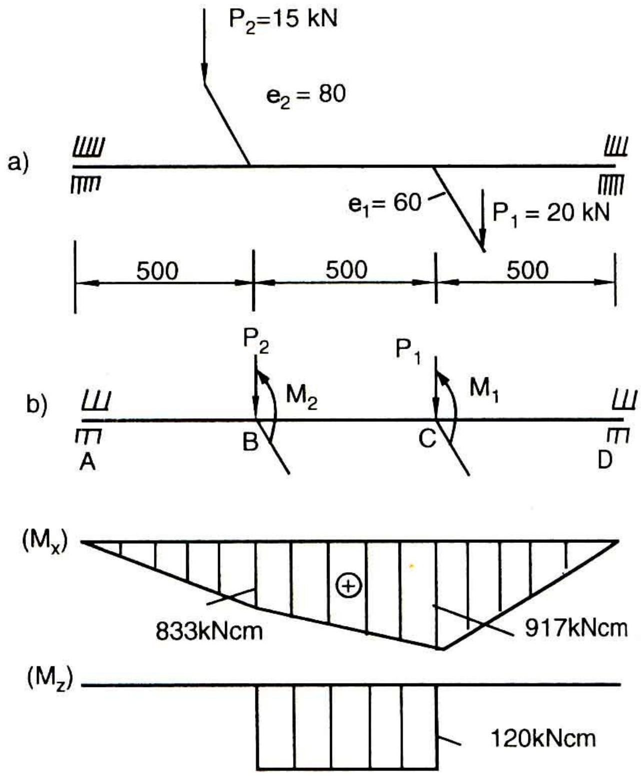


Figure 6.18

Solution: We can draw the load-resistant structure of shaft as in the figure 6.18b with P_1 = 20kN; P_2 = 15kN

$M_1 = P_1.e_1$ = 120kNcm

$M_2 = P_2.e_2$ = 120kNcm.

Bending moment diagram M_x and torque M_z are shown as in the figure 6.18c. According to the diagram, dangerous section is at C and its magnitudes are:

M_{xmax} = 917kNcm

M_{zmax} = 120kNcm

According to the third reliability theory (the formula (6.23)), we have:

$$\sigma_{t3} = \frac{1}{W_x}\sqrt{M^2_{x\max} + M^2_{z\max}} = \frac{32}{\pi.10^3}\sqrt{917^2 + 120^2} = 9,3kN/cm^2 < [\sigma] = 16kN/cm^2$$

Hence, the shaft has enough strength.

6.5. Round shaft is subjected to general loadings

6.5.1. Concept

Round shaft is subjected to general loadings when on its cross-sections, there are six internal forces: N_z, Q_x, Q_y, M_z, M_x, M_y. If we ignore shear force and combine $\vec{M}_u = \vec{M}_x + \vec{M}_y$, there are only three internal forces: N_z, M_u, M_z.

6.5.2. Stress on cross-section

- Normal stress:

$$\sigma_z = \frac{M_u}{J_u}v + \frac{N_z}{F} \tag{6-26}$$

It has the maximum value at the points which are the furthest from neutral axis to two sides:

$$\begin{aligned} \sigma^k_{\max} &= \frac{N_z}{F} + \frac{|M_u|}{W_u} \\ \sigma^n_{\max} &= \frac{N_z}{F} - \frac{|M_u|}{W_u} \end{aligned} \tag{6-27}$$

- Shear stress:

Besides normal stress, there are also shear stress τ_ρ and its maximum value is at the points on the perimeter of section:

$$\tau_{\max} = \frac{M_z}{W_p} \tag{6-28}$$

6.5.3. The condition of strength

The condition of strength of dangerous elements is also similar to those in item 6.4.3. Here, we do not mention any more. Thanks to this condition of strength, we also have three basic problems.

Example7: Determine the diameter of the gear shaft of a reducer thanks to the third reliability theory. The structure of shaft is in the figure 6.19a. The gear 1 has diameter D_1, at the joints with other gears, there are loads P_{t1} = 9700N; P_{r1} = 3530N. The gear 2 is a helical gear having diameter D_2 and loads acting on it are P_{t2} = 2700N; P_{r2} = 1000N; P_{d2} = 460N; D_2 = 299mm. The distances are a = 62mm; b = 72mm; c = 52mm. Know that $[\sigma]$ = 5000N/cm^2.

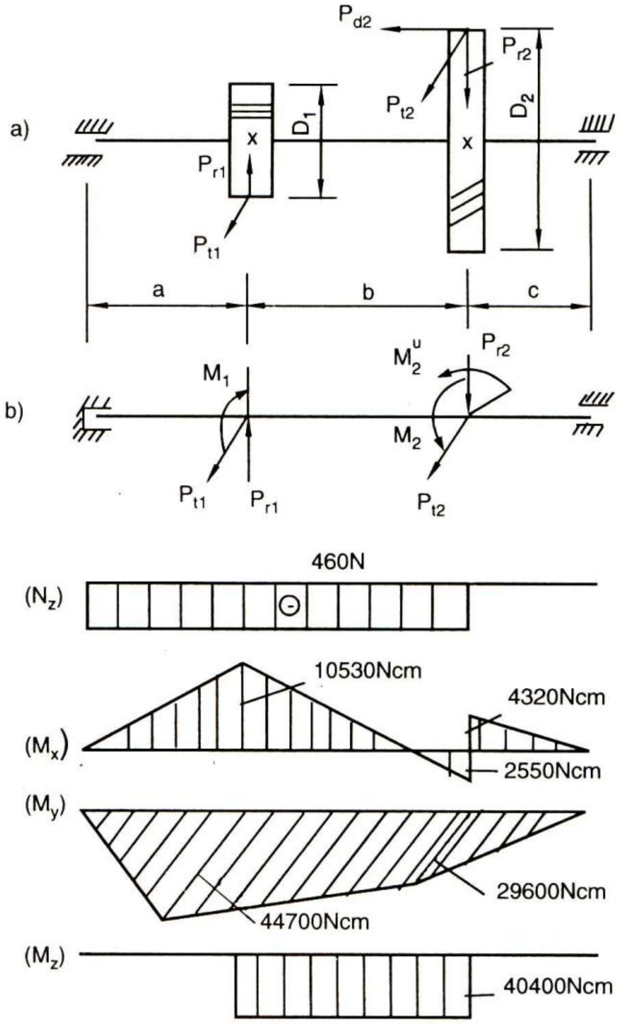


Figure 6.19

<u>Solution</u>: The load-resistant layout of shaft is in the figure 6.19b.

In this layout, the magnitudes of concentrated moments are:

$$M_1 = M_2 = P_{t2}.\frac{D_2}{2} = 2700.\frac{29,9}{2} \approx 40400 Ncm$$

$$M_z^u = P_{d2}.\frac{D_2}{2} = 460.\frac{29,9}{2} = 6870 Ncm$$

Internal force diagrams are shown as in the figure 6.19. According to the diagrams, we realize that dangerous sections are at the position of the gear 1 which has:

N_z = -460N
M_{xmax} = 10530Ncm
M_{ymax} = 44700Ncm
M_z = 40400Ncm

Preliminarily choose the diameter of shaft thanks to bending moment, torque and the third reliability theory, we get:

$$\sigma_{t3} = \frac{32}{\pi d^3}\sqrt{M_{x\max}^2 + M_{y\max}^2 + M_z^2} \le [\sigma]$$

$$d \ge \sqrt[3]{\frac{32\sqrt{M_{x\max}^2 + M_{y\max}^2 + M_z^2}}{\pi[\sigma]}} = \sqrt[3]{\frac{32\sqrt{10530^2 + 44700^2 + 40400^2}}{\pi.5000}} = 5cm$$

Check the strength of element in case of considering longitudinal force:

$$\sigma_{\max}^n = -\frac{M_{x\max}^2 + M_{y\max}^2}{W_x} + \frac{N_z}{F} = -3767 N/cm^2$$

$$\tau_{\max} = \frac{M_z}{W_p} = \frac{40400.16}{\pi d^3} = \frac{40400.16}{\pi.125} = 1616 N/cm^2$$

$$\sigma_{t3} = \sqrt{\sigma_{z\max}^2 + 4\tau_{zy\max}^2} = \sqrt{3767^2 + 4(1616)^2}$$

$$\sigma_{t3} = 4963,4 N/cm^2 < [\sigma]$$

Hence, the diameter of shaft will be d = 5cm.

Theoretical questions

1. Define the bar subjected to general loadings (oblique bending, bending and tension (compression), bending and torsion, general loadings). Raise practical examples about the bar subjected to complicated loadings.
2. Write formula to calculate stress on cross-section in case of the bar subjected to complicated loadings. Obviously explain paramaters in the formula.
3. Raise the method to determine the points having maximum stress on cross-section and the value of maximum stress in case of the bar subjected to complicated loadings.
4. Raise the condition of strength and the way to solve three basic problems in case of the bar subjected to complicated loadings.
5. Which kinds of bar are not subjected to oblique bending. Explain reasons.
6. What is the properties of the problem of eccentric tension (compression)? Why can we say that neutral axis on cross-section of the bar subjected to eccentric tension (compression) only depends on the position of load-setting point and it does not depends on the magnitude of load?
7.

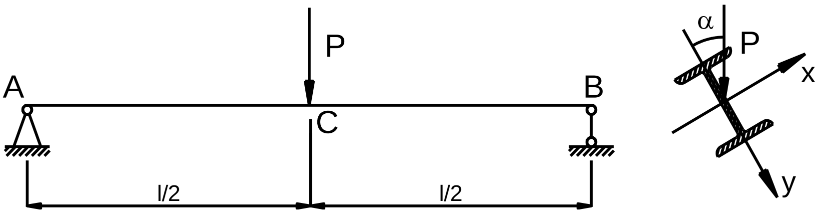
Numerical problems

<u>Exercise 1</u>:

A steel beam is made from IN^{o}24.

Know that: l = 4 m; P = 10 kN; α = 30^o, $[\sigma]$ = 160 MN/m^2, E = 2.10^{4}kN/cm^2.

Check the strength of beam, determine the maximum total deflection.

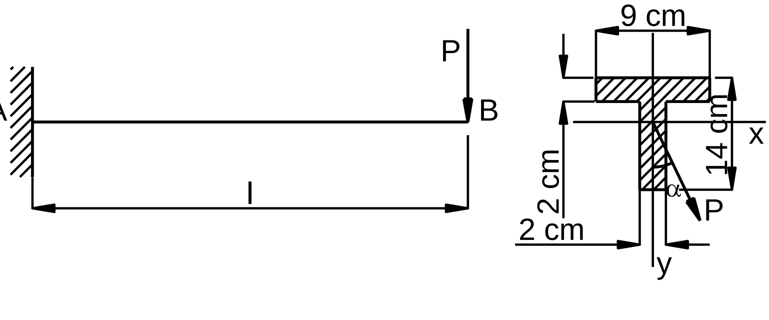


<u>Exercise 2</u>:

Know that: l = 1 m, α = 30^o; P = 2 kN, $[\sigma]_k$ = 6 KN/cm^2, $[\sigma]_n$ = 18 kN/cm^2, $\left[\frac{f}{l}\right] = \frac{1}{500}$; E = 1,2.10^5 MN/m^2.

Draw stress diagram at the dangerous section.

Check strength and stiffness.

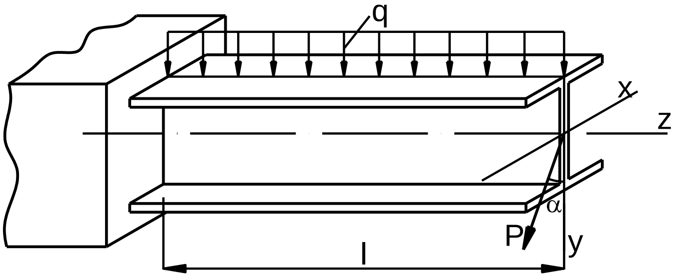


Determine allowable load [P].

Exercise 3:

Know that: $[\sigma] = 160\ MN/m^2$; $l = 3$ m, $\alpha = 30^o$; $E = 2.10^5\ MN/m^2$, $P = 10$ kN; $q = 10$ kN/m.

- Choose the sign number of I-steel.
- Thanks to the selected sign number, determine deflection at the free end.
- Draw normal stress diagram (σ_z) at dangerous section.



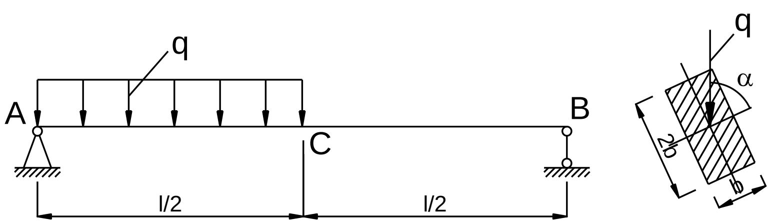
Exercise 4:

Check the strength of beam.

Know that: $q = 20$ kKN/m; $l = 4$ m; $b = 4$ cm; $[\sigma] = 120\ MN/m^2$; $E = 2.10^4 kN/cm^2$, $\alpha = 60^o$.

Determine allowable load.

Thanks to the allowable load, determine deflection at the point C.



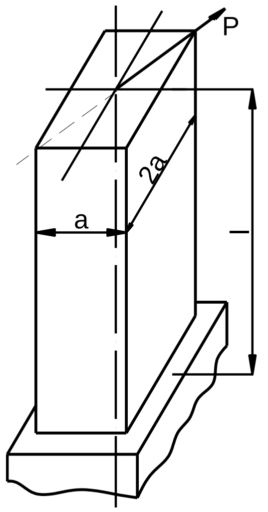
Exercise 5:

The concrete column has specific gravity γ. It is subjected to load P which is put to concide with the diagonal of section.

Know that: $a = 10$ cm; $\gamma = 25\ kN/m^3$, $[\sigma]_k = 70\ N/mm^2$, $[\sigma]_n = 800\ N/mm^2$, $l = 2$ m.

Determine P to ensure that the column does not has tensile stress.

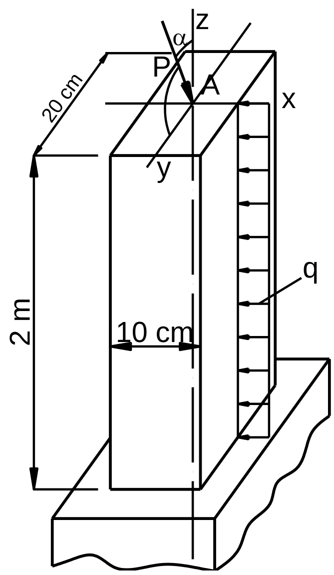
Thanks to the determined load P, check the strength of column.



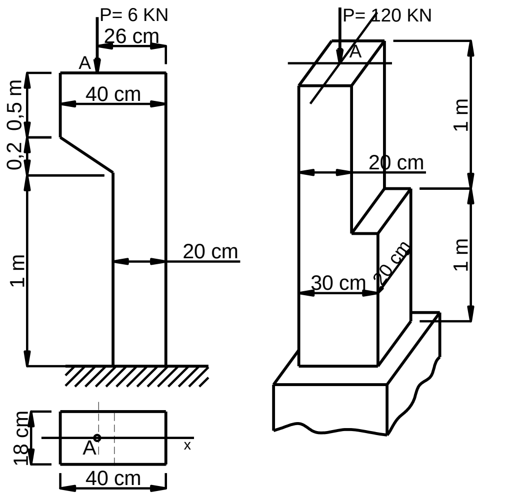
Exercise 6:

Check the strength of concrete column subjected to compressive load in two cases (a) and (b).

Know that: the load P is put at the point A, $[\sigma]_k = 70\ N/cm^2$, $[\sigma]_n = 700\ N/cm^2$.

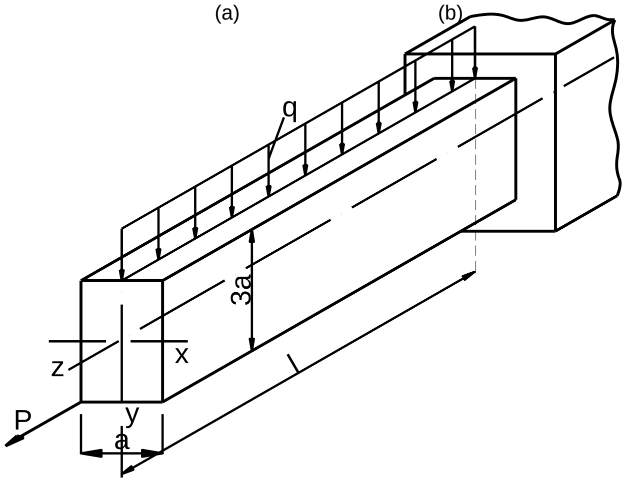


Exercise7:
Draw stress diagram at the dangerous section of column.
Know that: P = 2500 N, q = 20 kN/m, $\alpha = 30^{o}$.

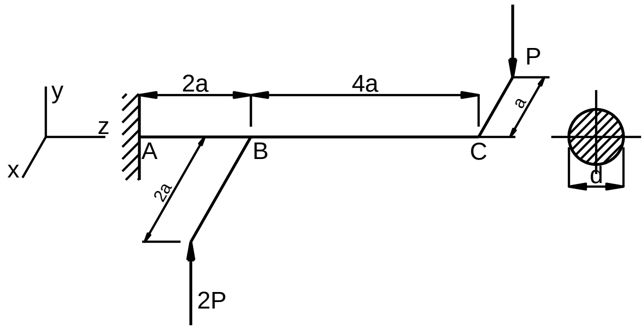


(a) (b)

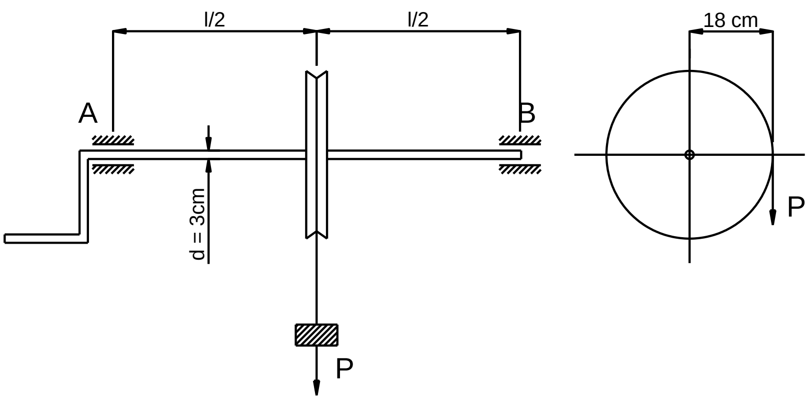
Exercise8:
Check the strength of bar.
Know that: P = 60 kN; q = 20 kN/m; l = 2,4 m; a = 6 cm, $[\sigma] = 120$ MN/m^2.
Know P = ql and the above datasl, a, $[\sigma]$.
Determine allowable load.



Exercise 9:
Determine allowable load.
Know that: a = 40 cm, d = 4 cm, $[\sigma] = 160$ MN/m^2.



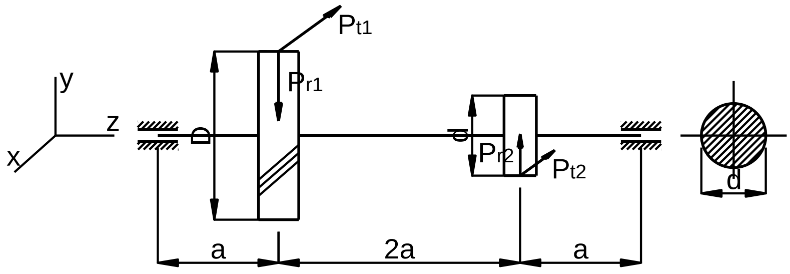
Exercise 10:
Thanks to the condition of strength conforming to the third reliability of theory, determine the allowable value of load P.
Know that: $[\sigma] = 8$ kN/cm^2, l = 1 m.



Exercise 11:

Determine the diameter of the shaft of reducer thanks to the forth reliability theory.

Know that: P_{t1} = 9700 N, P_{t2} = 2700 N, P_{r1} = 3500N, P_{r2} = 1500N, D = 300 mm, a = 60 mm, [σ] = 50MN/m^2.

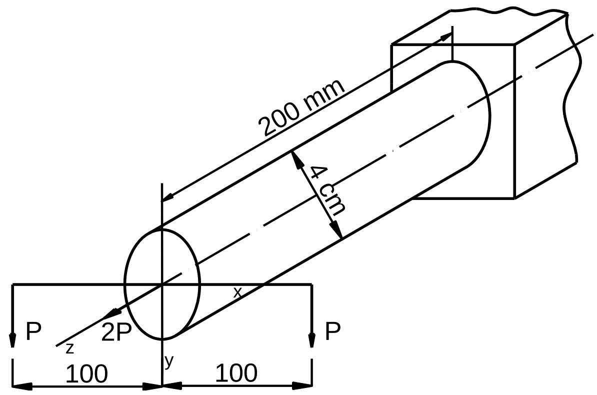


Exercise 12:

Thanks to the third reliability theory, the forth reliability theory and the reliability theory Morh, determine the calculating stress of a steel bar which is subjected to the load P = 1 kN in two cases:

a, P is put at Aand B, 2P is put at C.

b, P is put at A, 2P is put at C.



CHAPTER 7: BUCKLING OF COLUMNS

7.1. Concept

7.1.1. Buckling of columns

To have concept about buckling of columns, we carry out the follwing experiment: assume that we have a straight column as in the figure 7.1 and it is subjected to an axial compressive load P.

Initially, we choose P to ensure that it is small and the column is straight. Use a small stimulating load to take the column out of its straight stateand after that, ignore the stimulatingload. The column will return to original position and after a period of fluctuating around vertical position, the column will return to the initial position. The vertical state of bar is called stable state. (figure 7.1a)

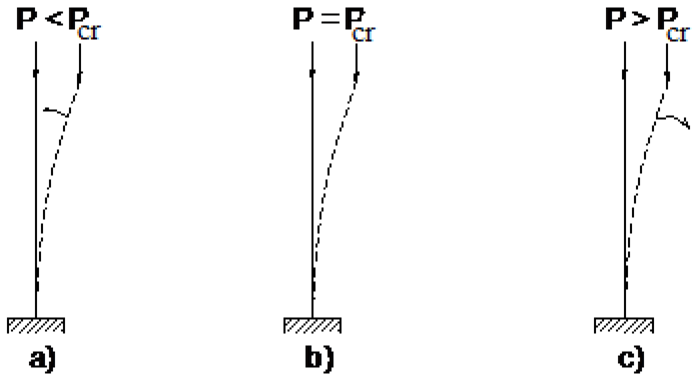


Figure 7.1

Increase load P to a value P_{cr} and push columnout of the straight position by a temporarily small stimulating load. At that moment, the column is at the new position and it does not return to the initial position. The initially vertical state of column is called neutral state. (figure 7.1b)

Continue to increase compressive load P to ensure that column is still in vertical state. We use a temporarily small stimulating load to push column out of the straight position, then we realize that column can not return to the initial position and it continues to be curved. We say that the initial straight position is unstable state. (figure 7.1c)

According to the above experiment, we realize that the column which is subjected to axial compressive load will be lost stability as the compressive load reaches P_{cr} and P_{cr} is called the critical load of the bar subjected to axial compressive load.

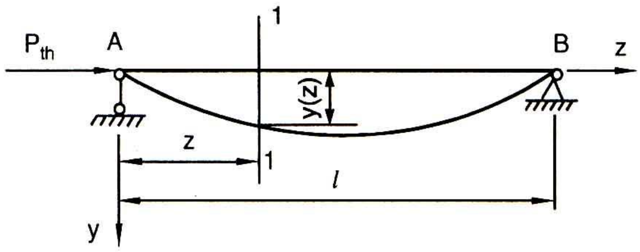
Therefore, the critical load of the column subjected to axial compressive load is the minimum compressive load which makes column lose its stability. The fact shows that as compressive load is higher than critical load or equals critical load, column will be lost its stability. When column is lost its stability, its deformation increases very fast and leads to the destruction of structures. Hence, as designing the column subjected to axial compressive load, besides requirements of strength and stiffness, engineers need to ensure the stability of column. To do the above requirements, we need to know the magnitude of the critical load of column.

Therefore, one of the tasks of calculating the stability of column is to determine critical load.

7.2. The Euler's formula to determine critical load

7.2.1. The Euler's problem to determine critical load in the column with hinged ends.

The problem is shown as below: Determine the critical load of a column with hinged ends. (figure 7.2)



As compressive load reaches P_{cr}, column will be curved to the side which has the minimum flexure-resistant stiffness.
Assume that we consider a section at a distance z from the origin 0. The deflection of this section is y(z) and on the section, there will be bending moment M_u (z):Figure 7.2

$$M_u(z) = P_{cr}.y(z) \qquad (a)$$

Assume that as lost stability, column still works in elastic region;hence, we can use the differential equation of elastic line as in the case of bended beam.

$$y'' = -\frac{M_u}{EJ_{min}} \qquad (b)$$

Substitute (a) in (b), we have the equation:

$$y'' = -\frac{P_{cr}}{EJ_{min}}y(z)$$

$$\text{Replace } \alpha^2 = \frac{P_{cr}}{EJ_{min}} \qquad (c)$$

We get the differential equation of elastic line:

$$y'' + \alpha^2 y = 0 \qquad (d)$$

The root of the equation (d) is:

$$y(z) = C_1 \sin\alpha z + C_2 \cos\alpha z \qquad (e)$$

Here $C_1$and $C_2$are the constants of integration determined by the conditions of the problem.

- As z = 0, y(0) = 0
- As z = l, y(l) = 0

Substitute in the root (e), we get a set of equations

$$\begin{cases} 0.C_1 + 1.C_2 = 0 \\ \sin\alpha l.C_1 + \cos\alpha l.C_2 = 0 \end{cases}$$

When lost stability, column is curved; hence, y(z) differs 0. It means that $C_1$and C_2can not equal 0 simultaneously. Thanks to this condition, we infer:

$$\Delta = \begin{vmatrix} 0 & 1 \\ \sin\alpha l & \cos\alpha l \end{vmatrix} = -\sin\alpha l = 0 \qquad (f)$$

According to the equation (f), we get:

$$\sin\alpha l = 0$$

$$\alpha l = k\pi$$

$$\alpha = \frac{k\pi}{l} \quad (\, k = 1, 2, 3\ldots..) \qquad (g)$$

Combine (g) with (c), we infer:

$$P_{cr} = \frac{k^2\pi^2 EJ_{min}}{l^2} \qquad (h)$$

At the different values of k, critical load has different values corresponding with the shape of different elastic line. We can realize that k is the number of half of the sine-shaped wavelength of elastic line as column is lost its stability.
Because critical load is the minimum load at which column is lost stability, we take k = 1. At that moment,

$$P_{cr} = \frac{\pi^2 EJ_{min}}{l^2} \qquad (7\text{-}1)$$

The expression (7-1) is the formula to determine critical load in column with hinged ends. The above formula is called the Euler’s formula.

7.2.2. The Euler's formula to determine the critical load of the column subjected to axial compression load.

The expression (7-1) is the formula to determine the critical load of the column with hinged ends. In case of columns with other end supports, we also can determine critical load by similar method to the Euler's problem and found result can write in the following general form:

$$P_{cr}^{Eul} = \frac{\pi^2 EJ_{\min}}{(\mu l)^2} \qquad (7\text{-}2)$$

μis coefficient depending on the types of supports in two ends of column.

$\mu = \dfrac{1}{m}$ with m is the number of half of the sine-shaped wavelength of elastic line as column is lost its stability.

The formula (7-2) is called the Euler's formula about the critical load of the column subjected to axial compressive load.

These are some types of supports which are easily seen in the column subjected to axial compressive load. (figure 7.3)

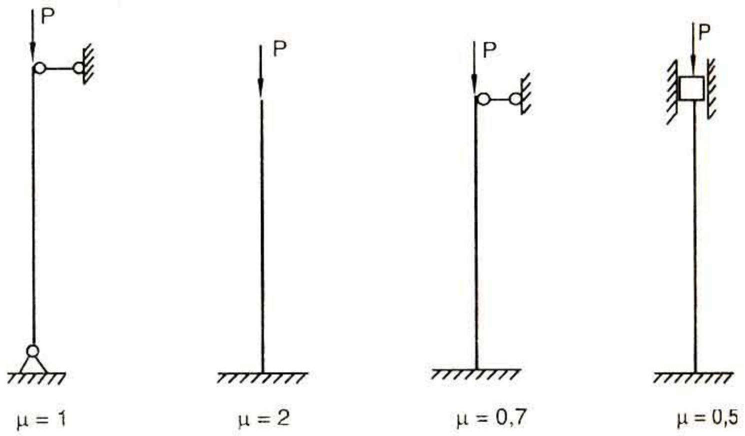


Figure 7.3

7.3. The Euler's formula to determine critical stress. Scope to use this formula

7.3.1. The Euler's formula to determine critical stress

As compressive load reaches critical load, column is still straight and compressed axially. Therefore, stress on cross-section will be:

$$\sigma_{cr} = \frac{P_{cr}}{F} = \frac{\pi^2 EJ_{\min}}{(\mu l)^2 F} \qquad (i)$$

We have: $\dfrac{J_{\min}}{F} = i_{\min}^2$

In which $i_{\min} = \min(i_x, i_y)$ and it is the least radius of gyration of cross-section.

And put $\dfrac{\mu l}{i_{\min}} = \lambda$ (λis called the slenderness ratio of the column subjected to axial compressive load. (7-2a)

At that moment, the expression (i) will be: $\sigma_{cr}^{Eul} = \dfrac{\pi^2 E}{\lambda^2}$ (7-3)

The expression (7-3) is the Euler's formula about critical stress. In the above expression, we realize that as λ is larger and larger, critical stress is smaller and smaller. It means that column is easy to lose its stability. Therefore, people call λ the slenderness ratio of column. According to the expression (7-2a), we realize that the slenderness ratio λ of column depends on the shape, dimension of column and the end conditions of supports at two ends of column.

It does not depend on the material of column. Each column has a value of the determining slenderness ratioλ.

7.3.2. Scope to use the Euler's formula

As establishing the Euler's formula, we rely on the basic assumption that the material of column still works in elastic region. Because the Euler's formula (7-2) or (7-3) only uses as stress in column is smaller than proportional limit. Hence, we have the following condition:

$\sigma_{cr}^{Eul} = \dfrac{\pi^2 E}{\lambda^2} \leq \sigma_{pr}$ ($\sigma_{pr}$is proportional stress determined by experiments.)

or $\lambda \geq \sqrt{\dfrac{\pi^2 E}{\sigma_{pr}}}$

If we put $\lambda_0 = \sqrt{\dfrac{\pi^2 E}{\sigma_{pr}}}$; $\lambda_0$is called critical slenderness ratio; it depends on material completedly.

The column having$\lambda \geq \lambda_0$is called the one which has large slenderness ratio. Therefore, the Euler's formula only uses in case of the column having large slenderness ratio or the column having $\lambda \geq \lambda_0$.

7.4. The formula to determine the critical stress of column as material works outside elastic region

In case of the columns having intermediate and small slenderness ratio($\lambda < \lambda_0$), it means thatcolumn works in inelastic region, there is no fully worked-out theoretic formula about critical load. Therefore, people expose some experimental formulas to calculate σ_{cr}.

a. The column having intermediate slenderness $\lambda_1 \leq \lambda \leq \lambda_0$

Use Iasinxki's formula:

$$\sigma_{cr} = a - b\lambda \qquad (7\text{-}4)$$

In which: a and b are constants depending on material and determined by experiments. We can find the magnitudes of a, b in technical handbooks. These are the magnitudes of λ_0, a and b of some materials.

Material	**λ_0**	**a, MN/m^2**	**b, MN/m^2**
Steel CT2, CT3	100	310	1,14
Steel CT5	100	464	3,26
Steel 48	100	460	2,56
Steel silicon (Steel 52)	100	578	3,75
Timber	75 ÷ 76	36,8	0,265
Cast iron	80	776	4,15

b. The column having small slenderness $0 < \lambda \leq \lambda_1$

In case of the columns having small slenderness ratio, they will be destroyed because of losing their strength before they are lost their stability.

Therefore, people consider $\sigma_{cr} = \sigma_0^{comp}$.

In whichσ_0^{comp} is dangerous compressive stress.

Ductile materials: $\sigma_0^{comp} = \sigma_{yiel}$

Brittle materials: $\sigma_0^{comp} = \sigma_{ulti}^{comp}$

Hence, we have three formulas to determine σ_{cr}. The use of one of three formulas depends on the slenderness ratio of column.

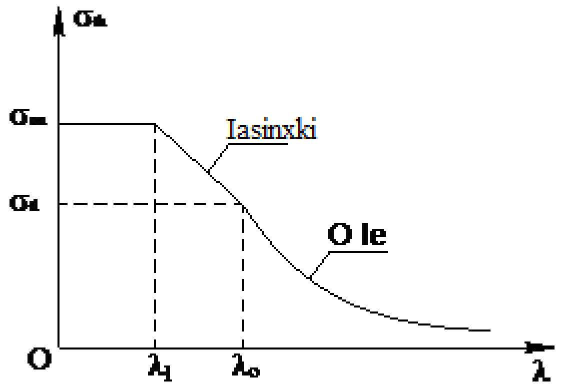


Figure 7.4

The figure 7.4is the chart expressing the relationship betweenσ_{cr} and λ at different slenderness ratios. According to the chart, we realize that $\sigma_{cr} \leq \sigma_0^{comp}$.

Example 1: Calculate the critical stress and critical load of the column made from steel CT5, $IN^0$24-cross section, hinged ends in two cases:

a) Column's length is 3m.

b) Column's length is 2m.

Solution:The cross-section of $IN^0$24-steel has area F = 34,8cm^2; $i_{min} = i_y$ = 2,37cm; E = 2.10^{4}kN/cm^2. The supports of column at two ends are hinged so μ = 1.

a) In case ofcolumn's length l = 3m = 300cm, its slenderness ratio is:

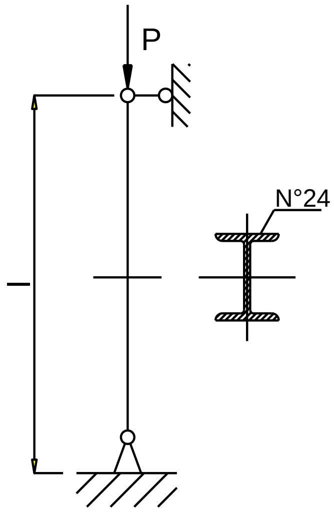
$$\lambda = \frac{\mu l}{i_{min}} \frac{1.300}{2{,}37} = 126{,}6$$

As material is steel CT3, consult index, we have λ_0 = 100. Compare, we realize that$\lambda > \lambda_0$ so we use the Euler's formula (7-3) to calculate critical stress.

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda} = \frac{3{,}14^2.2.10^4}{126{,}6^2} = 12{,}3kN/cm^2$$

The critical load of column will be:

$P_{cr} = \sigma_{cr}.F$ = 12,3.34,8 = 428kN



b) In case of column's length l = 2m = 200cm:

Figure 7.5

$$\lambda = \frac{\mu l}{i_{min}} = \frac{1.200}{2{,}37} = 84$$

Consult index as material is steel CT3, we have λ_0 = 100, λ_1 = 70. Compare, we realize that $\lambda_1 < \lambda < \lambda_0$.

We use the Iasinxki's formula to calculate λ_{cr} : λ_{cr}= a - bλ

with a = 464MN/cm^2 = 46,4 kN/cm^2; b = 3,26 MN/m^2 = 0,326kN/cm^2, critical stress will be:

σ_{cr} = a - bλ = 46,4 - 0,326 . 84 = 19kN/cm^2

The critical load of column will be:

$P_{cr} = \sigma_{cr}.F$ = 19.34,8 = 661,2 kN

7.5. Calculate the stability of the column subjected to axial compressive load thanks to factor of safety about stability (K_{buck})

7.5.1. Buckling condition thanks to factor of safety about stability

To ensure buckling condition for the column subjected to axial compressive load, stress in column does not exceed allowable stress about stability $[\sigma]_{buck.}$

Here, allowable stress about stability is determined by the following expression:

$$[\sigma]_{buck} = \frac{\sigma_{cr}}{K_{buck}} \qquad (7\text{-}5)$$

$K_{buck}$is factor of safety about stability, people usually choose its magnitude larger than n which is factor of safety about strength because the faculty of losing stability usually occurs earlier than destruction thanks to strength. Hence, bucking condition will be:

$$\sigma_z = \frac{N_z}{F} \le [\sigma]_{buck} = \frac{\sigma_{cr}}{K_{buck}} \qquad (7\text{-}6)$$

7.5.2. Three basic problems

Thanks to the buckling condition (7-6), we also have three following basic problems:

a. Test problem

In case of this problem, we know the load, dimensions, shape, material and supports of column. Factor of safety K_{buck} is given. We have to check whether column has enough stability.

To solve this problem, we follow the following procedure:

- Calculate the slenderness ratio of column by the formula (7-2a).
- Compare slenderness ratio λ which has just been calculated with λo, λ_1 to choose formula to determine $\sigma_{cr.}$
- Determine critical stress σ_{cr} by the chosen formula.
- Check stability by the formula (7-6) and conclude.

<u>Example 2</u>***:*** The column has rectangular cross-section (bxh) = (10x15)cm, two ends are hinged. (figure 7.6). It is subjected to axial compressive load P = 200 kN(figure 7.6). Check the stability of column. Know that K_{buck} =2, the material of column has E = 2.10^4 kN/cm^2, λ_1= 60, λ_o= 100, l = 4 m.

<u>Solution</u>: We determine the geometric properties of the cross-section of column: F= bh = 10.15 = 150 cm^2

$$i_{\min} = i_y = \frac{b}{\sqrt{12}} = \frac{10}{\sqrt{12}} = 2,89cm$$

The slenderness ratio of column is:

$$\lambda = \frac{\mu l}{i_{\min}} = \frac{1.400}{2,89} = 138,4$$

We realize that$\lambda > \lambda_o$ so critical stress is calculated by the Euler's formula:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda} = \frac{3,14^2.2.10^4}{138,4^2} = 11kN/cm^2$$

We have $\dfrac{|N_z|}{F} = \dfrac{P}{F} = \dfrac{200}{150} = 1,33kN/cm^2$

$$\frac{\sigma_{cr}}{K_{buck}} = \frac{11}{2} = 5,5kN/cm^2$$

Compare and realize that $\dfrac{|N_z|}{F} < \dfrac{\sigma_{cr}}{K_{buck}}$. In conclusion, the column has enough stability.

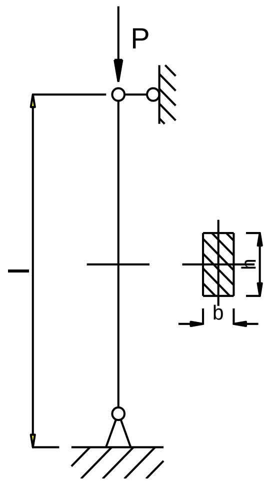


Figure 7.6

b. The problem of determining allowable load

In case of this problem, we know the shape, dimensions, material, supports of column as well as factor of safety about stability. However, we do not know compressive load. We have to determine the maximum compressive load acting on column so that the column still ensures stability.

To solve this problem, we follow the following procedure:

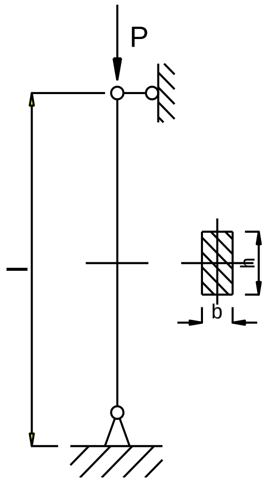
- Calculate the slenderness ratio of column by the formula (7-2a).

- Compare slenderness ratio λ which has just been calculated with λo, λ_1 to choose formula to determine σ_{cr}.
- Determine critical stress σ_{cr} by the chosen formula.
- Determine allowable load thanks to buckling condition by the formula (7-6).

$$[P] = \frac{F.\sigma_{cr}}{K_{buck}} \tag{7-7}$$

Example 3: A steel column has IN^o18-section. An end is fixed and an end is hinged. Its length is 3m. Determine allowable load thanks to buckling condition. Know that $\lambda_o = 100$, $\lambda_1 = 60$, $K_{buck} = 2$, $E = 2.10^4$ kN/cm^2.

Solution: Consult the index of I-steel, N^o18 has $F = 23,8$ cm^2, $i_{min} = i_y = 1,99$cm.



The slenderness ratio of column is:

$$\lambda = \frac{\mu l}{i_{min}} = \frac{0,7.300}{1,99} = 105,5$$

We realize that $\lambda > \lambda_o$ so critical stress is calculated by the Euler's formula:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda} = \frac{3,14^2.2.10^4}{105,5^2} = 17,7 kN/cm^2$$

The allowable load of column is:

$$[P] = \frac{F.\sigma_{cr}}{K_{buck}} = \frac{23,8.17,7}{2} = 210,6kN$$

Figure 7.7

c. The problem of determining the dimension of the cross-section of column

In case of this problem, we know the load, length, the shape of cross-section, material, supports of column as well as factor of safety about stability. We have to determine the minimum dimension of the cross-section of column so that the column still ensures stability. According to the formula about buckling condition (7-6), we realize that the area of cross-section F and critical stress σ_{cr} depends on the dimension of cross-section, so we have to use the gradually correct method to determine the dimension of cross-section.

Firstly, we assume that column still works in elastic region and we use the Euler's formula about critical load

$$P_{cr} = \frac{\pi^2 E J_{min}}{(\mu l)^2}$$

and assume that $[P] = \dfrac{P_{cr}}{K_{buck}} = P$

we infer $J_{min} = \dfrac{P(\mu l)^2 K_{buck}}{\pi^2 E}$

From J_{min}, we can calculate the dimension of cross-section. Now, we check result again by determining the slenderness ratio λ.

- If $\lambda > \lambda_o$, the found dimension is the result of problem.
- If $\lambda < \lambda_o$, we calculate critical stress again by the formula (7-3) or (7-4).

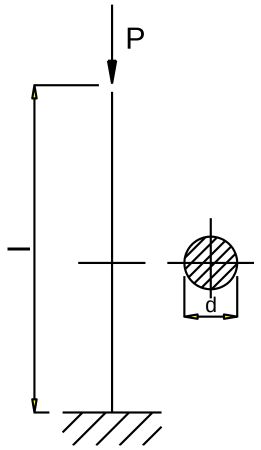
Thanks to buckling condition (7-6), we calculate area and then the dimension of cross-section.

$$F_1 = \frac{P.K_{buck}}{\sigma_{cr}}$$

From this new dimension, we determine new slenderness ratio λ_1.

- If λ_1 does not much differ from λ, the found dimension is the result of problem.

- If λ_1 much differ from λ, we use average magnitude and continue to calculate until discrepancy between new and old slenderness ratio does not exceed 5%.



Example 4: A circular steel column has structure as shown in the figure 7.8 with P= 50kN, l = 1 m, K_{buck} = 4, λ_o= 100, λ_1= 60, E = 2.10^4 kN/cm^2. Determine the diameter of cross-section.

Solution: Assume that the column works in elastic region, so:

$$J_{\min} = \frac{P(\mu l)^2 K_{buck}}{\pi^2 E} = \frac{50.(2.100)^2.4}{3{,}14^2.2.10^4} = 40{,}57 cm^4$$

The diameter of the cross-section of column is:

$$d = \sqrt[4]{\frac{J_{\min}.64}{\pi}} = \sqrt[4]{\frac{64.40{,}57}{3{,}14}} = 5{,}36 cm$$

Radius of gyration: $i_{\min} = \frac{d}{4} = 1{,}35 cm$

The slenderness ratio of column: $\lambda = \frac{\mu l}{i_{\min}} = \frac{2.100}{1{,}35} = 148{,}4 > \lambda_o$ Figure 7.8

Hence, the diameter of the cross-section of column is d = 5,36cm.

7.6. Calculate the stability of the column subjected to axial compressive load thanks to standard code (Use coefficient φ)

7.6.1. Strength condition and buckling condition

As we knew in chapter 2, when the column subjected to axial compressive load satisfies the strength condition,

$$|\sigma_Z| = \frac{|N_Z|}{F} \le [\sigma]_{comp} = \frac{\sigma_{0comp}}{n} \tag{7-8}$$

In which:

σ_{ocomp} is dangerous compressive stress

n is factor of safety about strength

Because σ_{ocomp} is selected by experiments, $[\sigma]_{comp}$ is supposed to be given by experiments.

On the other hand, if column satisfies buckling condition,

$$|\sigma_Z| = \frac{|N_Z|}{F} \le [\sigma]_{buck} = \frac{\sigma_{cr}}{K_{buck}} \tag{7-9}$$

In which: σ_{cr} is determined by the slenderness ratio of column.

According to two above conditions, it can be realised that as calculating strength, we are given allowable stress $[\sigma]_{comp}$ but as calculating stability, we have to determine $[\sigma]_{buck}$ và σ_{cr}. The exposed problem is how to use $[\sigma]_{comp}$ to calculate the stability of column.

To do this problem, we put the ratio $[\sigma]_{buck}$ / $[\sigma]_{comp}$ by a coefficient φ

$$\varphi = \frac{[\sigma]_{buck}}{[\sigma]_{comp}} = \frac{\sigma_{cr}}{\sigma_{ocomp}} . \frac{n}{K_{buck}} \tag{7-10}$$

Consider the magnitude of φ: Because $\frac{\sigma_{cr}}{\sigma_{ocomp}} \le 1, \frac{n}{K_{buck}} \le 1$, so φ ≤ 1. Hence, φ is called allowable stress-reducing coefficient.

According to the formula (7-10), we realise that φ depends on the material and the slenderness ratio of column. The magnitude of φ is given in the following index:

The slenderness ratio λ	The magnitude of φ				
	Steel CT 2,3,4	Steel CT5	Aloy steel	Cast iron	Timber
10	0,99	0,98	1	1	1

20	0,96	0,95	0,95	0,91	0,97
30	0,94	0,92	0,91	0,81	0,93
40	0,92	0,89	0,87	0,69	0,87
50	0,89	0,86	0,83	0,54	0,80
60	0,86	0,82	0,79	0,44	0,71
70	0,81	0,76	0,72	0,34	0,6
80	0,75	0,7	0,65	0,26	0,48
90	0,69	0,62	0,55	0,2	0,38
100	0,6	0,51	0,43	0,16	0,31
110	0,52	0,43	0,35		0,25
120	0,45	0,36	0,3		0,22
130	0,4	0,33	0,26		0,18
140	0,36	0,29	0,23		0,16
150	0,32	0,26	0,21		0,14
160	0,29	0,24	0,19		0,12
170	0,26	0,21	0,17		0,11
180	0,23	0,19	0,15		0,10
190	0,21	0,17	0,14		0,09
200	0,19	0,16	0,13		0,08

According to the above index, we realize that λdepends on slenderness ratio by complicated rules; however, to be convenient for interpolating to find φ, we suppose that in a 10-unit gap between λ, the rule of the change of φ is linear. Therefore, we have formula of interpolation as below:

with: $\lambda_1 \le \lambda \le \lambda_2$

$$\varphi = \varphi_2 + \frac{\varphi_1 - \varphi_2}{10}(\lambda_2 - \lambda) \qquad (9\text{-}13)$$

or

$$\varphi = \varphi_1 - \frac{\varphi_1 - \varphi_2}{10}(\lambda - \lambda_1) \qquad (9\text{-}13')$$

According to the expression (7-10), we get:

$$[\sigma]_{buck} = \varphi[\sigma]_{comp}$$

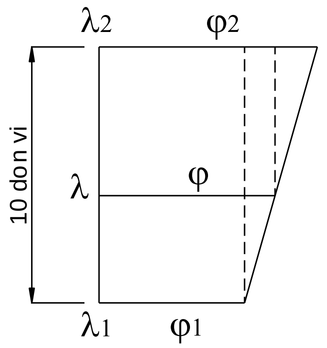


Figure 7.9

7.6.2. Buckling condition follows standard code

Subsitute in the expression (7-9), we get buckling condition following standard code as below:

$$|\sigma_Z| = \frac{|N_Z|}{F} \le \varphi[\sigma]_{comp} \qquad (7\text{-}11)$$

7.6.3. Three basic problems

Thanks to the buckling condition (7-11), we also have three following basic problems:

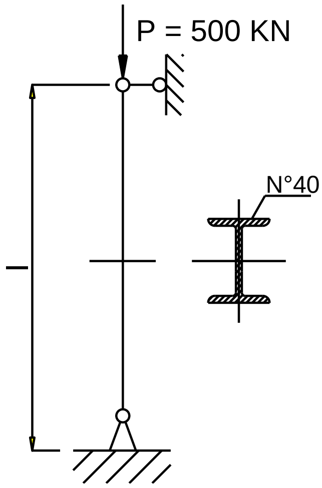
a. Test problem

In case of this problem, we know the load, shape, dimensions, material, supports of column. We have to check whether column has enough stability.

To solve this problem, we follow the procedure below:

- Calculate the slenderness ratio λ of column by the formula (7-2a).
- Consult and interpolate, we find φ.
- Check stability by the condition (7-11) and conclude.

Example 5: Check the stability of column which has the structure as in the figure 7.10. Know that l = 3m, $[\sigma]_{comp}$ =10 kN/cm^2.



Solution:Consult the index of IN°40-steel, we have F = 72,6cm², i_{min} =i_y = 3,03 cm. In case of two hinged ends, μ =1 and the slenderness ratio of column will be:

$$\lambda = \frac{\mu l}{i_{\min}} = \frac{1.300}{3,03} = 99$$

Consult the index of factor φ, we have:
Atλ_1= 90: φ_1 = 0,69
Atλ_2= 100:φ_2 = 0,60
Hence,

$$\varphi = \varphi_2 + \frac{\varphi_1 - \varphi_2}{10}(\lambda_2 - \lambda)$$

$$= 0,60 + \frac{0,09}{10}(100 - 99) = 0,609$$

Figure 7.10

Check stability, we have :

$$\left|\sigma_z\right| = \frac{\left|N_z\right|}{F} = \frac{P}{F} = \frac{500}{71,9} = 6,954 kN/cm^2 > \varphi[\sigma]_n = 0,609.10 = 6,09 kN/cm^2$$

Hence, the given column is unstable.

b. The problem of determining allowable load

In case of this problem, we know the load, shape, dimensions, material, supports of column. We have to determine the maximum compressive load to ensure that column has enough stability.

To solve this problem, we follow the following procedure:

- Calculate the slenderness ratio λ of column by the formula (7-2a).
- Consult and interpolate, we find φ.
- Calculate allowable compressive load from the buckling condition (7-11).

$$[P] = \varphi F [\sigma]_{comp} \tag{7-12}$$

Example 6:The IN°30a-steel column's length is l = 3 m with two hinged ends. Determine the magnitude of the allowable compressive load of column. Know that $[\sigma]_{comp}$ = 140 MN/m².

Solution:We have: F = 49,9cm², i_{min} = i_y=2,95cm. The column has two hinged ends, soμ=1

The slenderness ratio of column will be:

$$\lambda = \frac{\mu l}{i_{\min}} = \frac{1.300}{2,95} = 101$$

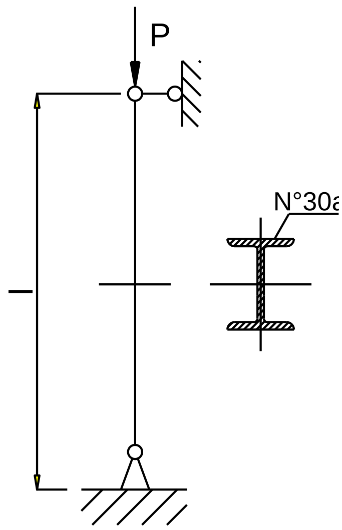
Consult the index of factor φ, we have:
Atλ_1= 100, φ_1 = 0,60
At λ_2= 110, φ_2 = 0,52

$$\varphi = \varphi_2 + \frac{\varphi_1 - \varphi_2}{10}(\lambda_2 - \lambda)$$

$$= 0,52 + \frac{0,60 - 0,52}{10}(110 - 101) = 0,592$$

According to buckling condition:

$$\left|\sigma_Z\right| = \frac{\left|N_Z\right|}{F} = \frac{P}{F} \le \varphi[\sigma]_{comp}$$



Hence allowable compressive load is

Figure 7.11

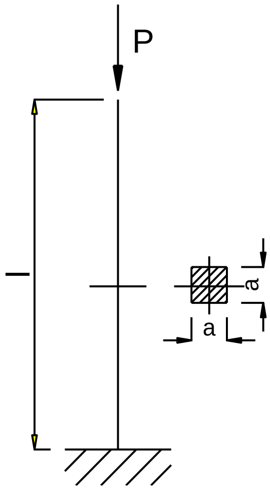
[P] = 0,592.49,9.14 = 413,7 kN.

c. The problem of deterining the dimensions (or sign number) of the cross-section of column

In case of this problem, we know the compressive load, shape, length, material, supports of column. We have to determine the minimum dimensions (or sign number) of column to ensure that column has enough stability.

In this problem, we do not know the area of cross-section and allowable stress-reducing factor φ, so we use the gradually correct method as below:

- Preliminarily select a value φ_o (usually $\varphi_o = 0,5$).
- From the formula (7-11), we calculate the area of the cross-section F of column.
- According to the calculated area, we preliminarily design cross-section.
- At the selected section, calculate slenderness ratio λ.
- Consult and interpolate, we find φ_1.
- If $\varphi_1=\varphi_o$, we finish the problem; if φ_1 differs from φ_o, we calculate again from the initial step which the value of φ_{20} is selected to equal the average of φ_1 and φ_o, we gain φ_{20}.
- Continue to do this until the found value of φ equaling the premilinarily selected value of φ. The problem will be finished. The dimension of the last round is the result of the problem.



Example 7:A square timber column is subjected to axial compressive load P = 100kN. Determine the length a of the cross-section of column. Know that l = 3m, $[\sigma]_{comp} = 1kN/cm^2$ (figure 7.12)

Solution:At structure as in the figure 7.12, we have μ =2.

Firstly, we choose $\varphi'_o = 0,5$. According to buckling condition, we determine the dimension of cross-section and then, slenderness ratio λ. Consult index, we have φ_1. If $\varphi_1 \neq \varphi'_o$, we choose $\varphi_1' = \dfrac{\varphi_o' + \varphi_1}{2}$ and return to the second round which is similar to the first round.

We continue to do this until round i to ensure that

$$\varphi_i = \varphi_{i-1}'$$

The results of rounds can see in the table below:Figure 7.12

Round	φ_{i-1}'	$F_i = \dfrac{P}{\varphi_{i-1}'[\sigma]_n}$	$a_i = \sqrt{F_i}$	$i_{i\min} = \dfrac{a_i}{\sqrt{12}}$	$\lambda_i = \dfrac{\mu l}{i_{i\min}}$	φ_i
1	0,5	200 cm^2	14,1 cm	4,05 cm	148	0,145
2	0,32	311	17,7	5,11	117	0,24
3	0,28	357	18,9	5,45	110	0,25
4	0,265	378	19,4	5,58	107	0,265

After finishing the 4th round, we realise that $\varphi_4 = \varphi_3' = 0,265$

Therefore, the dimension of the cross-section of column is a = 19,4 cm.

If the cross-section of column is shaped steel, because the dimension of cross-section changes by levels, as the value of φ in premilinary calculation and in test approximates to each other, we need to check buckling condition again and choose the minimum section which satisfies buckling condition.

Example 8:Choose the sign number of I-steel for the CT2-steel column which has l = 2 m, [σ] = 140 MN/m^2, and is subjected to an axial compressive load P = 230 kN.

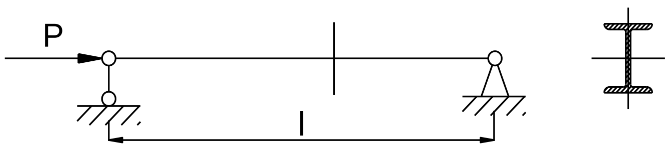


Figure 7.13

Solution:

- Chooseφ = 0,5, substitute in buckling condition, we have:

$F \geq \dfrac{P}{\varphi_0[\sigma]_{comp}} = \dfrac{230}{0{,}5.14}$ = 32,85 cm^2

Consult the index of shaped-steel, choose IN^{o}22a having F = 32,4 cm^2.

$\rightarrow$ $i_{min} = i_y$ = 2,5 cm

In case of the section IN^{o}22a, we calculate $\lambda = \dfrac{\mu l}{i_{min}} = \dfrac{1.200}{2,5}$ = 80.

Fromλ = 80 and material is CT2 steel, consult index, we find φ_1 = 0,75.

Compare and realise that $\varphi_1$much differs fromφ_0, so we have to choose again.

Takeφ_{20} = $\dfrac{\varphi_0 + \varphi_1}{2} = \dfrac{0,5 + 0,75}{2}$ =0,625.

Subsitute in buckling condition, we have: $F \geq \dfrac{P}{\varphi_0[\sigma]_{comp}} = \dfrac{230}{0{,}625.14}$ = 26,28 cm^2.

Consult index and choose IN^{o}20 having F = 26,4 cm^2, $i_{min} = i_y$ = 2,06 cm.

In case of the section IN^{o}20, we calculate $\lambda = \dfrac{\mu l}{i_{min}} = \dfrac{1.200}{2,06}$ = 97.

Fromλ = 80 and material is CT2 steel, consult index and interpolate, we have

$\lambda_1 = 90 \rightarrow \varphi_1 = 0,69$

$\lambda_2 = 100 \rightarrow \varphi_2 = 0,60$ At l = 97, interpolate and findφ = 0,627.

Compare and realise that φ = 0,627 approximates φ_{20} = 0,625.

Cheack buckling condition again in case of the section IN^{o}20, we have:

$$\left|\sigma_z\right| = \frac{|P|}{F} = \frac{230}{26,4} 8,712 \frac{KN}{cm^2} < 0,627.14 = 8,778 \frac{KN}{cm^2}$$

Hence, we choose I-section having sign number IN^{o}20.

7.7. The suitable shape of cross-section and the way to choose material

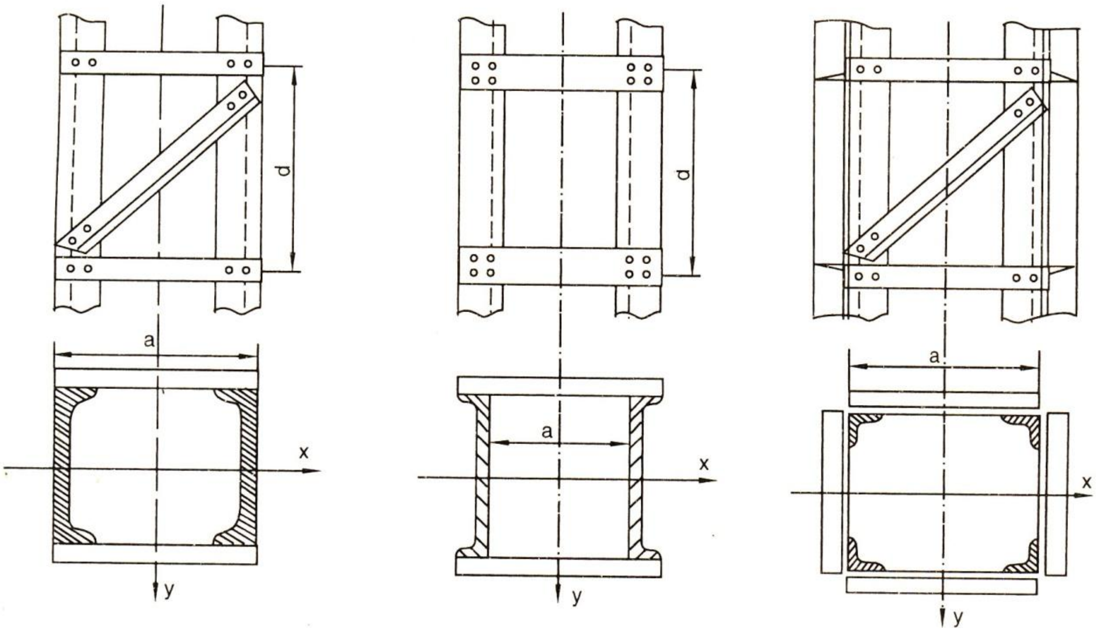


Figure 7.14

The column subjected to axial compressive load is ensured safety following the condition of strength as its cross-section has an arbitrary minimum area while its shape can choose arbitrarily. However, to ensure the stability of that column, besides ensuring the area of cross-section, we need to pay attention to its shape. We have to choose the shape of cross-section so that in a certain area, column can afford the maximum compressive load. That shape is called the suitable shape of cross-section because it not only ensures safety but also economizes most. It takes advantage of the load-resistant ability of material. As we know, to increase the stability of column, we need to reduce slenderness ratio λ.

To reduce slenderness ratio λ, we can decrease the length l of column, change supports in two ends of column so that μ is smaller (if the working condition of column allows) or increase the magnitude of i_{min}. Therefore, if we want cross-section to have suitable shape, we need to choose its shape in order to:

a. $i_{min} = i_{max}$, it means that $J_{min} = J_{max}$. Hence, column will combat against losing stability in all directions. As a result, the section of column is usually circle, square or regular polygon.

b. The centroidally principal moment of inertia of cross-section is as big as possible. Hence, people usually use hollow shape. However, it should not be too thin to avoid losing stability partially.

c. People usually use compound sections such as two I-letters, two [-letters or four L-letters… As joining, we need to ensure $J_{min} = J_{max}$and those centroidally principal moments of inertia are as big as possible.

As we know, in case of the column having big slenderness, the only mechanical property affecting σ_{cr} is the modulus of elasticity of material E. In case of the column having intermidiate and small slenderness, yielding limit or ultimate limit has big influence on σ_{cr}.

Therefore, as using material to make compressively loaded column, we need to notice that using high-intensive material does not bring interest in all cases. For example, although steel columns have different intensitiy, modulus of elasticity E is almost constant. Hence, in case of column with high slenderness, we should not use high-intensive steel because it is very wasteful. On the contrary, in case of column with intermediate and small slenderness, using high-intensive steel brings many benefits because it increases critical stress, it means that it rises the stabiliy of column.

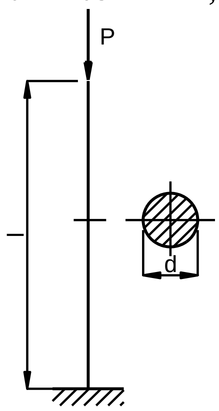
Theoretical questions

1. Present concept about the stable equilibrium and unstable equilibrium, critial state, critical load of a elastically deformed set.
2. Raise the sign of losing stability of the straigh column subjected to axial tensile (compressive) load in three Euler's problems. As losing stability, which plane will column curve in?
3. Define the slenderness ratio of column. Why can we say that the slenderness ratio of column is one of the important properties of column in calculating stability? Which factors doo the slenderness ratio of column depend on?
4. Write formula to calculate critical load aand critical stress as material works in elastic region (column has big slenderness) and as column works outside elastic region (column has intermediate and small slenderness). Present the way to determine slenderness ratio λ_0, λ_1.
5. Draw the chart expressing the relationship between σ_{cr} and λ. Comment that chart.
6. Present buckling condition following factor of safety about stability.
7. Raise the content of the method which calculates stability following standard code. Present buckling condition following stress-reducing factor φ.
8. In two ways to write buckling condition, which way is more reliable and accurate? Why?
9. Raise the way to solve three basic problems to calculate the column subjected to axial compressive load from buckling condition following standard code.
10. Raise the suitable shape of cross-section and the way to choose material.

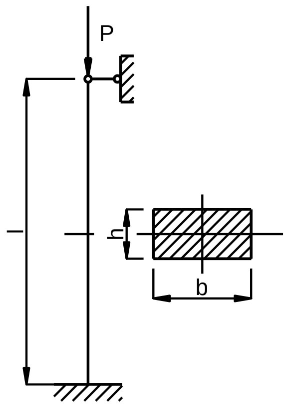
Numerical problems

Exercise 1:Determine critical load and critical stress for the columns as in the figure:

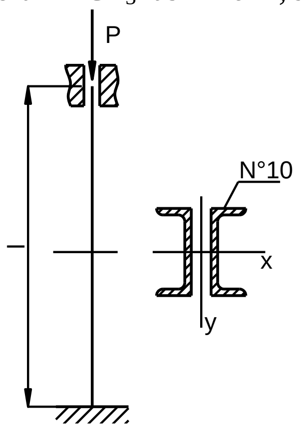
a, Timber columnhas l = 1 m, d = 6 cm.



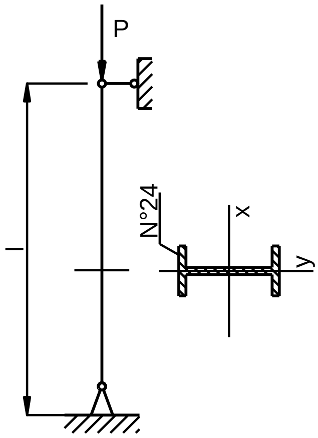
b, Cast-iron column has l = 2,5 m, h = 20 cm, b = 8 cm.



c, Steel column CT_3has l = 6 m, $J_x = J_y$.



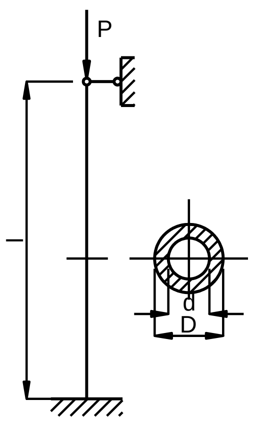
d, Steel column CT_3 has l = 3 m.



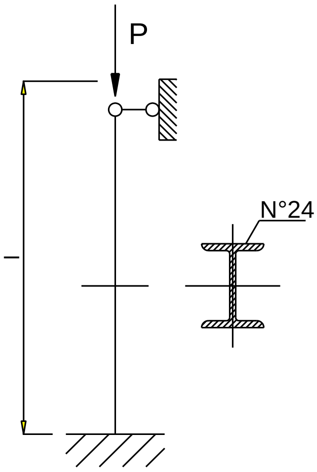
Exercise 2:

a, Timber column has l = 4 m, D = 8 cm, d = 6,4 cm, $[\sigma]_{comp}$ = 12 MN/m^2, P = 50 kN.

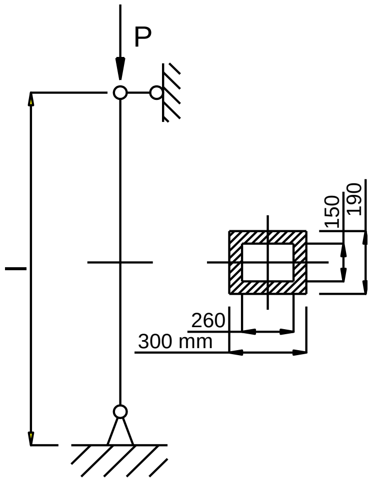
- Check buckling condition for the column?
- Determine factor of safety about stability as the column is still working?



b, Steel column CT3 has l = 4 m, $[\sigma]$ = 160MN/m^2, P = 300 kN.



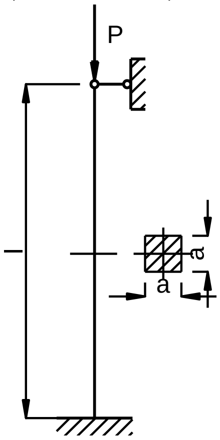
c, Cast-iron column has $[\sigma]_{comp}$ = 148 MN/m^2, P = 470 kN, l = 6 m.



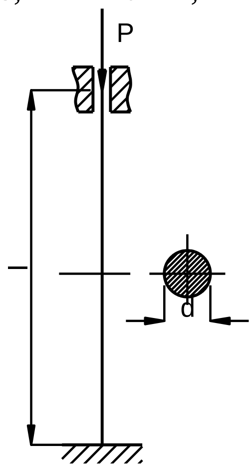
Exercise 3:

- Determine the dimension of cross-section for timber column subjected to load as in the figure a, b and choose the sign number of I-steel for steel column CT3 subjected to load as in the figure c.
- Calculate factor of safety about stability as the column works with chosen dimension.

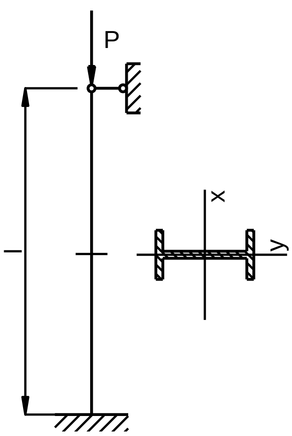
a, P = 60 kN, l = 4 m, $[\sigma]$ = 10MN/m^2.



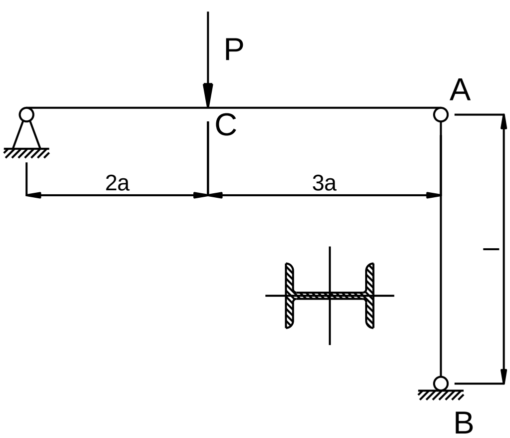
b, P = 70 kN, l = 5 m, $[\sigma]$ = 10MN/m^2.



c, P = 240 kN, l = 3 m, $[\sigma]$ = 160MN/m^2.

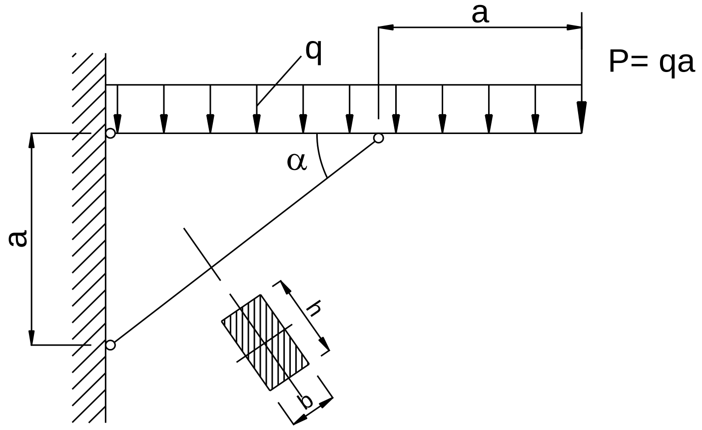


d, P = 950 kN, l = 2 m, $[\sigma]$ = 160 MN/m^2.

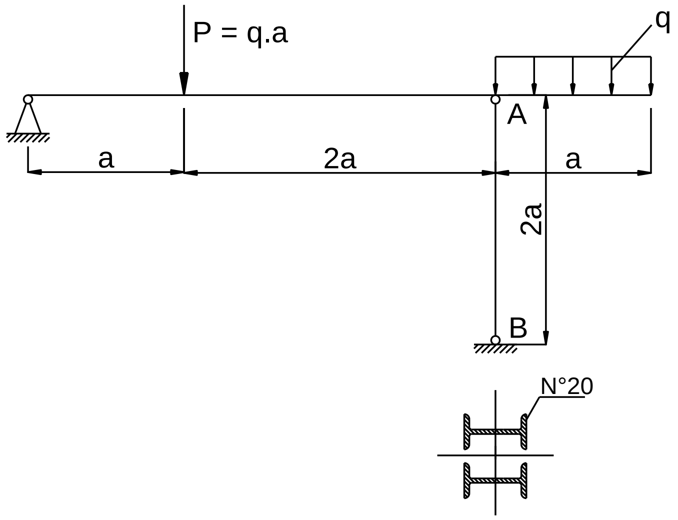


Exercise 4: Determine allowable load for the structure subjected to load as in the figure a, b, c.

a, The column CB is made from timber and has dimensions (bxh) = (10x20) cm^2, $\alpha = 30^o$ $[\sigma]_{comp} = 10\ MN/m^2$, a = 1,5 m, P = qa.



b, The column AB is made from steel CT3 and has the cross-section containing two IN^o20-steel combined together at a suitable distance a = 3 m, $[\sigma] = 150 MN/m^2$.



Exercise 5:

- Compare allowable load in case of the section containing two $[N^o14$-steel combined together by 4 projects:

a, Combine closely

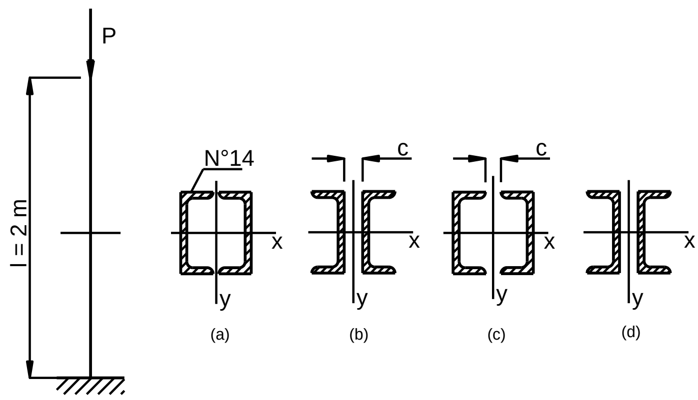
b, Combine at a distance c = 10 cm.

c, Combine at a distance c = 5 cm

d, Combine to be $J_x = J_y$

- Determine factor of safety as the column works at allowable load.

Know that $[\sigma] = 140\ MN/m^2$.



P
l = 2 m
N°14
c
c
x
x
x
x
y
y
y
y
(a)
(b)
(c)
(d)

CHAPTER 8: DYNAMIC LOAD

8.1. Concept, research direction

In the previous chapters, we considered the problems relating to static load whose magnitude increases gradually, lightly and does not create inertial force as acting on structures. In this chapter, we will research the problems relating to dynamic load. In these problems, the magnitude of load acting on structures increases suddenly and changes by time because of the appearance of inertial force on structures. Therefore, dynamic load is the one which creates inertial force as acting on an elastic structure. Inertial force which is arised on structures is caused by many different types of motion such as weights put on structures, translational motion, rotational motion, impact, oscillation…

Direction to research the problems relating to dynamic load is to determine different factors between impact of dynamic load and impact of static load correspondingly. Those different factors will be expressed by factors which are called dynamic factors and signed K_d. As we had dynamic factors, stress, deformation, deflection in problem will be calculated by multiplying stress, deformation, deflection in static problem with dynamic factor.

Therefore, the target of the problems relating to dynamic load is to determine dynamic factor K_d. To determine dynamic factors, besides using the general hypotheses of the subject, we also have to use principle of energy conservation, principle of momentum conservation, D'Alambert's principle.

Hereafter, we will research the types of dynamic problems in practice in turn.

8.2. The problem of translational motion with constant acceleration

Weight P is hung in an end of cable moving with constant acceleration a. The specific weight of material used to make cable is γ, the area of cross-section is F, the length of cable is l.

Acceleration a is considered to be positive if the direction of this acceleration is upward and it is considered to be negative as it is downward.

Now, we calculate internal forces at the section which is at a distance x from the end of cable. On each the cross-section of cable, there are the effect of weight P, the weight of cable and inertial force. The inertial force of weight P is considered as a concentrated force which has value $\frac{P}{g}a$. g is gravitational acceleration. The inertial force of cable is distributed along the length of cable. Its intensity is $\frac{\gamma F}{g}a$.

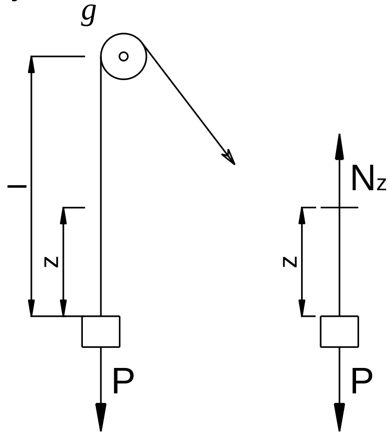


Figure 8.1

According to D'Alambert's principle, if there is inertial force, we can write static equilibrium for the part surveyed.

N_l is called longitudinal force on cross-section which is at a distance x from the end of cable. Its magnitude is:

$$N_l = (P + \gamma Fz) + \left(\frac{P}{g} a + \frac{\gamma Fz}{g} a \right)$$

Stress on the cross-section of cable is:

$$\sigma_l = \frac{N_l}{F} = (\frac{P}{F} + \gamma z)\left(1 + \frac{a}{g} \right)$$

If system is in static state, stress's magnitude on cable is:

$$\sigma_s = \frac{P}{F} + \gamma z$$

We put: $\left(1 + \frac{a}{g} \right) = k_d$

$K_d$is called dynamic factor. Hence: $\sigma_d = \sigma_s \left(1 + \frac{a}{g} \right) = \sigma_s . K_d$

The maximum normal stress is at the topmost end of cable. It means that at z = l, max $\sigma_s = \frac{P}{F} + \gamma l$

And the maximum stress's magnitude is:$\max \sigma_d = \max \sigma_s . k_d$

The condition of strength at dangerous section is:

$$\max \sigma_d = \max \sigma_s . k_d \le [\sigma]$$

Thanks to the expression of K_d, we realize that $K_d > 1$ if a > 0, it means that weight P moves upward and steadily fast. $K_d < 1$ if a < 0, it means that weight P moves upward and steadily slowly or moves downward and steadily fast.

Example 1: A $IN^0 30$-steel beam is raised in vertical direction at acceleration a = 5m/s (structure as in the figure 8.2a). The cross-section's area of cable is F = 1cm^2. Determine the maximum normal stress in the cable and beam. Know that a = 1m; l = 8m. (as calculating, we ignore longitudinal force)

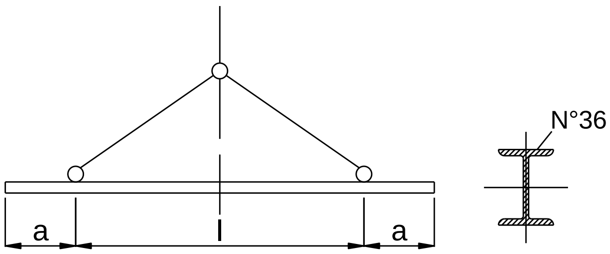
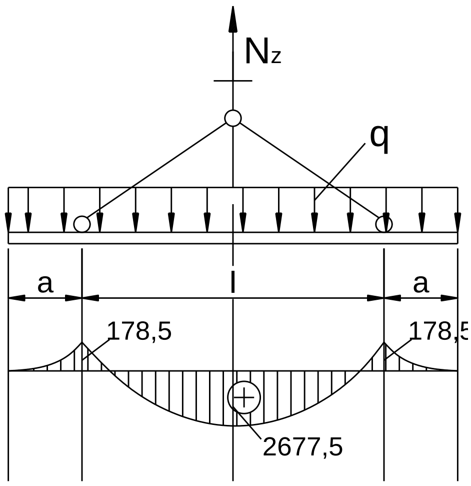


Figure 8.2a



Solution: The weight of beam acts on beam as uniformly distributed load with intensity q = γ.F.

Consult the index of $IN^0 30$, we have q = 357N/m; W_x = 472cm^3.

Dynamic factor here is:

$$K_d = 1 + \frac{a}{g} = 1 + \frac{5}{9,8} \approx 1,5$$

$N_z^s = q(l+2a) = 3570N$

The bending moment diagram of beam is in the figure 8.2b. The maximum static bending moment is in the middle of beam.

$M_x^{max} = 2677,5Nm$

The maximum normal stress in the cable is:Figure 8.2b

$$\sigma^d_{z\max} = \sigma^s_{z\max}.K_d = \frac{3570}{1}.1{,}5 = 5355 N/cm^2$$

The maximum stress in the beam is:

$$\sigma^d_{z\max} = \sigma^s_{z\max}.K_d = \frac{2677{,}5.10^2}{472}.1{,}5 = 850{,}9 N/cm^2$$

8.3. The problem of rotational motion with constant angular velocity

In case of the problem of rotational motion, we consider a rim having thickness t << D, the area of the cross-section of the rim is F. The specific gravity of material is γ, it rotates around axis 0-0 with constant angular velocity ω.

In fact, between the rim and the rotational center, there are spokes; however, we ignore their effects here (figure 8.3a). As the rim rotates with constant angular velocity, the points of the rim only have centrifugal acceleration which equals $\frac{1}{2}\omega^2.D$ while tangential acceleration equals 0.

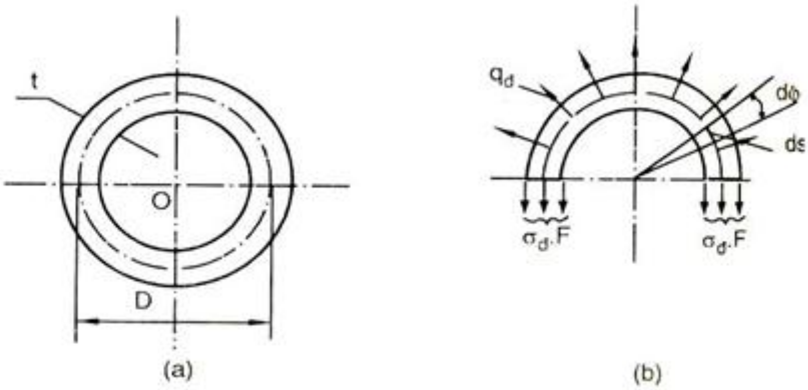


Figure 8.3

On each unit of the length of the rim, there is a centrifugal force:

$$q_d = \frac{\gamma F}{g}.\frac{1}{2}\omega^2 D \qquad \text{(a)}$$

Assume that we cut the rim by a plane going through the center and dividing the rim into halves (figure 8.3b). Because the area of the cross-section of the rim is very small, we consider that stresses are distributed uniformly on it. Consider the equilibrium of half the rim, we have equation:

$$2\sigma_d F - \int_0^\pi q_d.ds.\sin\varphi = 0 \qquad \text{(b)}$$

Differental length is: $ds = \frac{D}{2}d\varphi$ (c)

Subtitute (a) and (c) in (b) and integrate, we get:

$$\sigma_d = \frac{\gamma\omega^2 D^2}{4g}$$

We realize that in the problem of rotational motion, there is not corresponding static problem. Hence, we can not have clear dynamic factor as in the problem of translational motion.

Example2: The column BC of governor has rectangular section which has dimensions 60x20mm. It is attached to the column AB which is considered to be absolutely hard. At the end C, we attach a weight P = 100N. This governor rotates around axis O_1 - O_2 with speed ω = $30s^{-1}$. Determine the maximum normal stress in the column BC and the lateral displacement of the point C. (figure 8.4)

Solution: As the governor rotates, there will be a centrifugal force P_{cent} acting on the weight P.

$$P_{cent} = \frac{P}{g}\omega^2 AB$$

Here CC' is the displacement of the point C and it is also the deflection of the beam BC.

$$P_{cent} = \frac{100}{981}30^2 x10 = 917{,}431 \text{ N}$$

At that moment, the column BC is pulled by the weight P and bended by P_{cent}.

Draw diagram (N_z), (M_x).

The maximum normal stress at the cross-section R and its magnitude is

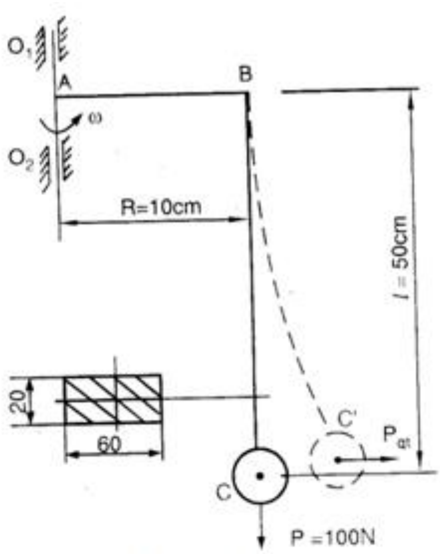


Figure 8.4

$$\sigma^{tens}_{z\max} = \frac{N_z}{F} + \frac{|M_x|}{W_x} = \frac{100}{6.2} + \frac{917{,}431.50}{2.\frac{6^2}{6}}$$

$$\sigma^{comp}_{z\max} = \frac{N_z}{F} - \frac{|M_x|}{W_x} = \frac{100}{6.2} - \frac{917{,}431.50}{2.\frac{6^2}{6}}$$

$$\sigma^{tens}_{z\max} = 3830{,}96 N/cm^2$$

$$\sigma^{comp}_{z\max} = -3814{,}29 N/cm^2$$

The lateral displacement CC' of the point C is the deflection at C of the beam BC and is calculated as below:

$$CC' = \frac{P_{cent}(BC)^3}{3EJ_x} \text{ với } J_x = \frac{2ab^3}{12} = 36 \text{ cm}^4$$

$E = 2.10^7$ N/cm^2

CC' = $5{,}31.10^{-2}$ cm

8.4. The problem of oscillation

8.4.1. The concept of oscillation

As we said at the beginning of the chapter, oscillation is one type of motion which has the acceleration of elastic system. Because it is acceleration-having motion, in oscillation, it appears inertial forces. These forces are added to increase the deformation and internal forces of system. Therefore, oscillation problem is the problem relating to dynamic load.

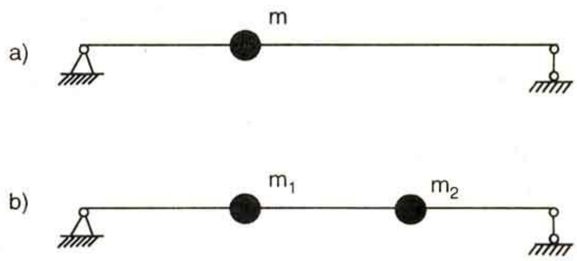


Figure 8.5

To research oscillation, we need to know some following concepts:

- The degree of freedom of an elastic system:

The degree of freedom of an elastic structure is the number of independent parameters to determine the position of structure. If we ignore the gravity of beam, in the figure 8.5a, the structure has single degree of freedom because the position of the beam is determined by the position of a mass m.

In the figure 8.5b, the structure has two degrees of freedom determined by the position of two masses m_1 và m_2.

If we include the gravity of beam, in both two beams, the structure has multi-degree of freedom.

- The oscillation frequency of system

The oscillation frequency of elastic system is the number of oscillation in a unit of time. It is usually the number of oscillation in one second and signed f. In technique, people use angular frequency which is the number of oscillation in time 2π and signed ω.

$$\omega = 2\pi f$$

Frequency is measured by Hz.

- The oscillation period of elastic system

The oscillation period of system is the time which system performs a full oscillation. Period is signed T:

$$T = \frac{1}{f}(s)$$

- Amplitude of oscillation:

Amplitude of oscillation is the distance which is furthest from equilibrium position in oscillation.

The oscillation of system is also divided into free oscillation and forced oscillation.

Free oscillation occurs when we affect to push system out of equilibrium position, then we do not affect any more. During the oscillation, there is not stimulating force.

Forced oscillation is the oscillation of system under the effect of external force which changes cyclically by time. For example, a motor is put on beam. Because the rotor of motor has eccentric weight, as rotating, it will cause centrifugal force which changes cyclically by time. This centrifugal force makes beam be oscillated forcedly.

Hereafter, we will consider the oscillation of the system which has single degree of freedom.

8.4.2. The free oscillation of the system having single degree of freedom

Consider a system which is brought back to a beam having an intermediate mass m (figure 8.6). Assume that we take the system out of equilibrium position. At that moment, the system will oscillate. During the oscillation, there will be an inertial force $m.\frac{d^2y}{dt^2}$ acting on m in opposite direction of oscillation. Here, y(t) is the displacement of mass m.

If we ignore resistive force, displacement y(t) is only caused by inertial force, so:

$$y(t) = -\delta.m\frac{d^2y}{dt^2} \quad (a)$$

Sign (-) shows that inertial force's direction is opposite to the direction of oscillation.

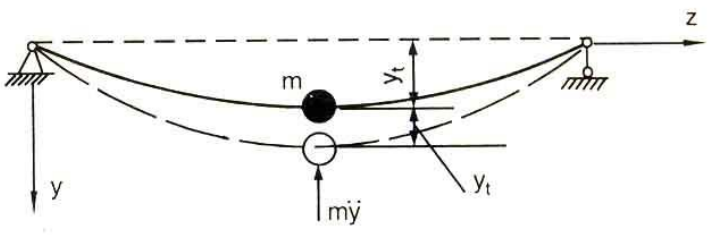


Figure 8.6

Put $\omega^2 = \frac{1}{\delta m}$. According to the expression (a), we have the equation of escillation without resistive force as below :

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad (b)$$

The root of equation is:

$$y(t) = C_1 \sin\omega t + C_2 \cos\omega t = A\sin(\omega t + \varphi) \quad (c)$$

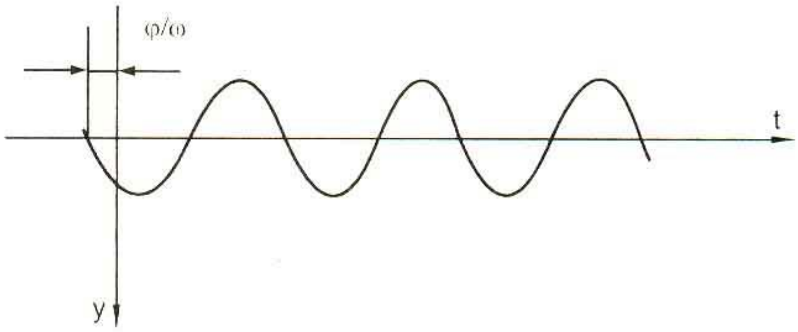


Figure 8.7

Here C_1, C_2or A, φis constants of integeration. The expression (c) expresses an oscillation with amplitude A and frequency ω. This oscillation can be expressed by the chart as in the figure 8.7. Frequency ω is called specific frequency of oscillation (or the frequency of free oscillation) of system and it is calculated by the following expression:

$$\omega = \sqrt{\frac{1}{\delta m}} = \sqrt{\frac{g}{\delta mg}} = \sqrt{\frac{g}{y_s}}$$

$y_s$is static displacement caused by the weight of mass m.

8.4.3. Free oscillation with resistive force

In fact, as system oscillates, because it is contacted with surrounding environment, system will be resisted by friction. Resistive force is very complicated. However, to be simple, people consider that resistive force is directly proportional to the speed of oscillation and it equals $\beta\dot{y}$. Its direction is opposite to oscillation's direction (figure 8.8). βis proportional coefficient and it depends on the property of surrounding environment. People consider that it is determined.

Therefore, the displacement of mass m is caused by inertial force and resistive force:

$$y(t) = -m\delta\frac{d^2y}{dt^2} - \beta\delta\frac{dy}{dt} \qquad \text{(d)}$$

We also put $\omega^2 = \dfrac{1}{\delta m}$ and $2\alpha = \dfrac{\beta}{m}$ and have equation:

$$\frac{d^2y}{dt^2} + 2\alpha\frac{dy}{dt} + \omega^2 y = 0 \qquad \text{(e)}$$

The root of equation is:

$$y(t) = Ae^{-\alpha t}\sin(\omega_1 t + \varphi_1) \qquad \text{(g)}$$

Here$\omega_1$is the frequency of free oscillation with resistive force and its magnitude is:

$$\omega_1 = \sqrt{\omega^2 - \alpha^2}$$

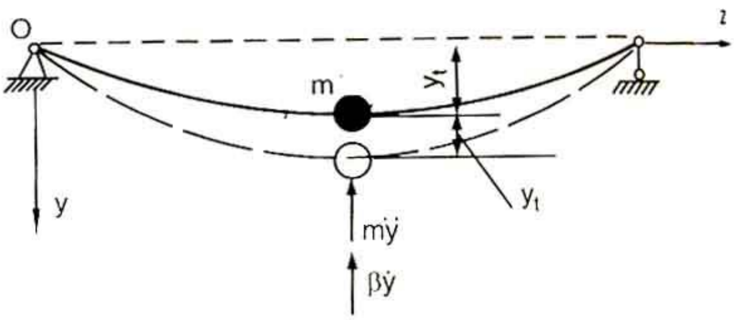


Figure 8.8

This amplitude of oscillation is $Ae^{-\alpha t}$ and it depends on time in inverse proportion. It means that the more time is, the smaller amplitude is. After a period of oscillation, amplitude reduces $\frac{e^{-\alpha t}}{e^{-\alpha(1+T)}} = e^{\alpha T}$

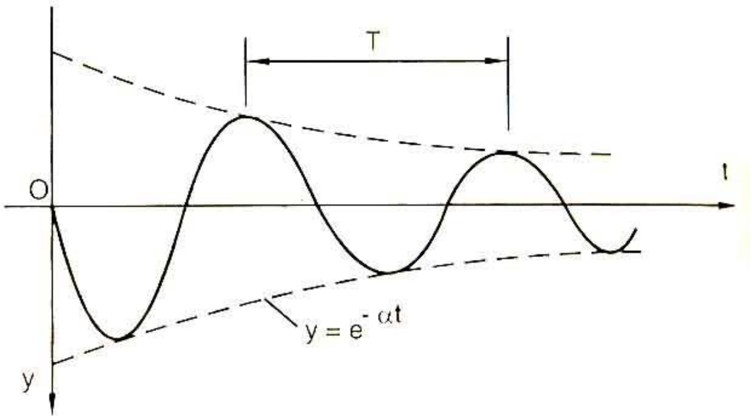


Figure 8.9

Hence, this oscillation is called damping vibration. We realize that damping becomes faster if resistive force becomes bigger, it means that α becomes bigger (figure 8.9).

8.4.4. Forced oscillation of system with single degree of freedom

We also consider oscillation of system with single degree of freedom which is brought back to a beam as in the figure 8.10. As system oscillates, besides inertial force and resistive force, there is also a stimulating force which changes by time $P(t) = P_0\sin\Omega t$ and acts on mass m in the same direction of motion of m.

P_0 is the maximum magnitude of stimulating force, Ω is the angular frequency of stimulating force. The displacement y(t) of mass m will be:

$$y(t) = \delta\left[P(t) - m\frac{d^2y}{dt^2} - \beta\frac{dy}{dt}\right]$$

We have the equation of oscillation:

$$\frac{d^2y}{dt^2} + 2\alpha\frac{dy}{dt} + \omega^2 y = \frac{P_0}{m}\sin\Omega t \qquad (h)$$

The root of above equation will be:

$$y = y_0 + y_1$$

in which:

y_0 is the general root of homogeneous equation as the expression (g).

y_1 is the specific root of inhomogeneous equation

$$y_1 = C_1\sin\Omega t + C_2\cos\Omega t \qquad (i)$$

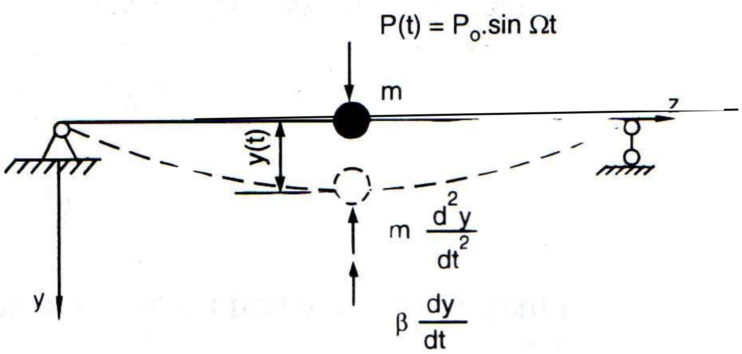


Figure 8.10

The constants C_1 and C_2 are determined by substituting the expression (i) in the equation (h) and interpolating the coefficients of $\sin\Omega t$ and $\cos\Omega t$ in both sides, we get:

$$C_1 = \frac{P_0}{m}\cdot\frac{\omega^2-\Omega^2}{\left(\omega^2-\Omega^2\right)^2+4\alpha^2\Omega^2}$$
$$C_2 = -\frac{P_0}{m}\cdot\frac{2\alpha\Omega}{\left(\omega^2-\Omega^2\right)^2+4\alpha^2\Omega^2} \qquad (k)$$

If we put:

$$\sin\psi = \frac{2\alpha\Omega}{\sqrt{\left(\omega^2-\Omega^2\right)^2+4\alpha^2\Omega^2}}$$
$$\cos\psi = \frac{\omega^2-\Omega^2}{\sqrt{\left(\omega^2-\Omega^2\right)^2+4\alpha^2\Omega^2}}$$

and substitute the expression C_1 and C_2 in the root (i), we get:

$$y_1 = \frac{P_0}{m}\cdot\frac{1}{\sqrt{\left(\omega^2-\Omega^2\right)^2+4\alpha^2\Omega^2}}\sin\left(\Omega t-\psi\right) \qquad (l)$$

$$\text{or } y_1 = \frac{P_0\delta}{\sqrt{\left(1-\frac{\Omega^2}{\omega^2}\right)^2+\frac{4\alpha^2\Omega^2}{\omega^4}}}\sin\left(\Omega t-\psi\right) \qquad (m)$$

The root (m) expresses an oscillation with the frequency which equals the frequency of stimulating force. Its amplitude is:

$$A_1 = \frac{P_0\delta}{\sqrt{\left(1-\frac{\Omega^2}{\omega^2}\right)^2+\frac{4\alpha^2\Omega^2}{\omega^4}}} = \frac{1}{\sqrt{\left(1-\frac{\Omega^2}{\omega^2}\right)^2+\frac{4\alpha^2\Omega^2}{\omega^4}}}.P_0\delta = K_d.\Delta t$$

Here, the magnitude $P_0\delta$ is displacement at the cross-section which brings the mass m. This displacement is caused by the force P_0 putting statically on the beam and is called static displacement $\Delta_t = P_0\delta$.

Therefore, the expression of dynamic factor will be:

$$K_d = \frac{1}{\sqrt{\left(1-\frac{\Omega^2}{\omega^2}\right)^2+\frac{4\alpha^2\Omega^2}{\omega^4}}}$$

We realize that dynamic factor depends on the ratio $\frac{\Omega}{\omega}$ and resistive coefficient α.

The figure 8.11 expresses the relationship between dynamic factor K_d and the ratio $\frac{\Omega}{\omega}$ with some values of α.

When $\frac{\Omega}{\omega}$ = 1, it means that as the frequency of stimulating force equals the specific frequency of oscillation, the value of K_d increases significantly. This phenomenon is called resonance. The fact shows that as the frequency of stimulating force does not

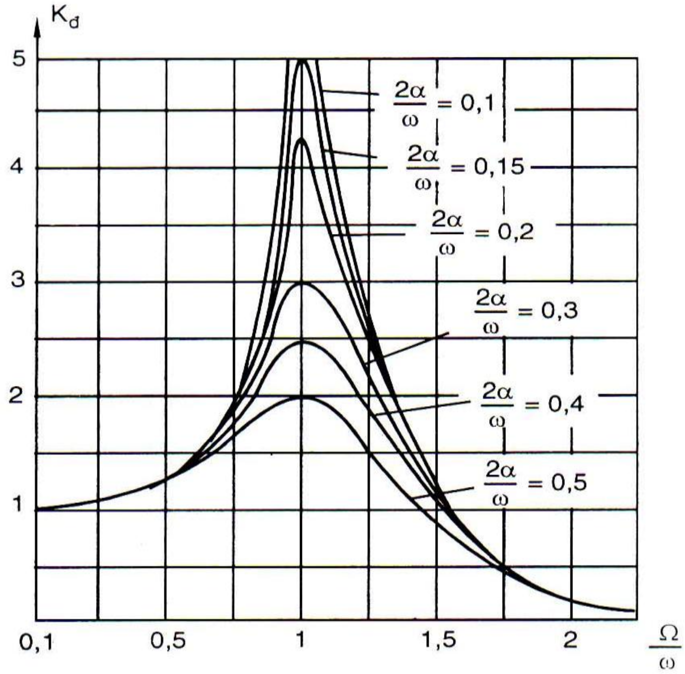


Figure 8.11

much differ from the frequency of free oscillation, the amplitude of oscillation increases clearly and creates a resonant region. In technique, to avoid resonance, the frequency of stimulating force has to much differ from the frequency of the free oscillation of system.

In the figure (8.11), we also see that as α is different, the curve of dynamic factor K_d will differ each other much in resonant region with the ratio $\frac{\Omega}{\omega}$ which is within 0,5 -2. On the other hand, as the above ratio is not in resonant region, K_d does not depend on resistive coefficient α. Therefore, if system works outside resonant region, dynamic coefficient can be calculated by the following formula:

$$K_d = \frac{1}{\left|1 - \frac{\Omega^2}{\omega^2}\right|}$$

In other problems relating to dynamic load, as we knew dynamic factor K_d, the stress and deformation of system are determined by multiplying stress and deformation caused by the maximum value of stimulating force putting statically on system with dynamic factor.

In oscillation problems, besides the mass m put on system, if we include the mass of beam and this mass is considered to be small in comparison with the mass m, we can substitute it by equivalent concentrated mass. This mass has the magnitude which equals converted factor μ. The magnitude of converted factor μ is determined by using principle of kinetic energy conservation. The magnitudes of μ and the corresponding positions to put the equivalent concentrated mass of system are shown as below:

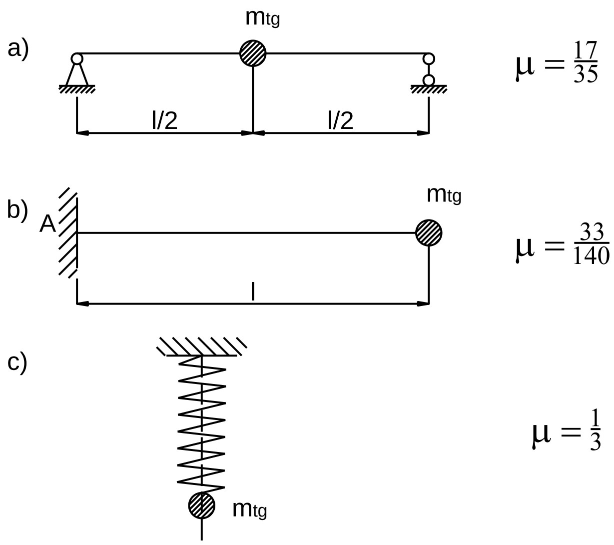
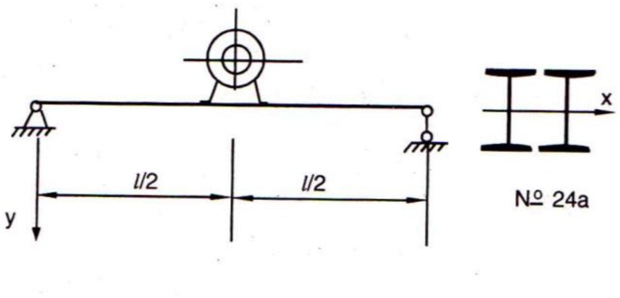


Figure 8.12

<u>Example3</u>: A beam has two hinged ends. Its length is 3m, its cross-section is $IN^0 24a$. In the middle of the beam, we put a motor which weighs $Q = 12.10^3$N and rotates with n = 1200 revolutions/minute. Centrifugal force is caused by the mass m = 2 kg which is put at a distance 0,3cm from axis. (we ignore resistive force and the gravity of beam). Determine the maximum stress in the beam (figure 8.13).Figure 8.13



<u>Solution</u>: Consult the index of $IN^0 24a$, we have

$J_x = 3800\text{cm}^4$; $W_x = 317\text{cm}^3$

The angular frequency of free oscillation is:

$$\omega = \sqrt{\frac{g}{y_s}}$$

Static deflection caused by Q is:

$$y_s = \frac{Ql^3}{48EJ_x} = \frac{12.300^3}{48.2.10^4.2.3800} = 4{,}44.10^{-2}\,cm$$

Substitute numerical value, we have the angular frequency of free oscillation:

$$\omega = \sqrt{\frac{981}{4,44.10^{-2}}} = 148,64(\frac{1}{s})$$

The angular frequency of motor is: $\Omega = \frac{2\pi.1200}{60} = 125,6(\frac{1}{s})$

The maximum value of stimulating force is:

$$P_0 = m\Omega^2 r = \frac{20}{9{,}81}(125{,}6)^2.0{,}03 = 0{,}972kN$$

Dynamic factor in case of not including resistive force is:

$$K_d = \frac{1}{\left|1-\frac{\Omega^2}{\omega^2}\right|} = \frac{1}{\left|1-\frac{(125,6)^2}{(148,64)^2}\right|} = 3,5$$

Bending moment caused by force P_0 put statically on beam at the section which puts load is:

$$M_x^{\max} = \frac{P_0 l}{4} = \frac{972.3}{4} = 72{,}9kN.cm$$

The corresponding static stress is:

$$\sigma_s^{\max} = \frac{M_x^{\max}}{W_x} = \frac{72{,}9}{2.317} = 0{,}115kN/cm^2$$

The stress caused by dynamic load is:

$$\sigma_d^{\max} = \sigma_s^{\max}.K_d = 0{,}115.3{,}5 = 0{,}402kN/cm^2$$

The static stress caused by the gravity of motor is:

$$\sigma_s^{'\max} = \frac{12.300}{4.2.317} = 1{,}42kN/cm^2$$

The total maximum stress on the beam is:

$$\sigma_{\max} = \sigma_s^{'\max} + \sigma_d^{\max} = 1{,}42 + 0{,}402 = 1{,}822kN/cm^2$$

8.5. The problem of impact

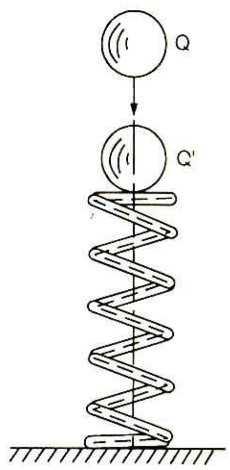


Figure 8.14

8.5.1. The concept of impact. Hypotheses

Assume that we have a weight Q moving to Q' which is put on elastic system (figure 8.14).

As moving to Q', Q collides with Q', attach to Q' and moves with Q' until elastic system is deformed. The above phenomenon is called impact. The problem which researches impact phenomenon is called the problem of dynamic load or impact problem. In this item, we only research the impact problem in case of system with single degree of freedom.

To be convenient in researching this problem, people publish the following hypotheses:

- Impact is flexible, central and this phenomenon occurs immediately.
- During the process, impact is not lost its energy to surrounding environment.

Thanks to two above hypotheses, we can apply principle of momentum conservation and principle of energy conservation in researching the problem of impact.

Hereafter, we will research two popular impact problems. They are the problem of vertical impact and the problem of horizontal impact.

8.5.2. The problem of vertical impact with single degree of freedom

Assume that we have an elastic system with single degree of freedom which is brough back to a horizontal beam attached a weight Q' as in the figure 8.15.

After falling from height h into the beam, the weight Q attaches to the weight Q' and continues to move and makes the beam have the maximum displacement y_d at the section occurring impact.

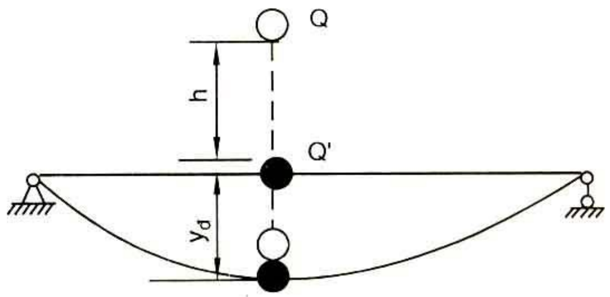


Figure 8.15

The speed of Q before the impact is v_0, its momentum will be $\frac{Q}{g}v_0$

The speed of Q and Q' after the impact is v, their momentum is: $\frac{Q+Q'}{g}v$.

According to principle of momentum conservation, we have:

$$\frac{Q}{g}v_0 = \frac{Q+Q'}{g}v \qquad \text{(a)}$$

Therefore, we infer the speed v:

$$v = \frac{Q}{Q+Q'}.v_0 \qquad \text{(b)}$$

The weight Q falls from height h, so $v_0 = \sqrt{2gh}$

The kinetic energy of Q and Q' after the impact is:

$$T = \frac{1}{2}.\frac{Q+Q'}{g}.v^2 \qquad \text{(c)}$$

Substitute (b) in (c), we get: $T = \frac{1}{2}.\frac{Qv_0^2}{\left(1+\frac{Q'}{Q}\right)g}$ (d)

The weights Q and Q' begin moving after the impact and lower their height y_d, so their potential energy reduces:

$\Pi = (Q + Q')y_d$ (e)

Now, we determine elastic strain potential energy.

Initially, on the beam, there is only Q'. The deflection of the beam is y'_s and elastic strain potential energy accumulating in the beam will be:

$$U_1 = \frac{1}{2}Q' y_s' = \frac{1}{2}\frac{y_s^2}{\delta} \qquad (g)$$

Here, δ isthe deflection caused by unit force which is put at the section containing Q', so $Q' = \frac{y_s^{'}}{\delta}$

Similarly, as the impact occurs, the beam have the total deflection $y_d + y'_s$so elastic strain potential energy will be:

$$U_2 = \frac{1}{2}\frac{\left(y_d + y_t^{'}\right)^2}{\delta} \qquad (h)$$

Therefore, elastic strain potential energy caused by the impact will be:

$$U = U_2 - U_1 = \frac{y_d^2}{2\delta} + \frac{y_s^{'} y_d}{\delta} = \frac{y_d^2}{2\delta} + Q' y_d \qquad (i)$$

According to principle of energy conservation, we have:

$U = T + \Pi$ (k)

Subtitute (d), (e) and (i) in (k), we get equation:

$$\frac{y_d^2}{2\delta} + Q' y_d = \frac{1}{2}.\frac{Qv_0^2}{g\left(1 + \frac{Q'}{Q}\right)} + (Q + Q')y_d \qquad (l)$$

y_s is the static deflection at the impact section caused by putting Q on the beam statically, so

$y_s = \delta.Q$ (m)

Subtitute the expression (m) in the equation (l), we get:

$$y_d^2 - 2y_s y_d - \frac{y_t v_0^2}{g\left(1 + \frac{Q'}{Q}\right)} = 0 \qquad (n)$$

The root of the equation (n) will be:

$$y_d = y_s \pm \sqrt{y_s + \frac{y_s v_0^2}{g\left(1 + \frac{Q'}{Q}\right)}}$$

Take the positive value of y_s. (o)

Take the positive value of $y_d$and subsitute $v^2{}_0 = 2gh$, we get:

$$y_d = y_s.\left(1 + \sqrt{1 + \frac{2h}{y_s\left(1 + \frac{Q'}{Q}\right)}}\right) \qquad (p)$$

According to the expression (p), we have the expression of dynamic factor:

$$K_d = 1 + \sqrt{1 + \frac{2h}{y_s\left(1 + \frac{Q'}{Q}\right)}}$$

As we include the effect of the weight of beam, we have to add converted weight into the weight Q'. In particular case, if there is not the weight Q' on beam:

$$K_d = 1 + \sqrt{1 + \frac{2h}{y_t}}$$

In case that Q is put suddenly on beam, it means that h =0, K_d = 2.
After we have K_d, the dynamic stress and deformation of system will be determined by the product of static stress, deformation and dynamic factor.

Example4: We have an $IN^0$22a-steel beam and a weight Q = 200N falling from height h = 4cm into the free end of the beam. The length of the span of beam is shown in the figure 8.16a. We ignore the gravity of the beam. Determine the maximum stress on beam. Know that E = 2.10^4 kN/cm^2.

Solution: Consult the index of $IN^0$22a-steel, we have the properties of section:

J_x = 2760 cm^4

W_x = 251 cm^3

Bending moment diagram caused by the weight Q which is put on beam statically is drawn as in the figure 8.16b.

The maximum bending moment is:

M^s_{xmax} =Q.b = 600Nm

The maximum static stress is:

$$\sigma^s_{\max} = \frac{M^s_{x\max}}{W_x} = 239 N/cm^2$$

To determine static deflection at the section occurring impact, we establish unit state. Bending moment diagram is shown as in the figure 8.16c. Multiply the diagram (b) and (c), we get:

$$y_s = \frac{Qb^2}{3EJ_x}(a+b) = \frac{200.300^2.800}{3.2.10^7.2790} = 8{,}695.10^{-2}\ \text{cm}$$

Dynamic factor is :

$$K_d = 1 + \sqrt{1 + \frac{2h}{y_s}} = 1 + \sqrt{1 + \frac{2.4}{86.10^{-3}}} = 10{,}644$$

The maximum normal stress at the moment of impact will be:

$\sigma^đ_{max} = \sigma^s_{max}.K_d$ = 0,234.10,644 = 2,544 kN/cm^2

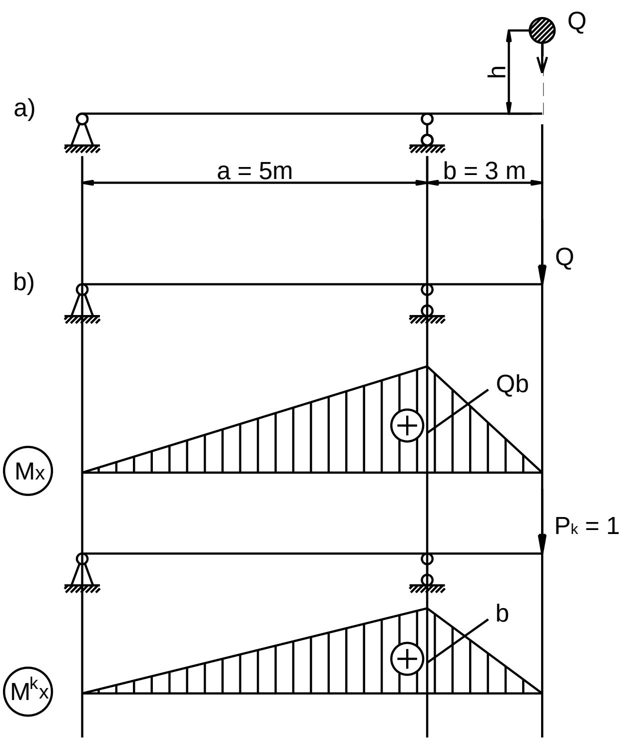


Figure 8.16

8.5.3.The problem of horizontal impact with single degree of freedom

Assume that there is an elastic system with single degree of freedom which is brought back to the vertical beam which is attached a weight Q' as in the figure 8.17. A weight Q moves in constant speed v_0 which follows horizontal direction and collides with Q', attaches to Q' and continues to move and makes the beam have displacement y_d. v is the speed of Q and Q' after the impact. According to principle of momentum conservation, we have:

$$\frac{Q}{g} v_0 = \frac{Q + Q'}{g} v$$

We infer: $v = \frac{Q}{Q + Q'} v_0$ (a)

The momentum of system after the impact is:

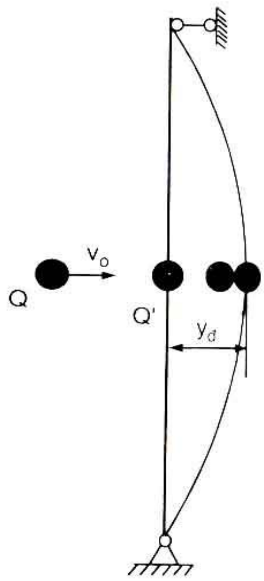


Figure 8.17

$$T = \frac{1}{2}.\frac{Q+Q'}{g}v^2 = \frac{1}{2}.\frac{Qv_0^2}{g\left(1+\frac{Q'}{Q}\right)} \tag{b}$$

According to the expression (g) in the item 8.5.2, the elastic strain potential energy accumulating in the beam is:

$$U = \frac{1}{2}.\frac{y_d^2}{\delta} \tag{c}$$

According to principle of energy conservation, we have:

$$U = T \tag{d}$$

Subsitute (b) and (c) in (d), we get equation:

$$\frac{1}{2}.\frac{y_d^2}{\delta} = \frac{1}{2}.\frac{Qv_0^2}{g\left(1+\frac{Q'}{Q}\right)} \tag{e}$$

$y_s$is the static displacement caused by the weight Q which is put at the impact section. We have $y_s = \delta.Q$. Subsitute in (e), we have:

$$y_d^2 = \frac{y_s.v_0^2}{g\left(1+\frac{Q'}{Q}\right)}$$

or $y_d = y_s.\frac{v_0}{\sqrt{gy_s\left(1+\frac{Q'}{Q}\right)}}$ (f)

Hence, dynamic factor will be:

$$K_d = \frac{v_0}{\sqrt{gy_s\left(1+\frac{Q'}{Q}\right)}}$$

If there is not the weight Q',

$$K_d = \frac{v_0}{\sqrt{gy_s}}$$

8.5.4. Method to reduce dynamic factor

According to the expression of dynamic factor in impact, we realize that to reduce the value of dynamic factor, we have to increase the static displacement y_s of beam. To increase static displacement without affecting the strength of beam, we have to replace the hard supports of system into elastic supports. The parts used in practice to decrease dynamic effects are usually cylindrical spring in trains or springs in cars…

We have many practical examples illustrating the above methods.

8.6. The critical speed of shafts

As designing the parts of machine with high speed, we need to pay attention to the effect of centrifugal force caused by the eccentricity of the weights put on shaft.

We consider a shaft bringing eccentric wheel (figure 8.18).

As shaft increases the number of revolutions to a certain value, shaft has the maximum deflection and rises noise.

If we continue to increase, noise and deflection reduce. As shaft has the maximum deflection, the speed of shaft is called critical speed.

y is deflection at the section bringing wheel; l is eccentric distance; Ω is the angular speed of shaft; m is the weight of wheel.

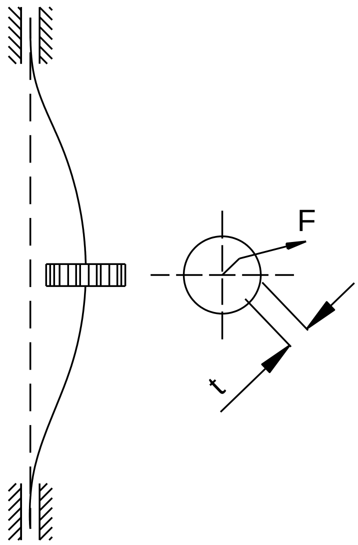
Centrifugal force is***:***

$F = m\Omega^2(e+y)$

δ is displacement caused by a unit of load which is put at the section bringing wheel. We have:

$y = F.\delta = m.\delta\Omega^2(e+y)$

or $y = \dfrac{e\Omega^2}{\dfrac{1}{m\delta} - \Omega^2}$



The deflection y is maximum when $\Omega^2 = \dfrac{1}{m\delta}$ or the critical speed of shaft equals the frequency of horizontal oscillation of system $\omega = \dfrac{1}{m\delta}$. Therefore, we need to notice:

- As designing shaft, we have to choose angular frequency which differs from critical speed.
- If $\Omega >> \omega$, $y_s \approx -e$, so the center of wheel is on the shaft. Figure 8.18

These notes play an important role in designing turbines and centrifugals.

Theoretical questions

1. Distinguish between dynamic load and static load? Raise direction to solve dynamic problem.
2. Raise the methoad to solve the problem of motion (translation and rotation) with constant acceleration.
3. Raise concept about the degree of freedom of an elastic system and the types of oscillation. Write the equation of oscillation, the chart of oscillation, the angular frequency of oscillation of system with single degree of freedom.
4. Write and explain the formula K_d in the problem of forced oscillation with single degree of freedom. Take specific example to illustrate this problem.
5. What is resonant phenomenon? Raise the method to solve resonant phenomenon.
6. Write and explain the formula K_d in the problem of impact. Raise the methods to reduce effects.
7. What is the converted factor of mass? Write formula to calculate converted factor in case of cantilever beam, simple beam and spring. As we calculate dynamic load, if we include the mass of elastic structure, what is the difference from ignoring the mass of beam?

Numerical problems

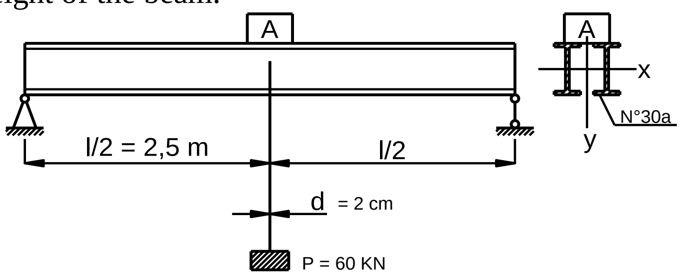
Exercise 1:

A structure A weighs 20 kN and is used to pull an object which weighs P = 40 kN in a steadily fast motion and in the first second, it moves a length 2.5m.

Check the strength of the string and beam. Know that $[\sigma] = 16$ kN/cm^2.

a, Ignore the weight of the beam.

b, Include the weight of the beam.



Exercise 2:

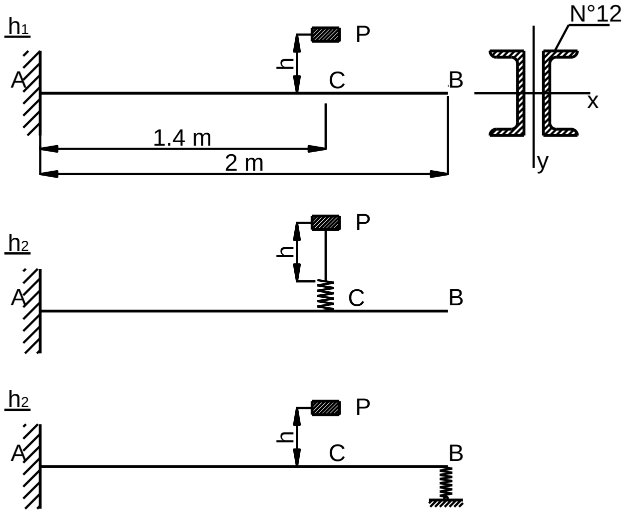
An object weighs P = 100 N and falls from height h into the beam AB and causes the maximum deflection f = 8 mm.

a, Determine the height h and the maximum normal stress on the beam.

b, With the found value of h, determine the maximum normal stress in the beam in two cases:

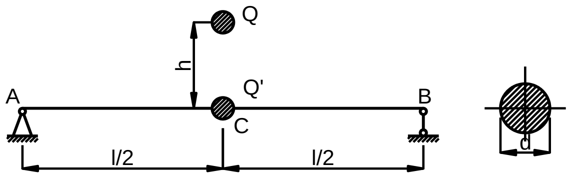
h_1-at C, we put a spring which has D = 6 cm; d = 0,5 cm; n = 10; G = 8.10^6 N/cm^2.

h_2-at B, we put a spring which has hardness C = 2 kN/cm.



Exercise 3:

Q falls from height h to collide with Q'. Know that Q' = 1200 N; Q = 5000 N; l = 2 m; h = 4 cm; d = 12 cm; [σ] = 160 MN/m^2; E = 2.10^5 MN/m^2; $\left[\frac{f}{l}\right] = \frac{1}{300}$.



Check the strength and stiffness of the beam:

a, Ignore the weight of the beam.

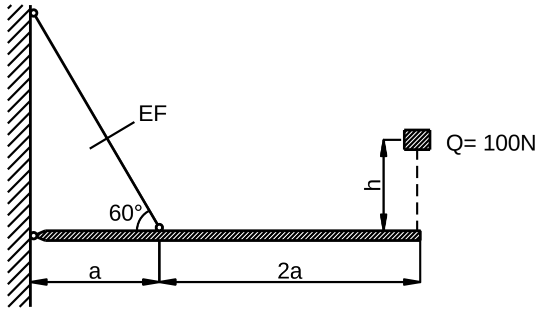
b, Include the weight of the beam.

Exercise4:

A structure is shown in the figure.

Know that E = 2.10^{11} N/m^2; F = 2 cm^2; [σ] = 160 MN/m^2; a = 0,5 m.

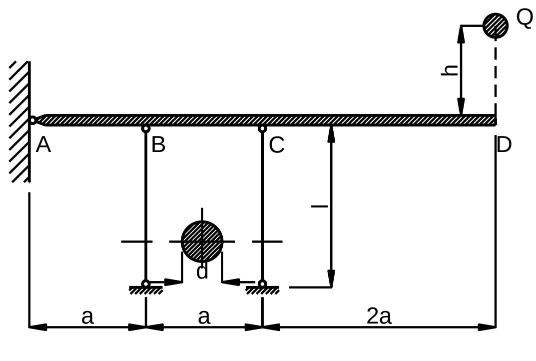
Determine h.



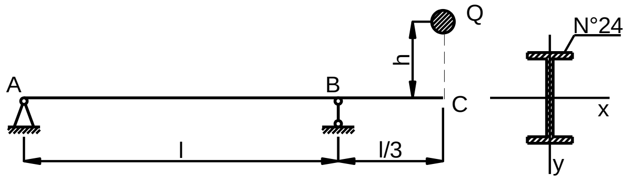
Exercise 5:

A weigh Q = 200 N falls from height h = 5 cm to collide with the section D of an absolutely hard bar AD. Know that l = 120 cm; d = 20 cm; [σ] = 12 MN/m^2; [ε_z] = 2.10^{-3}; E = 10^4 MN/m^2.

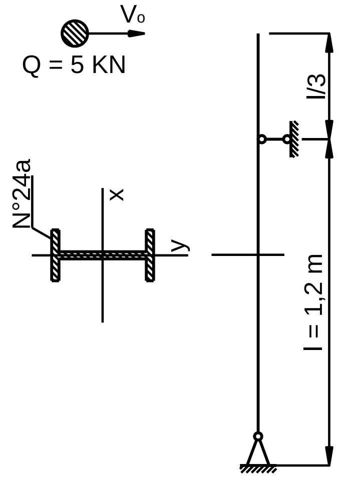
Check the strength and stiffness of the structure.



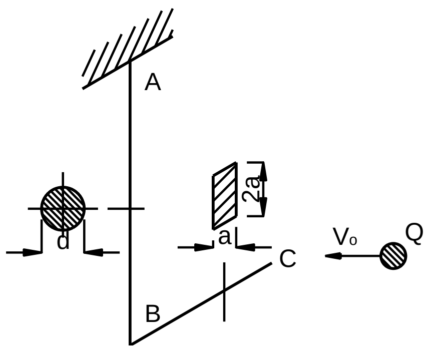
Exercise 6:
Determine the maximum stress and deflection in the beam in the figure. Know that Q = 5 kN; h = 10 cm; l = 2 m; l = 2 m; E = 2.10^5 MN/m^2



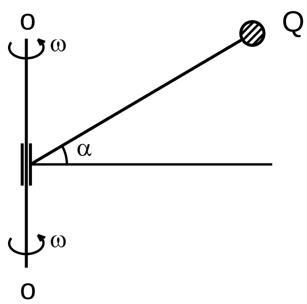
Exercise 7:
Determine the maximum value of V_o so that the beam satisfies the condition of strength and stiffness. Know that [σ] = 160 MN/m^2; $\left[\frac{f}{l}\right] = \frac{1}{500}$; E = 10^4 MN/m^2.



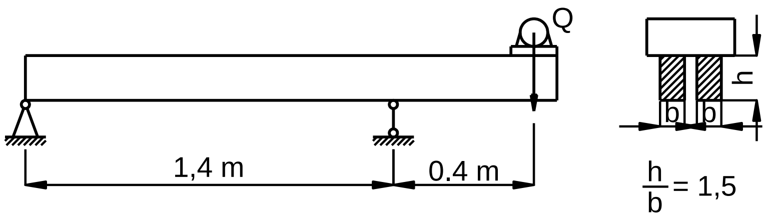
Exercise8:
A weigh Q = 500 N flies with speed V_o = 5 m/s to collide with the bar ABC as in the figure. Know that d = 12 cm; a = 8 cm; AB = 2 m; BC = 60 cm; [σ] = 160 MN/m^2; E = 10^4 MN/m^2.
Check the strength of the bar ABC.



Exercise 9:
At the free end of the bar having the length l and sloping to create with horizontal axisan angle α, people attach an object with weight Q. The bar rotates aound vertical axis o-o in constant angular speed ω (in the figure).
Determine the maximum dynamic normal stress at the dangerous section of the bar. Know that the unit weight of the bar is q, the area of cross-section is F and flexure-resistant moment is W.

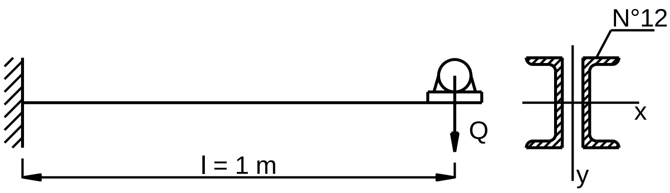


Exercise 10:
A motor is put on two timber beams and rotates in speed n = 1400rev/min, the weight of motor is Q = 1,6 kN. Determine the dimension of the cross-section of the beam so that the frequency of specific oscillation is 30% larger than thee frequency of stimulating force.
Know that the maximum value of stimulating force is 400N; [σ] = 10 MN/m^2. Check the strength of the beam (ignore the weight of the beam and resistive force).



Exercise 11:

A motor is put on a steel beam and rotates in n = 800 rev/min. The weight of motor is Q = 6 kN. As rotating, the motor creates a centrifugally inertial force P_0 = 4 kN. Determine the maximum deflection and the maximum normal stress in the beam.



Determine the number of revolutions of motor to occur resonant phenomenon.

Calculate in two cases:

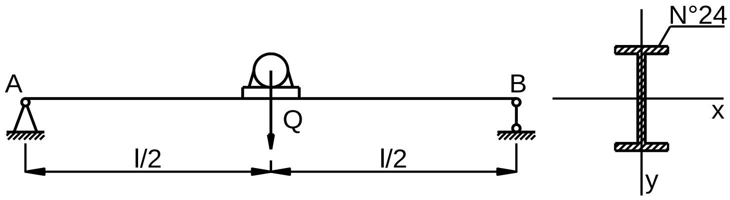
a, Ignore the weight of the beam and resistive force.

b, Include the weight of the beam and resistive force.

Know that: E = 2.10^7 N/cm^2; α = 2 (1/s).

Exercise 12:

A motor has weight Q = 48 kN and is put in the middle of IN^{o}24 beam. The beam's length is 4 m, the rotational speed of the motor is 510 rev/min. As rotating, the motor creates a centrifugally inertial force P_0 = 4,8 kN.



- Determine defletion and the maximum normal stress in the beam.
- Determine the number of revolutions of motor to occur resonant phenomenon.
- Determine the length of the beam to occur resonant phenomenon.

Calculate in two cases:

a, Ignore the weight of the beam and resistive force.

b, Include the weight of the beam and resistive force. (know that α = 2 (1/s))

CHAPTER 9: CURVED BAR

9.1. Concept – Internal force diagram

9.1.1. Concept

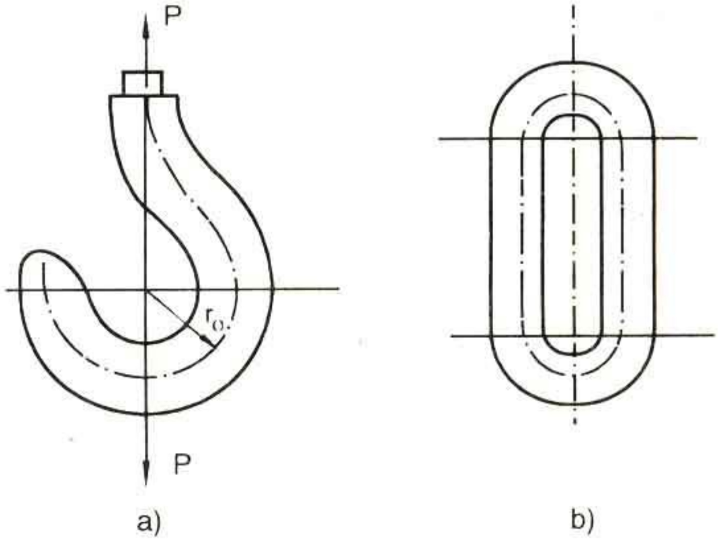
Besides the straight bars researched in the previous chapters, in fact, we also have seen bars with curved axis. They are called curved bars.

If the axis of bar is in plane, it is called planely curved bar.

In this chapter, we research the planely curved bars whose symmetric plane contains the axis of bar and external forces which affect the bar are in that plane.

Example: crane hooks, chain links, rings…

To evaluate the curvature of beam, people rely on the ratio between the radius of curvature (r_0) and the depth of the beam (h).Figure 9.1



- The beam has large initial curvature if $\frac{r_0}{h} \leq 4 \div 5$

- The beam has small initial curvature if $\frac{r_0}{h} > 4 \div 5$

To be simple, people can use the formula which calculates normal stress in straight bar to calculate stress for beams with small curvature.

9.1.2. Internal forces and internal force diagram

In case of planely curved beams, as loads are in the symmetric plane of the beam (yOz), on its cross-sections, there are three internal forces: longitudinal force N_z, shear force Q_y and bending moment M_x. To determine the above internal forces, we use the known section method. The position of each section will be the trigonometric function of that co-ordinate.

The sign convention of longitudinal force and shear force is similar to straight bar. Only bending moment M is positive if its length makes bar more curved.

Example 1: Draw the internal force diagram of curved bar AB supported and subjected to loads as in the figure 9.2a.

Solution: Use the section (1-1) to cut the bar into haves. Consider the right part as in the figure 9.2b.

The angle created by the section (1-1) and vertical direction is $\varphi(0 \leq \varphi \leq \frac{\pi}{2})$.

On the section, there are three internal forces:

- Longitudinal force: N = -Psinφ
- Shear force: Q = P.cosφ (1)
- Bending moment: M = P.r.sinφ

Internal force diagram is drawn as similarly as in the straight bar.

Here, the directrix of the diagram is the curvature which is parallel to the axis of bar.

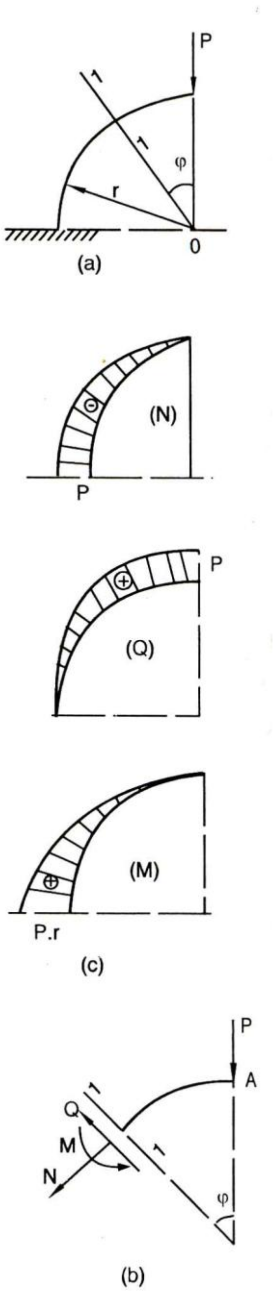


Figure 9.2

From (1), we have: at$\varphi = 0$, N = 0, Q = P, M = 0

$$\varphi = \frac{\pi}{2}, \; N = -P, \; Q = 0, \; M = Pr$$

The diagrams N, Q, M of the bar AB are drawn as in the figure 9.2c.

Example 2: Draw the internal force diagrams of curved bar AB with the radius of curvature r_0. It is subjected to load P at the point C. Load P creates with vertical direction an angle 30^0. The distance of two supports A, B is l = 10m. The bar is supported as in the figure 9.3a.

Solution:

1. Determine the reactions of the supports

Establish co-ordinate system x-y with the origin A. The projections of P in direction x and y are P_x, P_y.

$$P_x = P.\sin 30^0 = 40.\frac{1}{2} = 20kN$$

P_y = P.coss30^0 = 40.0,866 = 34,6kN

Take the moment of the load and reactions with the point B and A, we get:

$$\Sigma M_B = R_A.l - P_y.\frac{1}{2} - P_x.f = 0$$

$$\Rightarrow R_A = \left(P_y.\frac{l}{2} + P_x.f \right)\frac{1}{l} = 23{,}3kN$$

$$\Sigma M_A = R_B.l - P_y.\frac{l}{2} + P_x.f = 0$$

$$\Rightarrow R_B = \left(P_y.\frac{l}{2} - P_x.f \right).\frac{1}{l} = 11{,}3kN$$

Establish projective equation on the direction of axis x:

$\Sigma X = -P_x + H_B = 0$

$H_B = P_x = 20kN$

2. Determine unknown quantities

The angle created by the line which goes through the center of curvature and the support A and vertical direction is φ.

According to the figure 9.3a, we have:

$$\sin\varphi = \frac{l}{2r_o}; r_0 = \frac{l}{2r_0}; \cos\varphi = \frac{r_0 - f}{r_0}$$

$$\sin^2\varphi + \cos^2\varphi = \left(\frac{l}{2r_o}\right)^2 + \left(\frac{r_0 - f}{r_0}\right)^2 = 1$$

Assume that we get :

r_o = 5,67cm

$$\sin\varphi = \frac{l}{2r_0} = 0{,}882 \text{ hay } \varphi = 61^0 54'$$

3. Determine internal forces in the bar

We divide the bar into two segments

- Consider the segment 1: $(0 \le \alpha \le \varphi)$

Use the section (1-1) to cut the bar into haves. Consider the equilibrium of the left part. x_1, $y_1$are the co-ordinates of the centroid of the section (1-1) in the co-ordinate system x-y.

$x_1 = 0{,}5l - r_0.\sin(\varphi-\alpha) = 5-5{,}67\sin(\varphi-\alpha)$

$y_1 = f-r_0[1-\cos(\varphi-\alpha)]=5{,}67\cos(\varphi-\alpha)-2{,}67$

The internal forces on this section are:

$N_1 = -R_A.\sin(\varphi-\alpha) = -23{,}3.\sin(\varphi-\alpha)$

$Q_1 = R_A.\cos(\varphi-\alpha) = 23{,}3.\cos(\varphi-\alpha)$

$M_1 = -R_A.x_1 = 23{,}3[5-5{,}67.\sin(\varphi-\alpha)]$

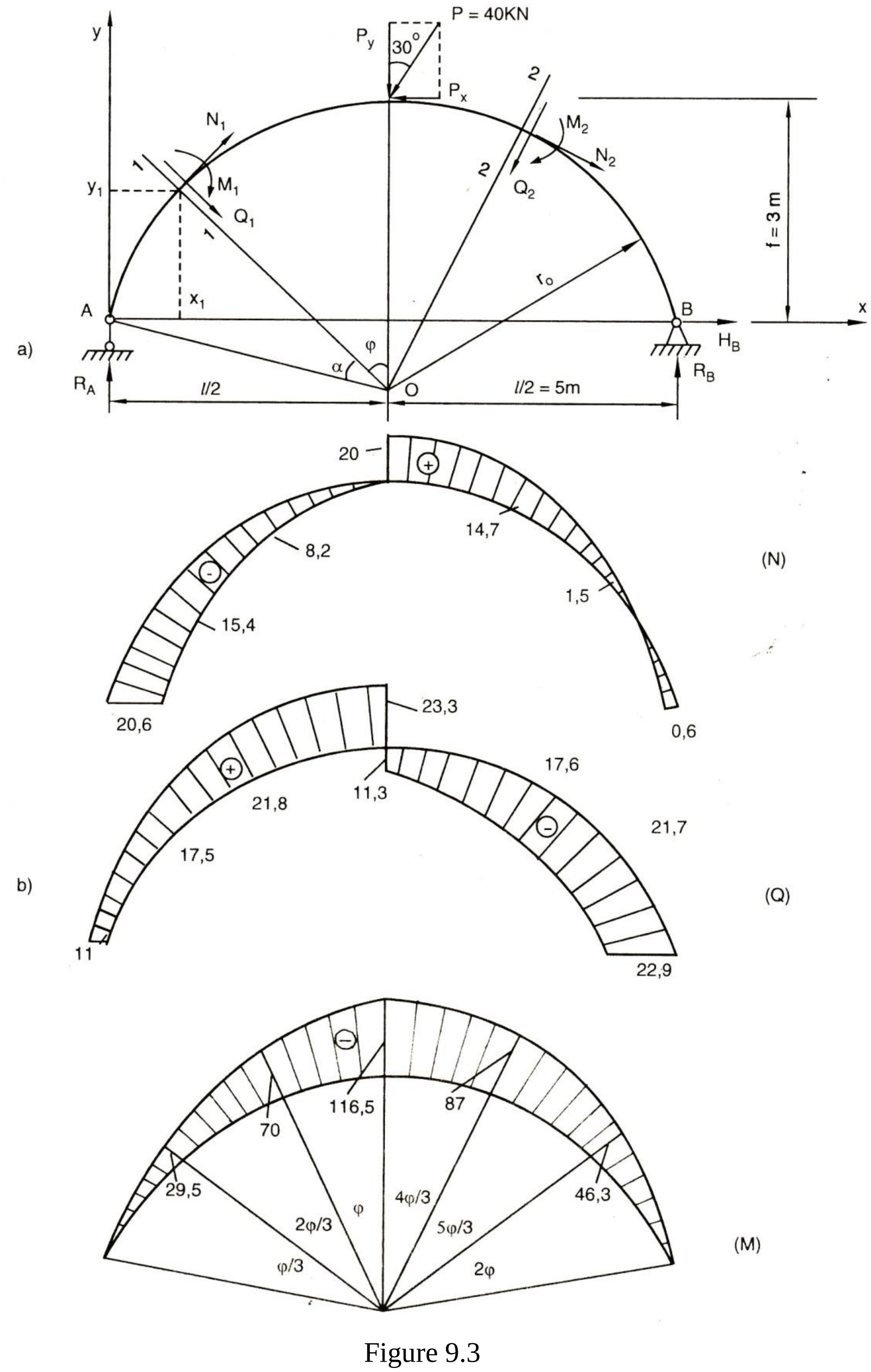


Figure 9.3

- Consider the segment 2: ($\varphi \leq \alpha \leq 2\varphi$)

Use the section (2-2). Consider the right part

x_2, y_2 are the co-ordinates of the centroid of the section (2-2) in the co-ordinate system x-y.

$$x_2 = \frac{l}{2} + r_0 \sin(\alpha - \varphi)$$

$$y_2 = f - r_0\left[1 - \cos(\alpha - \varphi)\right]$$

The internal forces on this section are:

$$N_2 = (R_A - P_y)\sin(\alpha - \varphi) + P_x \cos(\alpha - \varphi)$$

$$Q_2 = (R_A - P_y)\cos(\alpha - \varphi) - P_x \sin(\alpha - \varphi)$$

$M_2 = -R_A.x_2 + P_y(x_2 - 0,5l) + P_x(f - y_2)$

or

$N_2 = -11,3.\sin(\alpha-\varphi) + 20.\cos(\alpha-\varphi)$

$Q_2 = -11,3.\cos(\alpha-\varphi) - 20.\sin(\alpha-\varphi)$

$M_2 = -3,1 + 64,1.\sin(\alpha-\varphi) - 113,4.\cos(\alpha-\varphi)$

4. Draw internal force diagrams N, Q, M

To draw internal force diagrams, we form the table expressing the changes of internal forces by the angle α.

	$\alpha = 0$	$\alpha = \frac{\varphi}{2}$	$\alpha = \frac{2\varphi}{3}$	$\alpha = \varphi$	$\alpha = \frac{4\varphi}{3}$	$\alpha = \frac{5\varphi}{3}$	$\alpha = 2\varphi$
N_1(kN)	-20,6	-15,4	-8,2	0			
Q_1(kN)	11	17,5	21,8	23,3			
M_1(kNm)	0	-29,5	-70	-116,5			
N_2(kN)				20	14,7	7,5	-0,6
Q_2 (kN)				-11,3	-17,6	-21,7	-22,9
M_2 (kNm)				-116,5	-87	-46,3	0

9.2. Calculate curved bar subjected to pure flexure

9.2.1. Concept

9.2.2. Stress on cross-section

a. Basic hypotheses

As calculating the curved bar subjected to pure flexure, people have to reply on two following hypotheses:

- Hypotheses about plane section: before deformed, section is plane and perpendicular to the axis of bar. After deformed, section is still plane and perpendicular to the axis of bar.

- During the process of deforming, layers do not affect each other. It means that they do not push or press each other.

b. The deformation of bar and the relationship between stress and deformation

Assume that we have a curved bar subjected to pure flexure as in the figure 9.4a. On the bar, we split a segment by two sections (1-2) and (3-4). These two sections create together an angle dθ. Thanks to the effect of bending moment M, the bar is curved more (bending moment M is positive). At that moment, the bar appears neutral layers. The line of intersection between neutral layers and cross-sections is neutral line (figure 9.4b).

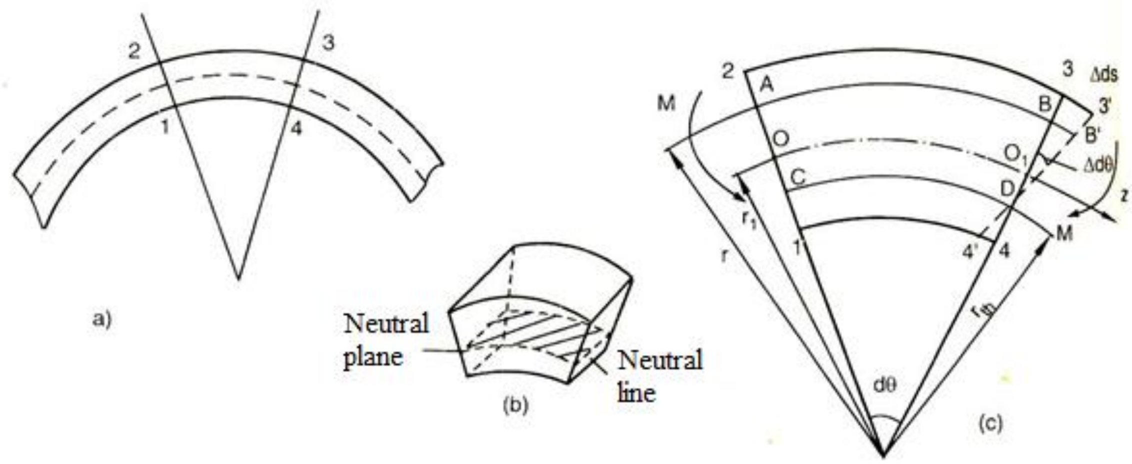


Figure 9.4

The radius of curvature of neutral layer CD is r_{th} and O, O_1 are the centroid of the sections (1-2) and (3-4) in turn.

After deformed, the section (3-4) is rotated. The relativeslope between the sections (1-2) and (3-4) is Δdθ (figure9.4c).

Consider the arbitrary layer AB of the segment having the radius of curvature r. The length of the layer before deformed is:

$$ds = r.d\theta$$

After deformed, the strain of the layer AB is:

$$\Delta ds = (r-r_{th})\Delta d\theta$$

The relative strain of the layer AB is:

$$\varepsilon = \frac{\Delta ds}{ds} = \frac{r - r_{th}}{r}.\frac{\Delta d\theta}{d\theta}$$

According to the hypothesis 2: the stress state of curved bar is single stress state.

According to Hook's law, we have: $\sigma = E.\varepsilon = E.\frac{\Delta d\theta}{d\theta}.\frac{r - r_{th}}{r}$ (a)

c. The position of neutral layer

According to the relationship between internal force and stress, we have:

$$N = \int_F \sigma dF \quad \text{(b)}$$

Substitute (a) in (b):

$$N = \int_F E.\frac{\Delta d\theta}{d\theta}.\frac{r - r_{th}}{r} dF \quad \text{(c)}$$

The ratio $\frac{\Delta d\theta}{d\theta}$ is constant in all points on section.

Bring constant quantities out of integration and note that N = 0. At that moment, (c) can write:

$$E.\frac{\Delta d\theta}{d\theta}\int_F \frac{r - r_{th}}{r} dF = 0$$

$$\int_F \frac{r - r_{th}}{r} dF = \int_F \frac{r}{r} dF - \int_F \frac{r_{th}}{r} dF = 0$$

Therefore:

$$r_{th} = \frac{F}{\int_F \frac{dF}{r}} \quad \text{(9-1)}$$

This is the formula to determine the radius of curvature of neutral layer. Here, neutral line does not go through the centroid of section. It turns to the center of curvature.

d. The expression of normal stress

We have: the sum of moments of internal forces towards the axis which is perpendicular to symmetric plane and goes through the center of curvature of bar equals bending moment M.

$$M = \int_F \sigma.r.dF \quad \text{(d)}$$

Subsitute (a) in (d):

$$M = \int_F E\frac{\Delta d\theta}{d\theta}.\frac{r - r_{th}}{r}.rdF \quad \text{(e)}$$

$$M = E\frac{\Delta d\theta}{d\theta}\int_F (r - r_{th})dF$$

Integration $\int_F (r - r_{th})dF$ is the static moment of section towards neutral axis.

Put: $\int_F (r - r_{th})dF = S$

S is calculated as below:

$$S = a.F \quad \text{(g)}$$

In which: $a = r_0 - r_{th}$ is distance between the centroid of section and neutral line.

Subsitute (g) in (e): $M = E\dfrac{\Delta d\theta}{d\theta}a.F$

$$E.\frac{\Delta d\theta}{d\theta} = \frac{M}{a.F} \quad \text{(h)}$$

Subsitute (h) in (a): $$\sigma = \frac{M}{a.F}\left(1 - \frac{r_{th}}{r}\right) \quad \text{(9-2)}$$

This is the formula to calculate normal stress at an arbitrary point which is at a distance r from the center of curvature.

In which:

M is bending moment on horizontal plane.

F is the area of cross-section.

r_{th} is the radius of curvature of neutral layer.

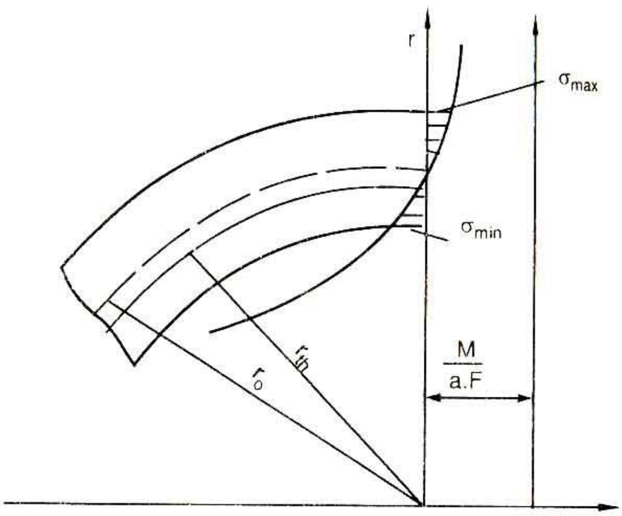
e. The diagram of stress distribution

According to the expression to calculate normal stress (8-2), we draw the diagram of stress distribution as in the figure 9.5.

It is the hyperbola having two asymptotes which are perpendicular each other.

- A line goes through the center of curvature and it is parallel to the normal of section.

- A line $\sigma = \dfrac{M}{a.F}$ is parallel to the symmetric axis of section.



Through the diagram, we realise that normal stresses increase slowly by the depth of beam from neutral layers to the outermost edge of section ($r > r_{th}$). In case of the points belonging to the center of curvature ($r < r_{th}$), normal stresses increase fast.

The direction of normal stress relates to the direction of bending moment.

The outermost points $r = r_{max}$ and the points with $r = r_{min}$ have the maximum compressive and tensile normal stress.

In case of the sections with constant width, the absolute value of the points inside is larger than the ones outside.

Figure 9.5

To reduce stress inside, people usually increase the horizontal dimension of section towards the center of curvature such as:

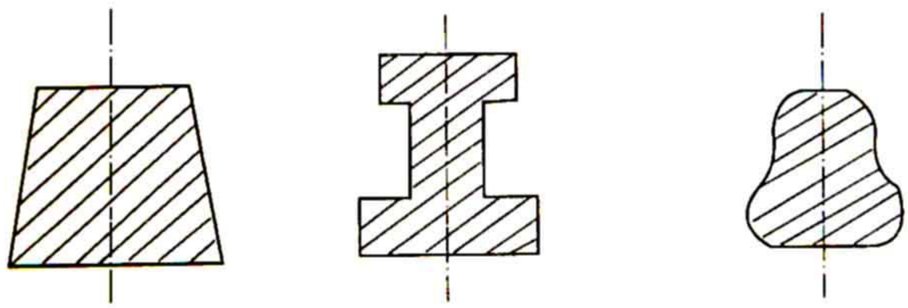


Figure 9.6

9.3. Determine the radius of curvature of neutral layer

In case of the cross-section having arbitrary shape, we use the formula $r_{th} = \dfrac{F}{\int_F \frac{dF}{r}}$ to calculate the radius of curvature of neutral layer.

9.3.1. Rectangular section

Assume that there is a rectangular section with the sides h, b. Its radiuses of curvature are $r_{max} = r_1$, $r_{min} = r_2$.

At radius r, we take a strip of area dF in horizontal direction. We have dF = b.dr

The area of section: F = b.h

Hence, according to (8-1), we have:

$$r_{th} = \frac{b.h}{b.\int_{r_2}^{r_1} \frac{dr}{r}}$$

$$r_{th} = \frac{b.h}{b.\ln\frac{r_1}{r_2}}$$

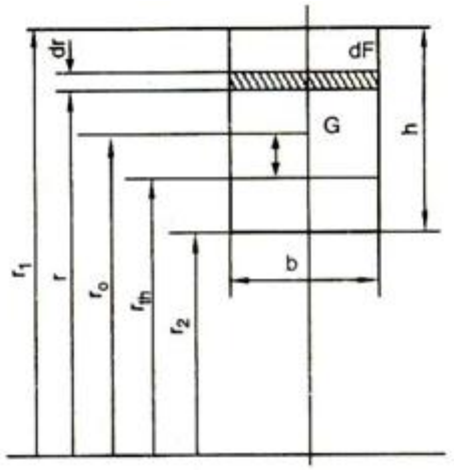


Figure 9.7

or $r_{th} = \dfrac{h}{\ln\frac{r_1}{r_2}}$ (9-3)

9.3.2. Circular section

Assume that a circle has diameter d.

The area of circle is $F = \dfrac{\pi d^2}{4}$. We take a strip of area dF in horizontal direction. We have dF = b_1.dr.

In which:

$$b_r = d.\cos\varphi$$

$$r = r_0 + \frac{d}{2}\sin\varphi$$

$$d_r = \frac{d}{2}\cos\varphi.d\varphi$$

$$dF = \frac{d^2}{2}\cos^2\varphi.d\varphi$$

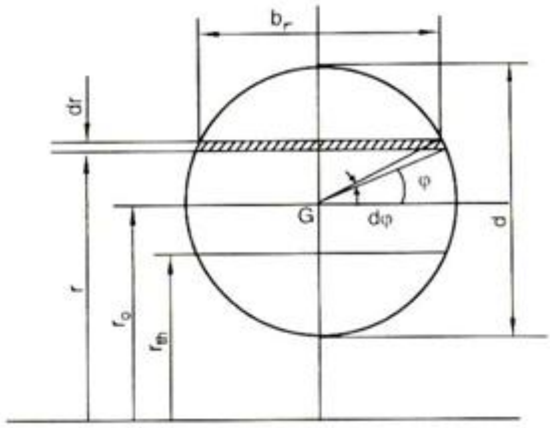


Figure 9.8

Subsitute the value dF, F and r in (8-1), we get:

$$r_{th} = \frac{d^2}{4\left(2r_0 - \sqrt{4r_0^2 - d^2}\right)} \quad (9\text{-}4)$$

9.3.3. Trapezoidal section

Assume that trapezium has height h, big bottom and small bottom b_2, b_1.

$$F = \frac{b_1 + b_2}{2}h$$

$$b_r = b_1 + (b_2 - b_1)\frac{r_1 - r}{r_1 - r_2}$$

$$dF = b_r.dr$$

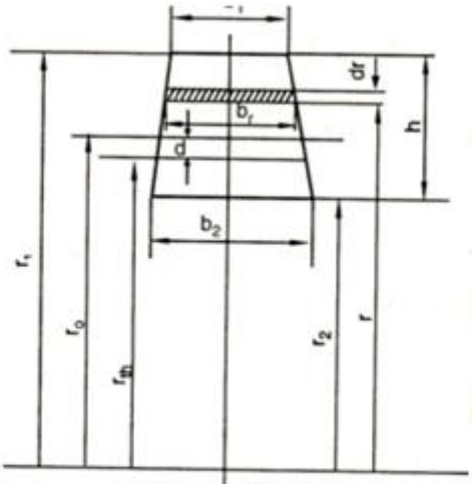


Figure 9.9

Apply the formula (8-1), we calculate r_{th}

$$r_{th} = \frac{\frac{b_1 + b_2}{2}.h}{\left[b_1 + r_1 \frac{b_2 - b_1}{h}\right]\ln\frac{r_1}{r_2} - (b_2 - b_1)} \qquad (9\text{-}5)$$

According to the formula (8-3), we get:

- As $b_1 = b_2$: this is rectangular section. This formula turns to the formula (9-3).

- As $b_1 = 0$; $b = b_2$, it means that triangular section has the bottom which rotates towards the center of curvature. At the moment, r_{th}will be:

$$r_{th} = \frac{b.h}{2r_1\frac{b}{h}\ln\frac{r_1}{r_2} - 2b} \qquad (9\text{-}6)$$

9.4. Calculate the curved bar subjected to complicated loads

In case of curved bar subjected to complicated loads, internal forces contain three components. They are longitudinal force N, shear force Q and bending moment M.

- Bending moment M creates normal stress and that stress is calculated by the formula (9-2). The normal stress caused by M is $\sigma_{(M)}$.

$$\sigma_{(M)} = \frac{M}{a.F}\left(1 - \frac{r_{th}}{r}\right)$$

- Longitudinal force N also creates normal stress and it is considered to distribute uniformly on section. The normal stress caused by N is $\sigma_{(N)}$.

$$\sigma_{(N)} = \frac{N}{F}$$

$$\sigma = \sigma_{(M)} + \sigma_{(N)} \qquad (9\text{-}7)$$

$$\sigma = \frac{N}{F} + \frac{M}{a.F}\left(1 - \frac{r_{th}}{r}\right)$$

- Shear force Q only creates shear stress and it does not affect the distribution of normal stress.

The value of shear stress can be calculated correctly gradually by Durapxki's formula for straight bar.

$$\tau = \frac{Q.S_x^c}{J_x.b^c} \qquad (9\text{-}8)$$

In which $J_x$is the static moment of section about axis x. Axis y concides with the symmetric axis of section and goes through the centre of curvature. x and y are centroidally principal axes of inertia of section.

Example 3: Build normal stress diagram on the cross-section of the bar having large curvature. Section is isosceles trapezium. Bending moment M = 40kNm (stretch outer layers), longitudinal force N = 0.

Solution:C is distance from the centroid of section to big bottom.

$$C = \frac{h}{3}.\frac{2b_1 + b_2}{b_1 + b_2} = \frac{20}{3}.\frac{2.10 + 20}{10 + 20} = 8{,}89\text{cm}$$

The radius of curvature of the bar's axis is:

$$r_0 = r_2 + C = 20 + 8{,}89 = 28{,}89\text{cm}$$

Accoding to the formula (9-5):

$$r_{th} = \frac{\frac{1}{2}(10+20)20}{\left[10+40.\frac{20-10}{20}\right]\ln\frac{20}{10}-(20-10)} = 27{,}79\text{cm}$$

a = r_o - r_{th} = 28,89 - 27,79 = 1,10cm

$$F = \frac{10+20}{2}.20 = 300\text{cm}^2$$

$$\sigma = \frac{M}{a.F}\left(1-\frac{r_{th}}{r}\right) = \frac{4000}{1{,}1.300}\left(1-\frac{27{,}79}{r}\right)$$

$$\sigma = 12{,}12 - \frac{337}{r}$$

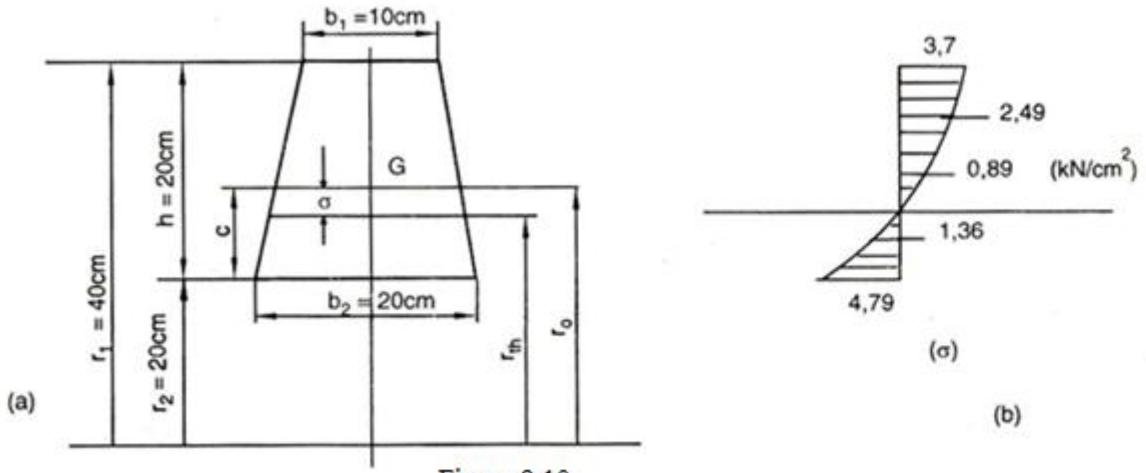


Figure 9.10

Atr = 40cm: $\sigma = 12{,}12 - \frac{337}{40} = 3{,}7\text{kN/cm}^2$

r = 35cm	σ = 2,49kN/cm^2
r = 30cm	σ = 0,89kN/cm^2
r = 25cm	σ = -1,36kN/cm^2
r = 20cm	σ = -4,73kN/cm^2

After having the value of stress, at each radius r, we draw stress diagram as in the figure 9.10b.

Example 4: Calculate the maximum stress and the minimum stress of crane hook as in the figure 9.11.

Know the weight P = 150kN.

The dimension of cross-section is given in the figure 9.11.

Solution: The most dangerous section is at AB. Because at that section, both M and N have the maximum value:

M = P.r_0; N = P

The area of cross-section F:

$$F = (8+3)\frac{12}{2} = 66\text{cm}^2 = 66.10^{-4}\text{m}^2$$

$$c = \frac{h}{3}.\frac{2b_1+b_2}{b_1+b_2} = \frac{12}{3}.\frac{6+8}{3+8} = 5{,}091\text{cm}$$

The radius of curvature $r_{th}$is:

$$r_{th} = \frac{\frac{8+3}{2}12}{\left(3+20\frac{5}{12}\right)\ln\frac{20}{8}-5} = \frac{66}{(3+8{,}333)(2{,}9957-2{,}0794)-5} = 12{,}265\text{cm}$$

Therefore:

$r_o = r_2 + c = 8 + 5{,}091 = 13{,}091$cm

$a = r_o - r_{th} \backslash\ 13{,}091 - 12{,}265 = 0{,}826$cm

$M = -P.r_o = -150.10^3.0{,}1309 = -19{,}63.10^3$Nm

$N = P = 150.10^3$N

The maximum compressive normal stress at the point B at the outer edge is

$r_{max} = r_1 = 20$cm

$$\sigma_{min} = \sigma_{(B)} = \frac{150.10^3}{66.10^{-4}} + \frac{(-19{,}63).10^3}{66.10^{-4}.82{,}6.10^{-4}}\left(1-\frac{12{,}265}{20}\right)$$

$\sigma_{min} = -115{,}8\text{MN/m}^2$

The maximum tensile normal stress at the point A:

$r_{min} = r_2 = 8$cm

$$\sigma_{max} = \sigma_{(A)} = \frac{150.10^3}{66.10^{-4}} + \frac{(-19{,}63).10^3}{66.10^{-4}.82{,}6.10^{-4}}\left(1-\frac{12{,}265}{8}\right)$$

$\sigma_{max} = 213{,}6\text{MN/m}^2$.

Note: *the value of a is very small in comparison with the value of $r_0$and r_{th}. Therefore, we need to accurately calculate the value of r_0 and $r_{th}$to ensure the accurate result of a.*

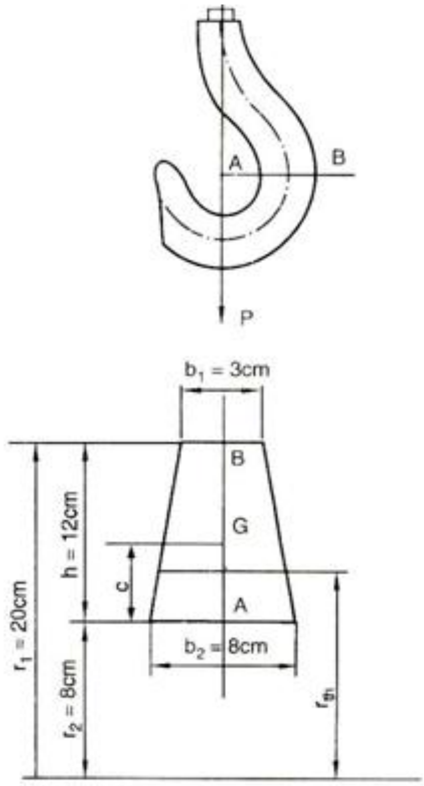


Figure 9.11

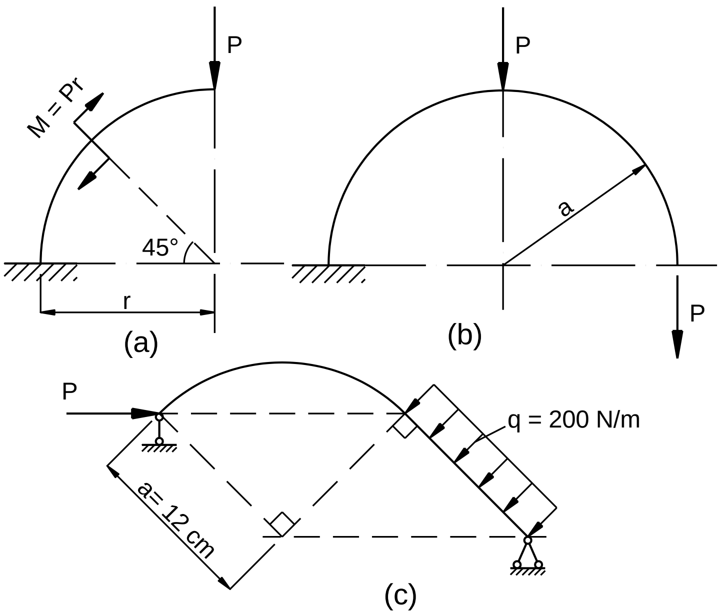
Theoretical questions

1. When we use section method to draw internal force diagram for curved bar, what is the difference from drawing internal force diagram for straight bar?
2. Clearly raise the distribution of stress on the cross-section of curved bar subjected to plane flexure. Write formula and explain the formula calculating normal stress on cross-section. Draw the diagram of stress distribution and raise the suitable shape of section.
3. Write formula and explain formula to determine the radius of curvature of neutral layer for rectangular section, circular section and trapezoidal section.
4. On the cross-section of curved bar subjected to complicated load, which stress exits? Write formula to calculate. Present the condition of strength of curved bar subjected to complicated load and the way to solve three basic problems.

Numerical problems

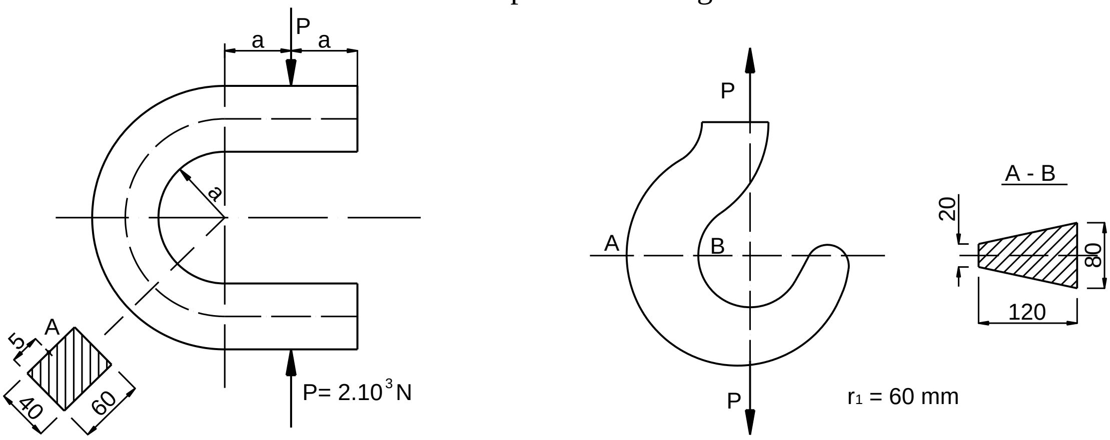
Exercise 1:

Draw bending moment diagram, longitudinal force diagram and shear force diagram of the following curved bars.



Exercise 2:

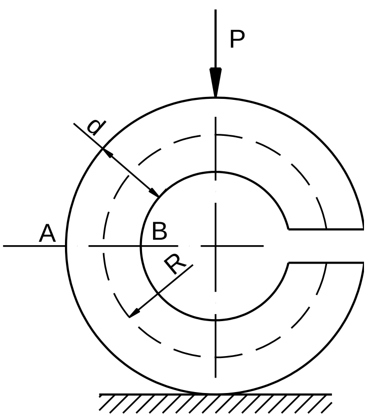
Determine the maximum tensile stress and the maximum compressive stress at dangerous section and the normal stress at the point A of dangerous section.



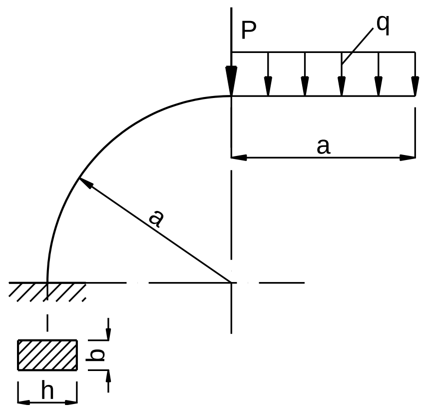
Exercise 3:

A cast iron ring has circular section as in the figure.

Determine allowable load. Know that $[\sigma]_{tens} = 6$ kN/cm^2; $[\sigma]_{comp} = 10$ kN/cm^2; cho R = 16 cm; d = 8 cm.



Exercise 4:A curved bar has rectangular section and it is subjected to load as in the figure. Know that P = 6 kN; q = 12 kN/m; a = 16 cm; $\frac{h}{b} = \frac{4}{3}$; [σ] = 200 MN/m^2.



Determine the dimension of cross-section.

APPENDIX 1 – LABORATORY MANUAL

TABLE OF CONTENTS

The number of experiments: …05………… The number of lessons: …05………

PART I - INTRODUCTION

1. The general target of experiments of the subject

Strength of materials as a basic subject in engineering field is defined as a branch of mechanics of deformable solids that deals with the behaviours of solid bodies subjected to various types of loadings. It provides the future civil engineers with the means of analyzing and designing so that all types of structures operate safely.

The method to study Strength of materials is association between theory and experiments. The study by experiments not only decreases or replaces some complicated calculations but also raises assumptions to establish formulas and checks the accuracy of the results found by theory.

The laboratory manual of Strength of materials is compiled in order to instruct students the most basic experiments of the subject. Hence, it helps students to get used to research method relying on experiments and understand the importance of experiments in study.

2. General introduction about equipment in the laboratory

The laboratory of Strength of materials has: versatilely tensile (compressive) machine, tensile (compressive) machine FM1000, torsion testing machine K5, two fatigue testing machines, two impact testing machines, deflection of string measuring machine, torsion testing table, plane bending testing table, axial compression testing table. Besides, there are measuring equipments: calipers, ruler, steel cutting pliers…

3. Progress and time to deploy experiments

After finishing the chapter "Torsion in round shaft", students start experimenting the first three lessons. After finishing the chapter "Buckling of columns", students experiment the last two lessons.

4. The assessment of the experimental results of students

The experimental results of students are assessed by answering questions in class, observing students during the process of experiments and checking reports.

5. The preparation of students

Before experimenting, students have to carefully study experiments. The leader of class prepares list of students, divide students into small groups and sends it to lecturer two weeks before experiments.

Experimen
tal curator

Pham Thi Thanh

PART II: DETAILED CONTENT OF EXPERIMENT
LESSON 4 -DETERMINE THE DEFLECTION OF STRAIGHT BAR SUBJECTED TO PLANELY HORIZONTAL FLEXURE

1.Thepurpose of experiment:

Determine the deflection and slope of the straight bar subjected to planely horizontal flexure by experiments. Hence, we can evaluate the accuracy of the deflection and slope calculated by theoretical methods.

2. Theoretical content

To determine the deflection and slope of the beam subjected to planely horizontal flexure, we have the following methods:

- Indefinite integration method
- Initial parameter method
- Artificial load method
- Morh's integration method associated with multiplying Veresaghin's diagram.

Use Morh's integration method associated with multiplying Veresaghin's diagram to calculate the deflection at B and the slope at A for the simple beam subjected to loads as in the figure 7. It is carried out by the following steps:

- Draw diagram M_x^m caused by the load.
- Establish state "k" and draw diagram $\overline{M}_x^k$
- Use Morh's integration method associated with multiplying Veresaghin's diagram, we have:

$$y_B = \frac{1}{EJ_x}\cdot\left(\frac{1}{2}\cdot\frac{Pl}{4}\cdot\frac{l}{2}\cdot\frac{2}{3}\cdot\frac{l}{4}\right).2 = \frac{Pl^3}{48EJ_x}$$

$$\varphi_A = \frac{1}{EJ_x}\left(\frac{1}{2}\cdot\frac{Pl}{4}\cdot\frac{l}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{Pl}{4}\cdot\frac{l}{2}\cdot\frac{1}{3}\right)$$

$$= \frac{Pl^2}{16EJ_x}$$

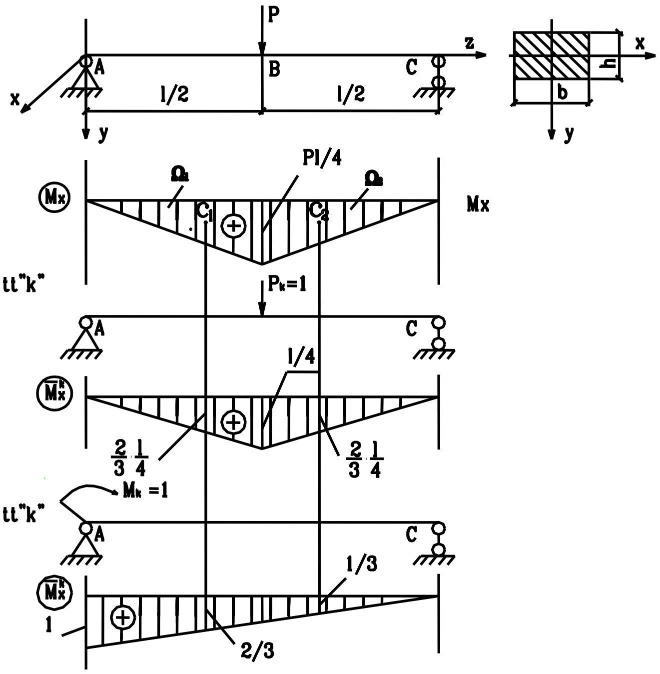


Figure 7

3. Experimental layout

The gauge K_1 determines the deflection at B. The gauge K_2 measures the displacement a. The slope at A is calculated by a, e.

$$\varphi_A = arctg\frac{a}{e}$$

4. Theprocedure of experiment

- Assemble experimental layout.
- Measure the dimensions of the bar: l, b, h, e.
- Put loads P in turn, read and write indexes on gauges.

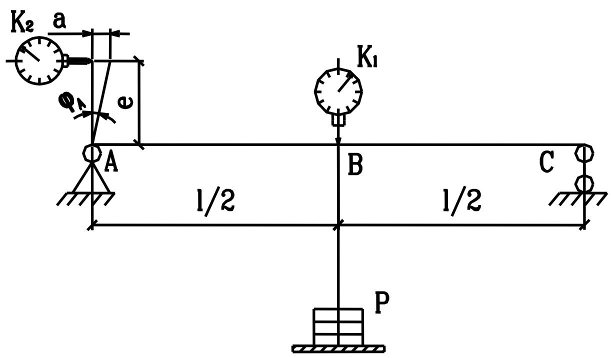


Figure 8

5. Experimental result

The experimental result is written in the table below:

P_i (N)	y_B^{th}	y_B^{ex}	φ_A^{th}	φ_A^{ex}	$\Delta y = \left\lvert \frac{y_B^{th} - y_B^{ex}}{y_B^{th}} \right\rvert 100\%$	$\Delta\varphi = \left\lvert \frac{\varphi_A^{th} - \varphi_A^{ex}}{\varphi_A^{th}} \right\rvert 100\%$
$P_1 = 4$ .... $P_5 = 20$						

6. Comments

- Comment the accuracy of experimental result.
- Analyse reasons.

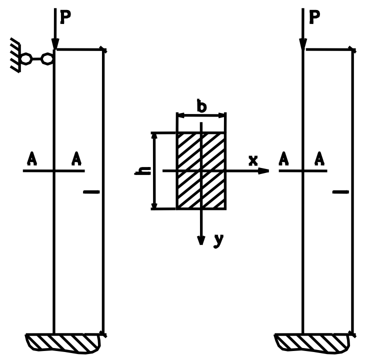
LESSON5-BUCKLING OF COLUMNS

1.Thepurpose of experiment:

Determine critical load by experiments and observe the stability of the column subjected to axial compressive load. Compare and check again the formula to calculate critical load by theory.

2. Theoretical content

Assume that there is the bar subjected to axial compressive load. The bar will be lost its stability as compressive load reaches critical load or compressive load is larger than critical load. Critical load is determined by Euler's formula:



$$P_{cr} = \frac{\pi^2 E J_{min}}{(\mu l)^2}$$

In which:

E - modulus of elasticity about tension or compression

J_{min}-the minimum moment of inertia

l -the length of bar

μ - the factor depending on supports

μ = 0.7 μ = 2

3. Experimental layout

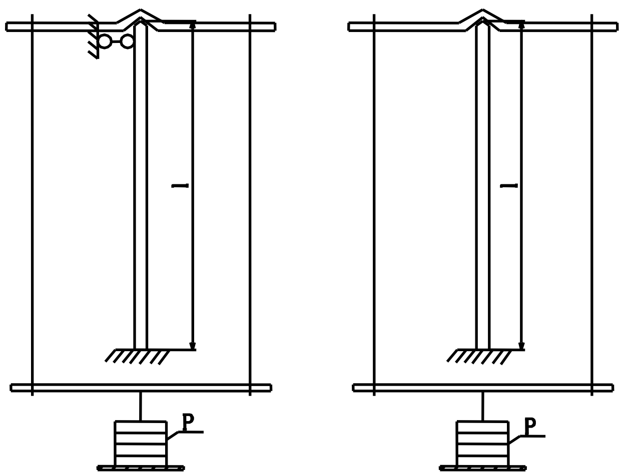


Figure 9

4. *The*procedure of experiment

- Measure the dimensions of the bar:l_1, b_1, h_1, l_2, b_2, h_2.
- Put loads P in turn until the bar is lost its stability. Write the value of the critical load of bar.

5. Experimental result

The experimental result is written in the table below:

Layout	P_{cr}^{ex} (N)	P_{cr}^{th} (N)	$\Delta P_{cr} = \frac{P_{cr}^{th} - P_{cr}^{ex}}{P_{cr}^{th}} 100\%$
1			
2			

6. **Comments**

- Comment the accuracy of experimental result.

- Analyse reasons.

LESSON 6-DETERMINE THE DEFLECTION OF THE BEAM SUBJECTED TO OBLIQUE BENDING

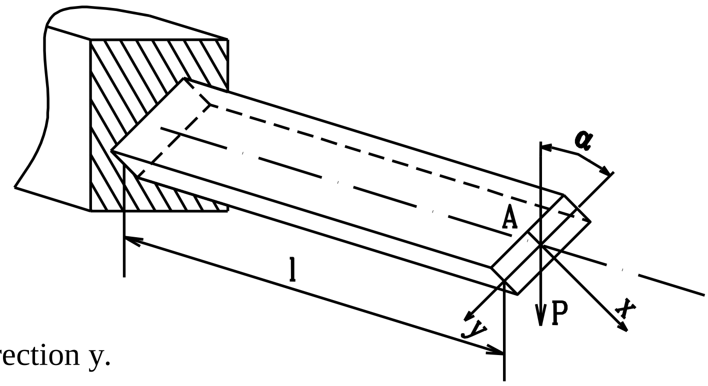
1. **ThePurpose of experiment**:

Determine the total deflection of the bar subjected to oblique bending by experiments. On that basis, we evaluate the accuracy of the total deflection determined by theory.

2. **Theoretical content**

Consider the cantilever beam subjected to oblique bending as in the figure 10. Consider section at free end A: $f_A = \sqrt{f_x^{A2} + f_y^{A2}}$. Use the methods of determining the displacement (deflection) of the beam subjected to plane flexure, we get:

$$f_x^A = \frac{P.\sin\alpha.l^3}{3EJ_y}$$



$$f_y^A = \frac{P.\cos\alpha.l^3}{3EJ_x}.$$

In which:

$f_y$is the deflection at A following the direction y.

$f^A$is the total deflection at A.Figure 10

3. **Experimental layout**

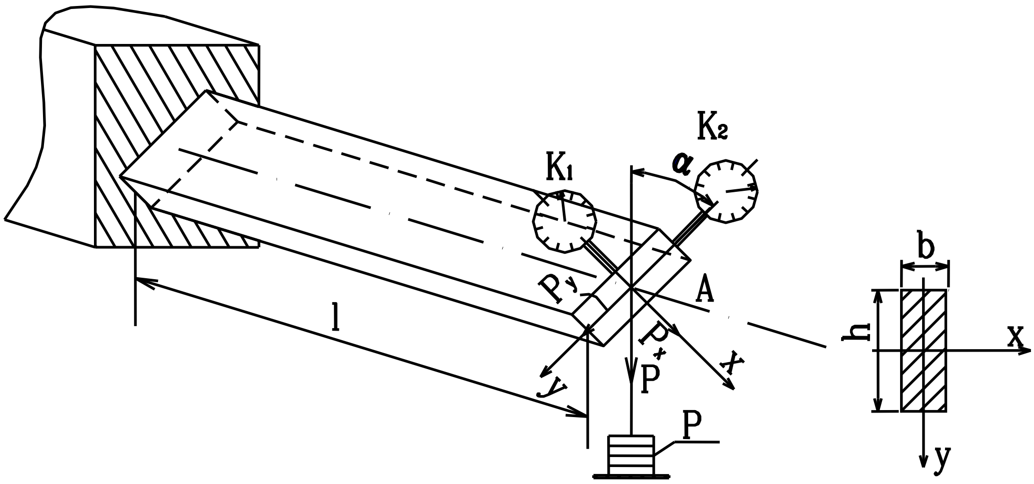


Figure 11

The gauge K_1 measures the deflection following the direction x. The gauge K_2 measures the deflection following the direction y. After measuring f_x, f_y by experiments,we determine the value of total deflection:

$$f = \sqrt{f_x^2 + f_y^2}$$

4.**The Procedure of experiment**

- Measure the dimensions of cross-section: l, b, h.
- Put loads P in turn, read and write indexes on gauges.

5. **Experimental result**

The experimental result is calculated and written in the table below:

P_i (N)	f_x^{th}	f_x^{ex}	f_y^{th}	f_y^{ex}	f_{tot}^{th}	f_{tot}^{ex}	$\Delta f = \frac{f_{tot}^{th} - f_{tot}^{ex}}{f_{tot}^{th}} 100\%$
$P_1 = 4$ .... $P_5 = 20$							

6. Comments

- Comment the accuracy of experimental result.
- Analyse reasons.

The sample of laboratory report

The cover of laboratory report is published the same sample. The content of report is handwritten and framed. The content of report needs to present purpose, content, internal force diagram, used methods to calculate theory clearly. The layout of experiment, necessary dimensions, the methods of calculation and the results of experiments, comments about the accuracy of theoretic formula are presented obviously in the report.

APPENDIX 02

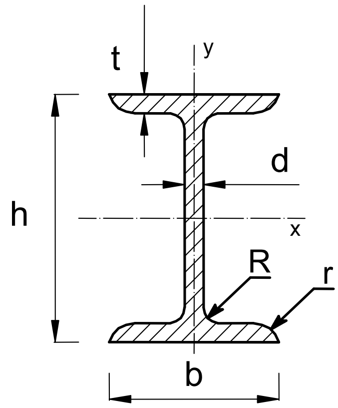
VIETNAM MARITIME UNIVERSITY

Subject of Strength of materials

PROPERTIES OF SHAPED STEEL

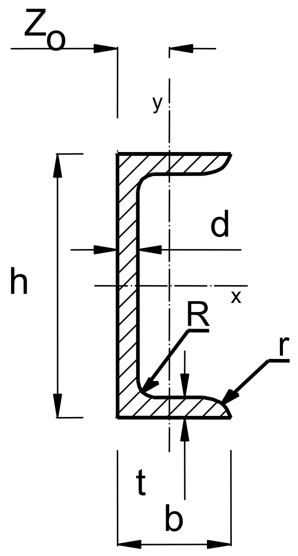
I -Section

ГОСТ 8239-56



The sign number of section	Gravity N/m	Dimensions (mm)						The area of section cm^2	The properities of area						
									x-x				y-y		
		h	b	d	t	R	r		J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm
10	111	100	70	4,5	7,2	7,0	3,0	14,2	244	48,8	4,15	28,0	35,3	10	1,58
12	130	120	75	5,0	7,3	7,5	3,0	16,5	403	67,2	4,94	38,5	43,8	11,7	1,63
14	148	140	82	5,0	7,5	8,0	3,0	18,9	632	90,3	5,78	51,5	58,2	14,2	1,75
16	169	160	90	5,0	7,7	8,5	3,5	21,5	945	118	6,63	67,0	77,6	17,2	1,90
18	187	180	95	5,0	8,0	9,0	3,5	23,8	1330	148	4,47	83,7	94,6	19,9	1,99
18a	199	180	102	5,0	8,2	9,0	3,5	25,4	1440	160	5,53	90,1	119	23,3	2,06
20	207	200	100	5,2	8,2	9,5	4,0	26,4	1810	181	8,27	102	112	22,4	2,17
20a	222	200	110	5,2	8,3	9,5	4,0	28,3	1970	197	8,36	111	148	27,0	2,29
22	237	220	110	5,3	8,6	10,0	4,0	30,2	2530	230	9,14	130	155	28,2	2,26
22a	254	220	120	5,3	8,8	10,0	4,0	32,4	2760	251	9,23	141	203	33,8	2,50
24	273	240	115	5,6	9,5	10,5	4,0	34,8	3460	289	9,97	163	198	34,5	2,37
24a	294	240	125	5,6	9,8	10,5	4,0	37,5	3800	317	10,1	178	260	41,6	2,63
27	315	270	125	6,0	9,8	11,0	4,5	40,2	5010	371	11,2	210	260	41,5	2,54
27a	339	270	135	6,0	10,2	11,0	4,5	43,2	5500	407	11,3	229	337	50,0	2,80
30	365	300	135	6,5	10,2	12,0	5,5	46,5	7080	472	12,3	268	337	49,9	2,69
30a	392	300	145	6,5	10,7	12,0	5,5	49,9	7780	518	12,5	292	346	60,1	2,95
33	422	330	140	7,0	11,2	13,0	5,5	53,8	9840	597	13,5	339	419	59,9	2,79
36	486	360	145	7,5	12,3	14,0	6,0	61,9	13380	743	14,7	423	516	71,1	2,89
40	561	400	155	8,0	13,0	15,0	6,0	71,9	18930	974	16,3	540	666	75,9	3,05
45	652	450	160	8,6	14,2	16,0	7,0	83,0	27450	1220	18,2	699	807	101	3,12
50	761	500	170	9,3	15,2	17,0	7,0	96,9	39120	1560	20,1	899	1040	122	3,28
55	886	550	180	10,0	16,5	18,0	7,0	113	54810	1990	20,2	1150	1350	150	3,46
60	1030	600	190	10,8	17,8	20,0	8,0	131	75010	2500	23,9	1440	1720	181	3,62
65	1190	650	200	11,7	19,2	22,0	9,0	151	100840	3100	25,8	1790	2170	217	3,79
70	1370	700	210	12,7	20,8	24,0	10,0	174	133890	3830	27,7	2220	2730	260	3,76
70a	1580	700	210	15,0	24,0	24,0	10,0	202	152700	4360	27,5	2550	3240	309	4,01
70b	1840	700	210	17,5	28,2	24,0	10,0	234	175350	5010	27,4	2940	3910	373	4,09

VIETNAM MARITIME UNIVERSITY
Subject of Strength of materials



PROPERTIES OF SHAPED STEEL
C -Section
ГОСТ 8240-56

The sign number of section	Gravity N/m	Dimensions (mm)						The area of section s cm^2	The properties of area							Z_0
									x-x				y-y			
		h	b	d	t	R	r		J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm	
6	54,2	50	37	4,5	7,0	6,0	2,5	6,90	26,1	10,4	1,94	6,36	8,41	3,59	1,10	1,36
6,5	65,0	65	40	4,5	7,4	6,0	2,5	8,28	54,5	16,8	2,57	10,0	11,9	4,58	1,20	1,40
8	77,8	80	45	4,8	7,4	6,5	2,5	9,91	99,9	25,0	3,17	14,8	17,8	5,89	1,34	1,48
10	92,0	100	50	4,8	7,5	7,0	3,0	11,7	187	37,3	3,99	21,9	25,6	7,42	1,48	1,55
12	108,0	120	54	5,0	7,7	7,5	3,0	13,7	313	52,2	4,78	30,5	34,4	9,01	1,58	1,59
14	123,0	140	58	5,0	8,0	8,0	3,0	15,7	489	69,8	5,59	40,7	45,1	10,9	1,70	1,66
14a	132,0	140	62	5,0	8,5	8,0	3,0	16,9	538	76,8	5,65	44,6	56,6	13,0	1,83	1,84
16	141,0	160	64	5,0	8,3	8,5	3,5	18,0	741	92,6	6,42	53,7	62,6	13,6	1,87	1,79
16a	151,0	160	68	5,0	8,8	8,5	3,5	19,3	811	101	6,48	58,5	77,3	16,0	2,00	1,98
18	161,0	180	70	5,0	8,7	9,0	3,5	20,5	1080	120	7,26	69,4	85,6	16,9	2,04	1,95
18a	172,0	180	74	5,0	9,2	9,0	3,5	21,9	1180	131	7,33	75,2	104	19,7	2,18	2,13
20	184,0	200	76	5,2	9,0	9,5	4,0	23,4	1520	152	8,07	87,8	113	20,5	2,20	2,07
20a	196,0	200	80	5,2	9,9	9,5	4,0	25,0	1660	166	8,15	95,2	137	24,0	2,34	2,57
22	209,0	220	82	5,3	9,9	10,0	4,0	26,7	2120	193	8,91	111	151	25,4	2,38	2,24
22a	225,0	220	87	5,3	10,2	10,0	4,0	28,6	2320	211	9,01	121	186	29,9	2,55	2,47
24	240,0	240	90	5,6	10,0	10,5	4,0	30,6	2900	242	9,73	139	208	31,6	2,60	2,42
24a	258,0	240	95	5,6	10,7	10,5	4.0	32,9	3180	265	9,84	151	254	37,2	3,78	2,67
27	277,0	270	95	6,0	10,5	11	4,5	35,2	4160	308	10,9	178	262	37,3	2,73	2,47
30	318,0	300	100	6,5	11,0	12	5,0	40,5	5810	387	12,0	224	327	43,6	2,84	2,52
33	365,0	330	105	7,0	11,7	13	5,0	46,5	7980	484	13,1	281	410	51,8	2,97	2,59
36	419,0	360	110	7,5	12,6	14	6,0	53,4	10820	601	14,2	350	513	61,7	3,10	2,68
40	483,0	400	115	8,0	13,5	15	6,0	61,5	15220	761	15,7	444	642	73,4	3,23	2,75

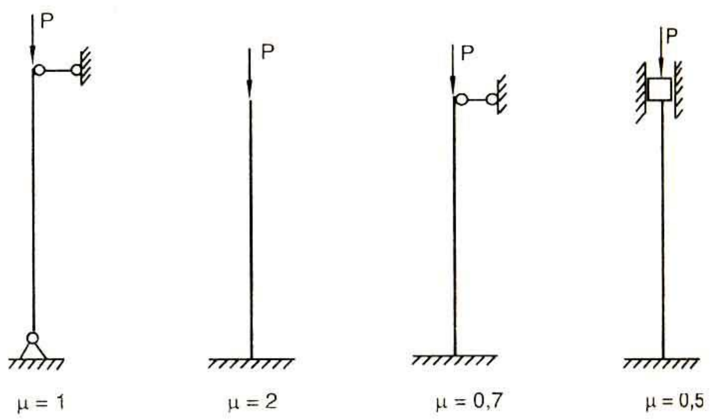
1. **THE INDEX OF λ**

The slenderness ratio λ	Trị số φ đối với				
	Steel CT 2,3,4	Steel CT5	Alloy steel	Cast iron	Timber
10	0,99	0,98	1	1	1
20	0,96	0,95	0,95	0,91	0,97
30	0,94	0,92	0,91	0,81	0,93
40	0,92	0,89	0,87	0,69	0,87
50	0,89	0,86	0,83	0,54	0,80
60	0,86	0,82	0,79	0,44	0,71
70	0,81	0,76	0,72	0,34	0,6
80	0,75	0,7	0,65	0,26	0,48
90	0,69	0,62	0,55	0,2	0,38
100	0,6	0,51	0,43	0,16	0,31
110	0,52	0,43	0,35		0,25
120	0,45	0,36	0,3		0,22
130	0,4	0,33	0,26		0,18
140	0,36	0,29	0,23		0,16
150	0,32	0,26	0,21		0,14
160	0,29	0,24	0,19		0,12
170	0,26	0,21	0,17		0,11
180	0,23	0,19	0,15		0,10
190	0,21	0,17	0,14		0,09
200	0,19	0,16	0,13		0,08

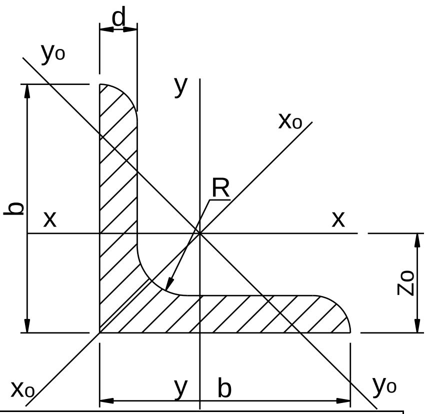
2. **THE MATERIAL INDEX (λ_o, λ_1, E, a, b)**

Material	λ_o	λ_1	E (KN/cm^2)	a (KN/cm^2)	b (KN/cm^2)
Steel CT2, CT3, CT4	100	70	2.10^4	31	0,114
Steel CT5	100	72	2.10^4	46,4	0,326
Timber	70	40	10^3	3,68	0,0265
Cast iron	80	30	$1,5.10^4$	77,6	0,415

3. **THE INDEX OF μ**



VIETNAM MARITIME UNIVERSITY
Subject of Strength of materials



PROPERTIES OF SHAPED STEEL
Equal angle steel
ГОСТ 8240-56

The sign number of section	Dimensions, mm				The area of section cm^2	Gravity per meter N	The properties of area							
							x - x		y - y		y_o - y_o		$x_0 - x_0$	
	b	d	r	R			J_x cm^4	i_x cm	J_{xomax} cm^4	i_{xomax} cm	J_{xomin} cm^4	i_{xomin} cm	J_{xomax} cm^4	Z_o cm
2	20	3	3.5	1.2	1.13	8.9	0.4	0.59	0.63	0.75	0.17	0.39	0.81	0.6
		4			1.46	11.5	0.5	0.58	0.78	0.73	0.22	0.38	1.09	0.64
2.5	25	3	3.5	1.2	1.43	11.2	0.81	0.75	1.29	0.95	0.34	0.49	1.57	0.73
		4			1.86	14.6	1.03	0.74	1.62	0.93	0.44	0.48	2.11	0.76
2.8	28	3	4	1.3	1.62	12.7	1.16	0.85	1.84	1.07	0.48	0.55	2.2	0.8
3.2	32	3	4.5	1.5	1.86	14.6	1.77	0.97	2.8	1.23	0.74	0.63	3.26	0.89
		4			2.43	19.1	2.26	0.96	3.58	1.21	0.94	0.62	4.39	0.94
3.6	36	3	4.5	1.5	2.1	16.5	2.56	1.1	4.06	1.39	1.06	0.71	4.64	0.99
		4			2.75	21.6	3.29	1.09	5.21	1.38	1.36	0.7	6.24	1.04
4	40	3	5	1.7	2.35	18.5	3.55	1.23	5.63	1.55	1.47	0.79	6.35	1.09
		4			3.08	24.2	4.58	1.22	7.26	1.53	1.9	0.78	8.53	1.13
4.5	45	3	5	1.7	2.65	20.8	5.13	1.39	8.13	1.75	2.12	0.89	9.04	1.21
		4			3.48	27.3	6.63	1.38	10.05	1.74	2.74	0.89	12.1	1.26
		5			4.29	33.7	8.03	1.37	12.7	1.72	3.33	0.88	15.3	1.3
5	50	3	5.5	1.8	2.96	23.2	7.11	1.55	11.3	1.95	2.95	1	12.4	1.33
		4			3.89	30.5	9.21	1.54	14.6	1.94	3.8	0.99	16.6	1.38
		5			4.8	37.7	11.2	1.53	17.8	1.92	4.63	0.98	20.9	1.42
5.6	56	3.5	6	2	3.66	30.3	11.6	1.73	18.4	2.18	4.8	1.12	20.3	1.5
		4			4.38	34.4	13.1	1.73	20.8	2.18	5.41	1.11	23.3	1.52
		5			5.41	42.5	16	1.72	25.4	2.16	6.59	1.1	29.2	1.57
6	63	4	7	2.3	4.96	39	18.9	1.95	29.9	2.45	7.81	1.25	33.1	1.69
		5			6.13	48.1	23.1	1.94	36.6	2.44	9.52	1.25	41.5	1.74

		6			7.28	57.2	27.1	1.93	42.9	2.43	11.2	1.24	50	1.78
7	70	4.5	8	2.7	6.2	48.7	29	2.16	46	2.72	12	1.39	51	1.88
		5			6.86	53.8	31.9	2.16	50.7	2.72	13.2	1.39	56.7	1.9
		6			8.15	63.9	37.6	2.15	59.6	2.71	15.5	1.38	68.4	1.94
		7			9.42	73.9	43	2.14	68.2	2.69	17.8	1.37	80.1	1.99
		8			10.7	83.7	48.2	2.13	76.4	2.68	20	1.37	91.9	2.02
7.5	75	5	9	3	7.39	58	39.5	2.31	62.6	2.91	16.4	1.49	69.6	2.02
		6			8.78	68.9	46.6	2.3	73.9	2.9	19.3	1.48	83.9	2.06
		7			10.1	79.6	53.3	2.29	84.6	2.89	22.1	1.48	98.3	2.1
		8			11.5	90.2	59.8	2.28	94.9	2.87	24.8	1.47	113	2.15
		9			12.8	101	66.1	2.27	105	2.86	27.5	1.46	127	2.18
8	80	5.5	9	3	8.63	67.8	52.7	2.47	83.6	3.11	21.8	1.59	93.2	2.17
		6			9.38	73.6	57	2.47	90	3.11	23.5	1.58	102	2.19
		7			10.8	85.1	65.3	2.45	104	3.09	27	1.58	119	2.23
		8			12.3	96.5	73.4	2.44	116	3.08	30.3	1.57	137	2.27
9	90	6	10	3.3	10.6	83.3	82.1	2.78	130	3.5	34	1.79	1.45	2.43
		7			12.3	96.4	94.3	2.77	150	3.49	38.9	1.78	1.69	2.47
		8			13.9	109	106	2.76	168	3.48	43.8	1.77	1.94	2.51
		9			15.6	122	118	2.75	186	3.46	48.6	1.77	2.19	2.55
10	100	6.5	12	4	12.8	101	122	3.09	193	3.88	50.7	1.99	214	2.68
		7			13.8	108	131	3.08	207	3.88	54.2	1.98	231	2.71
		8			15.6	122	147	3.07	233	3.87	60.9	1.98	265	2.75
		10			19.2	151	179	3.05	284	3.84	74.1	1.96	330	2.83
		12			22.8	179	209	3.03	331	3.81	86.9	1.95	402	2.91
		14			26.3	206	237	3	375	3.78	99.3	1.94	472	2.99
		16			29.7	233	264	2.98	416	3.74	112	1.94	542	3.06
11	110	7	12	4	15.2	119	176	3.4	279	4.29	72.7	2.19	308	2.96
		8			17.2	135	198	3.39	315	4.28	81.8	2.18	353	3
12.5	125	8	14	4.6	19.7	155	294	3.87	467	4.87	122	2.49	516	3.36
		9			22	173	327	3.86	520	4.86	135	2.48	582	3.4
		10			24.3	191	360	3.85	571	4.84	149	2.47	649	3.45
		12			28.9	227	422	3.82	670	4.82	174	2.46	782	3.53
		14			33.4	262	482	3.8	764	4.78	211	2.45	916	3.61
		16			37.8	296	539	3.78	853	4.75	224	2.44	1051	3.68
14	140	9	14	4.6	24.7	194	466	4.34	739	5.47	192	2.79	818	3.78
		10			27.3	215	512	4.33	814	5.46	211	2.78	914	3.82

		12			32.5	255	602	4.21	957	5.43	248	2.76	1097	3.9
16	160	10	16	5.3	31.4	247	774	4.96	1229	6.25	319	3.19	1356	4.3
		11			34.4	270	844	4.95	1341	6.24	348	3.18	1494	4.35
		12			37.4	294	913	4.94	1450	6.23	376	3.17	1633	4.39
		14			43.3	340	1046	4.92	1662	6.2	431	3.16	1911	4.47
		16			49.1	385	1175	4.89	1866	6.17	485	3.14	2191	4.55
		18			54.8	430	1299	4.87	2061	6.13	537	3.13	2472	4.63
		20			60.4	474	1419	4.85	2248	6.1	589	3.12	2756	4.7
18	180	11	16	5.3	38.8	305	1216	5.6	1933	7.06	500	3.59	2128	4.85
		12			42.2	331	1317	5.59	2093	7.04	540	3.58	2324	4.89
20	200	12	18	6	47.1	370	1823	6.22	2896	7.84	749	3.99	3182	5.37
		13			50.9	399	1961	6.21	3116	7.83	805	3.98	3452	5.42
		14			54.6	428	2097	6.2	3333	7.81	861	3.97	3722	5.46
		16			62	487	2326	6.17	3755	7.78	970	3.96	4264	5.54
		20			76.5	601	2871	6.12	4560	7.72	1182	3.93	5355	5.7
		25			94.3	740	3466	6.06	5494	7.63	1432	3.91	6733	5.89
		30			111.5	876	4020	6	6351	7.55	1688	3.89	8130	6.07
22	220	14	21	7	60.4	474	2814	6.83	4470	8.6	1159	4.38	4941	5.93
		16			68.6	538	3157	6.81	5045	8.58	1306	4.36	5661	6.02
25	250	16	24	8	78.4	615	4717	7.76	7492	9.78	1942	4.98	8286	6.75
		18			87.7	689	5247	7.73	8337	9.75	2158	4.96	9342	6.83
		20			97	761	5765	7.71	9160	9.72	2370	4.94	10401	6.91
		22			116.1	833	6270	7.69	9961	9.69	2579	4.93	11464	7
		25			119.7	940	7006	7.65	11125	9.64	2887	4.91	13064	7.11
		28			133.1	1045	7717	7.61	12244	9.59	3190	4.89	14674	7.23
		30			142	1114	8117	7.59	12965	9.56	3389	4.89	15753	7.31