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Requirements and detailed content

Name of module: Strength of materials 1

Module code: 18502

a. Number of credits: 03 credits **ASSIGNMENT** ☐ **PROJECT** ☐

b. Department: **Strength of materials**

c. Time distribution:

- Total: 48 lessons. - Theory: 26 lessons.
- Experiment: 6 lessons. - Exercise: 14 lessons.
- Assignment/Project instruction: 0 lesson. - Test: 2 lessons.

d. Prerequisite to register the module: After studying theoretical mechanics.

e. Purpose and requirement of the module:

Knowledge:

Supply students with knowledge about the fundamental concepts of the subject, the geometric properties of cross-sections, the mechanical properties of materials and methods to determine them. As a result, the subject brings out methods to calculate the strength, the stiffness of construction parts and machine parts in basic load-resistant manners.

Skills:

- Be able to correctly think, analyse, evaluate the load-resistant state of construction parts, machine parts.
- Be capable of applying the knowledge of the subject to solve practical problems.
- Be able to solve the basic problems of the subject proficiently.

Job attitude:

- Obviously understand the important role of the subject in technical fields. As a result, students have serious, active attitude and try their best in study.

f. Describe the content of the module:

Strength of materials 1 module consists of content below:

- Chapter 1: The fundamental concepts
- Chapter 2: Axially loaded bar
- Chapter 3: The properties of areas
- Chapter 4: Torsion in round shaft
- Chapter 5: Flexure of initially straight beam

g. Compiler: **MSc Nguyen Hong Mai, Strength of materials Department – Basic Science Faculty**

h. Detailed content of the module:

CHAPTER	LESSON DISTRIBUTION					
	SUM	THEORY	EXERCISE	EXPERIMENT	INSTRUCTION	TEST
Chapter 1. The fundamental concepts	**9**	**6**	**3**			
1.1. Objective and the researched object of the subject		0.5				
1.2. Assumptions and fundamental concepts		0.5				

CHAPTER	LESSON DISTRIBUTION					
	SUM	THEORY	EXERCISE	EXPERIMENT	INSTRUCTION	TEST
1.3. External force		1				
1.4. Internal force		3				
1.5. Stress		1				
Exercises			3			
Self-taught content (16 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Do exercises at the end of the chapter (in detailed lecture notes.*						
Chapter 2. Axially loaded bar	**10**	**5**	**3**	**2**		
2.1. Concepts		0.5				
2.2. Stress on cross-section		0.5				
2.3. The strain and deformation of axially loaded bar		1				
2.4. The mechanical properties of materials		1				
2.5. Compute axially loaded bar		2				
Exercises			3			
Self-taught content (20 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 2.6 and 2.7 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes.*						
Chapter 3 : Properties of areas	**7**	**4**	**2**			**1**
3.1. The properties of areas		0.5				
3.2. Moment of inertia of some popular cross-sections.		0.5				
3.3. The formulas of transfering axes parallel to the initial axes of static moment and moment of inertia.		0.5				
3.4. The formulas of rotating axes of moment of inertia – principal axes of inertia		0.5				
3.5. Determine moment of inertia of compound sections		2				
Exercises			2			
Periodic test						1
Self-taught content (10 lessons): *- Read the content of lessons (in detailed lecture notes) before school.*						

CHAPTER	LESSON DISTRIBUTION					
	SUM	THEORY	EXERCISE	EXPERIMENT	INSTRUCTION	TEST
- Read item 3.4, 3.5, 3.6 in lecture notes [1] in section k by yourselve. *- Do exercises at the end of the chapter (in detailed lecture notes.*						
Chapter 4: Torsion in round shaft	**9**	**4**	**3**	**2**		
4.1. Concepts		0.5				
4.2. Stress on cross-section		0.5				
4.3. The strain and displacement of cross-section		1				
4.4. Compute the spring helical, cylindrical, having small pitch		1				
4.5. Compute the round shaft subjected to torsion		1				
Exercises			3			
Self-taught content (16 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 4.3 and 4.8 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes.*						
Chapter 5: Flexure of initially straight beam	**13**	**7**	**3**	**2**		**1**
5.1. Concepts		0.5				
5.2. Pure bending or simple bending		1				
5.3. Plane bending		2				
5.4. The strength of planely bended beam		2				
5.5. The deflection of beam		0.5				
5.6. Compute planely bended beam thanks to the condition of stiffness		1	3			
Exercises				3		
Periodic test						1
Self-taught content (16 lessons): *- Read the content of lessons (in detailed lecture notes) before school.* *- Read item 5.4 and 5.5.3 in lecture notes [1] in section k by yourselve.* *- Do exercises at the end of the chapter (in detailed lecture notes.*						

i. Describe manner to assess the module

- To take the final exam, students have to ensure all three conditions:

+ Attend class 75% more than total lessons of the module.

+ Experiment meets the requirements.

+ X ≥ 4

- The ways to calculate X : X = X_2

  - X_2 is average mark of two tests at the middle of term (the mark of each test includes incentive mark of attitude at class, self-taught ability of students).

- Manner of final test (calculate Y):

  Written test in 90 miniutes.

- Mark for assessing module: **Z = 0,5X + 0,5Y**

In case students aren't enough conditions to take final test, please write **X = 0** and **Z = 0.**

In case **Y < 2**, **Z = 0**.

X, Y, Z are calculated by marking scheme of 10 and round up one numberal after comma.

After calculated by marking scheme of 10, Z is converted into marking scheme of 4 and letter-marking scheme A+, A, B+, B, C+, C, D+, D, F.

k. Textbooks:

[1]. Nguyen Ba Duong, *Strength of materials*, Construction Publishing House, 2002.

l. Reference materials:

[1]. Le Ngoc Hong, *Strength of materials*, Science and Technique Publishing House, 1998

[2]. Pham Ngoc Khanh, *Strength of materials*, Construction Publishing House; 2002

[3]. Bui Trong Luu, Nguyen Van Vuong, *Strength of materials exercises*, Education Publishing House, 1999.

[4]. I.N. Miroliubop, XA. Engalutrep, N.D. Xerghiepxki, Ph. D Almametop, N.A Kuristrin, KG Xmironop - Vaxiliep, L.V iasina, *Strength of materials exercises*, Construction Publishing House; 2002.

m. Approved day: 30/5/2015

n. Approval level:

Dean	Head of Department	Compiler
Ph.D Hoang Van Hung	**MSc Nguyen Hong Mai**	**Msc Nguyen Hong Mai**

CHAPTER I - THE FUNDAMENTAL CONCEPTS

1.1.Objective and the researched object of the subject

1.1.1. Concept of strength of materials

Strength of materials as a basic subject in engineering field is defined as a branch of mechanics of deformable solids that deals with the behaviours of solid bodies subjected to various types of loadings. It provides the future civil engineers with the means of analyzing and designing so that all types of structures operate safely.

1.1.2. Objective of the subject

It determines essential dimensions and chooses suitable materials for structures so that they ensuretechnical requirements about strength, stiffness and stability at the minimum cost.

- Strength: structures do not crack, break or are destroyed under the effects of loads.
- Stiffness: the amount of deformation they suffer is acceptable.
- Stability: structures ensure the initally geometric shape according to design.

Besides the below requirements, some structures demand fatigue...

1.1.3. The researched object of the subject

a. *Object*: The objects researched in strength of materials are deformable solids under the effects of loads.

b. Shape of object

Load-resistant structures in practice have different shapes. However, they are classified according to relatively geometric dimensions in space.

- Cube objects: these objects have dimensions in three directions which are equivalently large.

Example: platform, foundation...

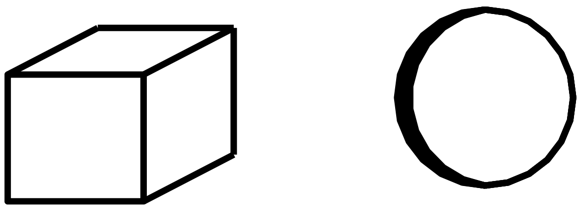


Figure 1.1

- Plate and shell objects: these objects have dimensions in two directions which are larger than the other dimension.

Example: hull, casing...

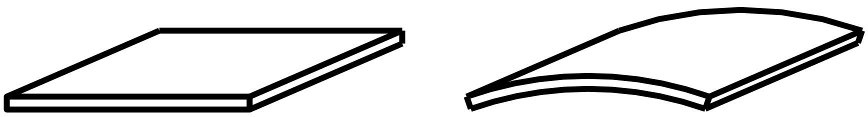


Figure 1.2

- Bar objects: these objects have dimension in one direction which is larger than the other two dimensions. Larger dimension is called axial direction or longitudinal direction.

Example: beam, refter, pillar...

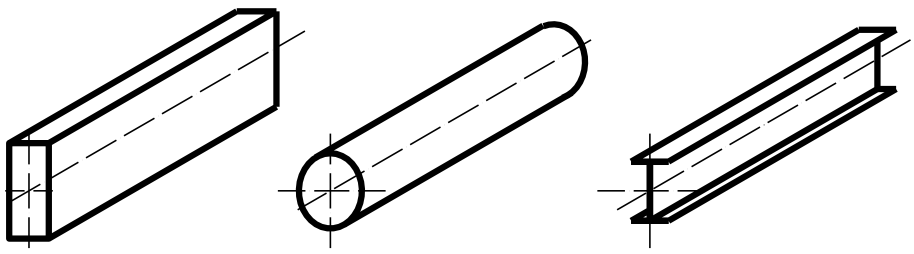


Figure 1.3

Bars are frequently used in construction; therefore, strength of materials primarily researches bars. Bar is illustrated by its axis and cross-section.

*Definition of bar: Bar is a structural component which has longitudinal dimension or axial dimension larger than the other two dimensions. The intersection of a plane normal to longitudinal direction of bar is defined the cross-section. The longitudinal direction is called axis of bar.

*If bars are classified according to the shape of axis, there will be straight bars, curved bars and space bars...

If bars are classified according to their cross-sections, there will be circular bars, rectangular bars, prismatic bars...

1.1.4. Study scope of the subject

a. Elasticity of materials

When an external force acts on the body, the body tends to undergo some deformations. If the external force is removed and the body comes back to its original shape and size (which means that the deformation disappears completely), the body is known as elastic body. This property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity.

The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus, there is a limiting value of force up to and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

If the external force is so large that the stress exceeds the elastic limit, to some extent bar will lose its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in the material.

b. Study scope of the subject

Strength of materials only researches the materials in elastic period.

1.1.5. Method to research the subject

The subject is researched by technical thought. It helps to solve practical problems by the less complicated methods but still ensures essential and appropriate accuracy. The results of strength of materials are checked and implemented by the studies of accurate sciences, including theory of plasticity, theory of oscillation...

1.2.Assumptions and fundamental concepts

1.2.1. The assumptions of materials

Structures are generated from many different materials. Therefore, their properties are also different. To exert generally calculative methods, strength of materials researches a conventional type of material which has the most general and popular properties of many materials. These properties are specified through three assumptions below:

*Assumption 1: Material is isotropic, constant and homogeneous.

-Isotropic: If the response of the material is independent of the orientation of the load distribution of the sample, then we say that the material is isotropic or in other words we can saythat isotropy of a material is a characteristic, which gives us the information that theproperties are the same in the three orthogonal directions x y z; on the other hand if theresponse is dependent on orientation, it is known as anisotropic.

Examples of anisotropic materials, whose properties are different in different directionsare wood, fibre reinforced plastic, reinforced concrete.

-Homogeneous: A material is homogenous if it has the same composition through ourbody. Hence the elastic properties are the same at every point in the body. Isotropic materials have the same elastic properties in all the directions. Therefore, thematerial must be both homogenous and isotropic in order to have the lateral strains to besame at every point in a particular component. It is likely to research a particular component, then to apply for the whole body.

-Constant: Thanks to constancy of materials, it is likely to use Maths, Theory of elasticity in strength of materials.

*Assumption 2: Material operates in elastic limit. This assumption allows to use Hook's law which expresses the relationship between stress and strain in linear.

*Assumption 3: The deformation of materials caused by the external forces is considered small since we presumably are dealing with strains of the order of one percent or less.

1.2.2. The fundamental concepts

1.2.2.1. Strain

a. Normal strain

If anelement is subjected to a direct load, and hence the element willchange in length. If the elementhas an original length dx and changes by an amount Δdx, thestrain produce is defined as follows:

$$\varepsilon_x = \frac{\Delta dx}{dx}; \varepsilon_y = \frac{\Delta dy}{dy}; \varepsilon_z = \frac{\Delta dz}{dz} \qquad (1\text{-}1)$$

Strain is thus, a measure of the deformation of the material and is a nondimensional quantity. It has no units. It is simply a ratio of two quantities with the same unit.Since in practice, the extensions of materials under load are very very small, it is oftenconvenient to measure the strain in the form of strain x 10^{-6} i.e. micro strain.Tensile strains are positive whereas compressive strains are negative. The strain definedearlier was known as linear strain or normal strain or the longitudinal strain.

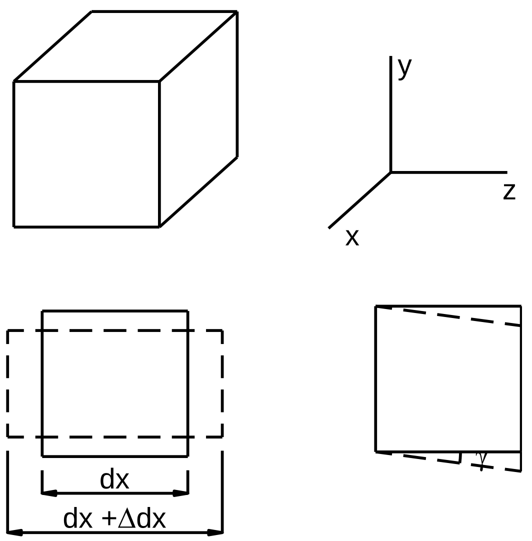


Figure 1.4

b.Shear strain

- An element which is subjected to a shear stress experiences a deformation asshown in the figure below. The tangent of the angle through which two adjacent sidesrotate relative to their initial position is termed shear strain. In many cases the angle isvery small and the angle itself is used in radians instead of tangent. It has no unit.

- Volumetric strain: the ratio of change of volume of the body to the original volume. It has no unit.

$$\theta = \frac{\Delta V}{V} \qquad (1\text{-}2)$$

1.2.2.1. Transposition

a. Longitudinal transposition: it is positional change of a point from A to A' after deformation.

b. Angle transposition: it is the angle formed by a line before and after deformation.

1.2.2.3. The prismatic bar subjected to variuos types of loading

a. Axial tension (compression) of a bar: after deformating, axis of bar is still straight. However, its length is lengthened or shortened.
b. Torsion of a shaft: after deformating, the axis of bar is still straight. However, its cross-section will rotate around axis.
c. Direct shear: after deformating, the axis of bar is still straight but interruptive. Its cross-section slides each other.
d. Bend of a beam: after deformating, the axis of bar is curved. Its cross-section slides each other and rotates an angle in comparison with initial angle.

Four above cases are the simplest cases. However, in practice, capacity to resist loads of bars is regarded as a collection of the fundametal above cases. At that moment, bars are called the ones resisting complicated loads.

1.3.External force

1.3.1. The concept of external force

External forces are the ones coming from environment surrounding or other bodies and acting on the researched body.

1.3.2. The classification of external force

External forces are classified into two main types: loads and reactions.

a. Loads: They are forces acting on bodies and their points, directions and magnitudes are known.

Loads can be classified as static loads or dynamic loads. Static load is the one whose magnitude, direction and point are time-independent. Inertial force is, therefore, negligible. Dynamic load is the one whose magnitude, direction and point are time-dependent and vary with time. Inertial force is, therefore, are non-zero.

Loads can be classified as concentrated loads and distributed loads.

A concentrated load is the one that is considered to act at a point, although, in practice, it must really be distributed over a small area. It means that the length of beam over which the force acts on is so small in comparison to its total length.

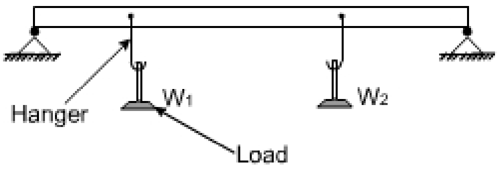


Figure 1.5

Distributed load is the kind of load which is made to spread over a entire span of beam or over a particular portion of the beam in some specific manner.

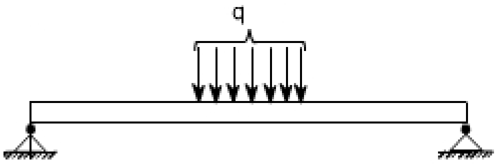


Figure 1.6

In the above figure, the rate of loading ‘q' is a function of z over a span of the beam, hencethis is a non uniformly distributed load.

The rate of loading ‘q' over the length of the beam may be uniform over the entire span ofbeam, then we call this as a uniformly distributed load (U.D.L). The U.D.L may be represented in either of the way on the beams.

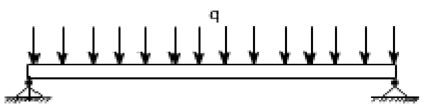


Figure 1.7

Sometimes, the load acting on the beams may be the uniformly varying as in the case ofdams or on inclind wall of a vessel containing liquid, then this may be represented on thebeam as below:

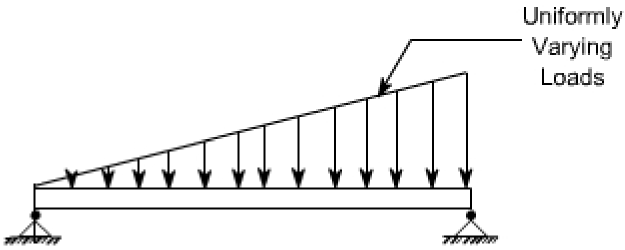


Figure 1.8

The U.D.L can be easily realized by making idealization of the ware house load, wherethe bags of grains are placed over a beam.

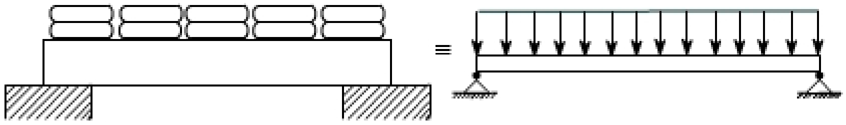


Figure 1.9

b. Reaction

Reaction is the force or moment appearing at point of contact between researched body and other body when there are loads acting on it. The magnitude and direction of reaction depend on not only loads but also manner of support. Therefore, we will consider types of support and their reactions.

1.3.3. Supports and reactions

a. The classification of supports

Cantilever Beam: A beam which is supported on the fixed support is termed as acantilever beam. Such a support isobtained by building a beam into a brick wall, casting it into concrete or welding the endof the beam. Such a support provides both the translational and rotational constrainmentto the beam, therefore the reaction as well as the moments appears, as shown in the figurebelow.

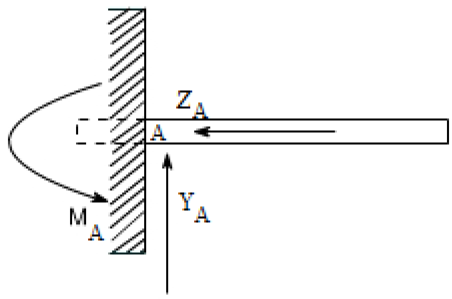


Figure 1.10

Simply Supported Beam: The beams are said to be simply supported if their supportscreate only the translational constraints.

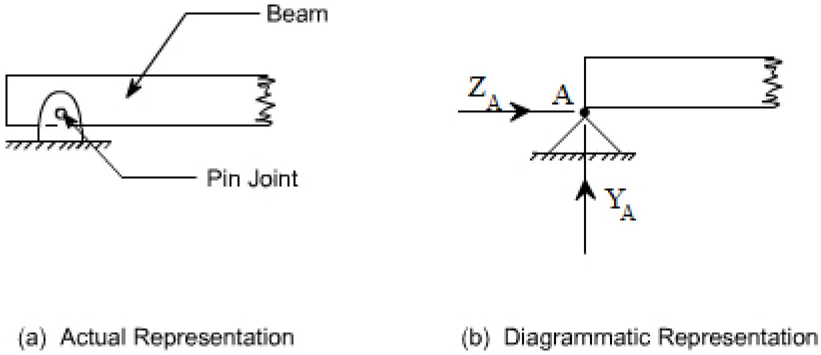


Figure 1.11

Sometimes, the translational movement may be allowed in one direction with the help ofrollers and can be represented like this:

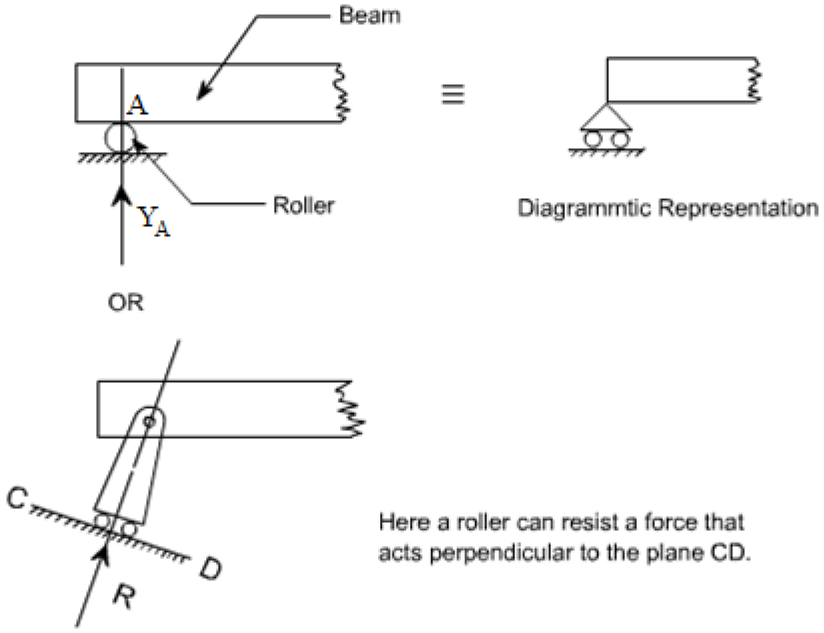


Figure 1.12

b. The determination of reactions

To determine reactions, it is necessary to consider body as an absolute solid and all external forces acting on it create a static equilibrium. If all external forces are in the same plane containing the axis of bar, it is called plane problem. At that moment, there are three equations for static equilibrium. Meanwhile, there are six equations for static equilibrium in space problem.

In term of plane problem, there are three types of equations of static equilibrium as shown below:

a. $\sum_{i=1}^{n} X(P_i) = 0; \sum_{i=1}^{n} Y(P_i) = 0; \sum_{i=1}^{n} M_A(P_i) = 0 (x \not\parallel y)$

b. $\sum_{i=1}^{n} U(P_i) = 0; \sum_{i=1}^{n} M_A(P_i) = 0; \sum_{i=1}^{n} M_B(P_i) = 0 (AB \not\perp u)$ (1-3)

c. $\sum_{i=1}^{n} M_A(P_i) = 0; \sum_{i=1}^{n} M_B(P_i) = 0; \sum_{i=1}^{n} M_C(P_i) = 0$ (A,B,C are not in the same line)

P_i: external forces, i = 1,2,...,n.

The beams can also be categorized as static determination or else it can be referred as statical indetermination. If all the external forces and moments acting on it can be determined from the equilibrium conditions alone then, it will be referred as a staticallydeterminate beam, whereas in the statically indeterminate beams, one has to considerdeformation (deflections) to solve the problem.

1.4.Internal force

1.4.1. The concept of internal force

The change of force among elements inside body when it is deformed is called internal force. According to the above concept, it is obvious that internal force only appears when body is deformed or there is external force acting on body.

1.4.2. Section method

It is used to determine internal force. Its content is shown as below:

Consider the bar subjected to balance-forces. To determine internal forces on arbitrary 1-1 section, it is imagined to cut that bar through 1-1 section. At that moment, the bar is divided into two parts A and B.

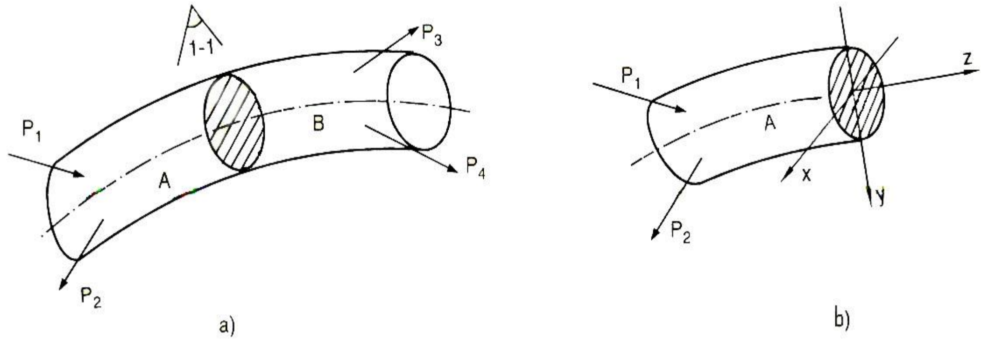


Figure 1.13

Consider equilibrium of part 1. This part has to be in static equilibrium. Therefore, internal forces on section and external forces acting on this part of bar creat a static equilibrium. According to static equations, internal forces on section 1-1 are determined.

1.4.3. Types of internal forces on a section

In case section 1-1 is cross-section, on section 1-1, co-ordinate system is chosen below: the normal of section is 0z, 0x and 0y are in the section and perpendicular each other; O concides with centroid of section. All points on the section have internal forces. When internal forces are fixed on 0, it will turn into a main force $\vec{R}$ and a moment $\overrightarrow{M}$ which have direction and magnitudedetermined.

$\vec{R}$ is divided into three components along three axes

- Component along axis z is signed $\overrightarrow{N_z}$ and called longitudinal force.
- Component along axis x is signed $\overrightarrow{Q_x}$ and called shear force.
- Component along axis y is signed $\overrightarrow{Q_y}$ and called shear force.

$\overrightarrow{M}$ is divided into three components rotating around three axes.

- The component rotating around axis z is signed $\overrightarrow{M_z}$ and called torque.
- The component rotating around axis x is signed $\overrightarrow{M_x}$ and called bending moment.
- The component rotating around axis y is signed $\overrightarrow{M_y}$ and called bending moment.

In general, there are six types of internal forces on section.

1.4.4. Sign convention of internal forces

- Longitudinal force N_z wil be positive if its direction is out the section.
- Shear forces Q_x, Q_y will be positive if its direction concides with the normal of section after normal is clockwise rotated an angle 90^0.
- Torque M_z will be positive if someone see section and recognise that M_z is rotating clockwise.
- Bending moment M_x will be positive if it pulls positive filaments of axis y. If positive direction of axis y is chosen to be downward, Mx will be positve when it pulls lower filaments.
- Bending moment $M_y$will be positive if it pulls positive filaments of axis x.

1.4.5. Determination of internal forces on section

Because considered part of bar is in static equilibrium, internal forces on cross-section and external forces acting on this part of bar creat a static equilibrium. The equations of static equilibrium are created below:

$$N_z + \sum_{i=1}^{n} Z(P_i) = 0$$

$$Q_x + \sum_{i=1}^{n} X(P_i) = 0$$

$$Q_y + \sum_{i=1}^{n} Y(P_i) = 0 \qquad (1-4)$$

$$M_z + \sum_{i=1}^{n} M_z(P_i) = 0$$

$$M_y + \sum_{i=1}^{n} M_y(P_i) = 0$$

$$M_x + \sum_{i=1}^{n} M_x(P_i) = 0$$

P_i: the external forces acting on considered part of bar, i = 1,2,...,n.

Six above equations show relationship between internal forces on cross-section and external forces.

This relationship is used to determine internal forces.

1.4.6. Internal force diagrams

a. *Concept*: The diagrams which illustrate the variations in the values of internal forces along the length of the beam for any fixed loading conditions are called internal force diagrams.

b. *Procedure for drawing internal force diagrams:*

Step 1: Choose co-ordinate system

Step 2: Determine reactions

Step 3: Divide bar into segments to ensure that along each segment, internal forces will vary at a continuous rule. In practice, split points are the ones that apply concentrated force, starting point and end of distributed force.

Step 4: Apply a free-body analysis and write equations for static equilibrium to determine the function of internal forces along each segment of bar.

Step 5: Draw diagrams to perform internal force functions determined above, sign and rule diagrams.

Note: bending moment diagrams M_x, M_y are drawn at tensile filaments.

Example 1: Draw internal force diagram of the bar shown in the figure under the given loads.

Choose co-ordinate system as figure 1.14, origin O is placed at A, axis z goes from left to right.

In this exercise, cantilever A only exists one reaction Z_A (its direction is assumed).

Condition of static equilibrium:

$$\sum F_z = 0$$

$$Z_A + P - 2P = 0$$

$$Z_A = P$$

(assumed direction is right.)

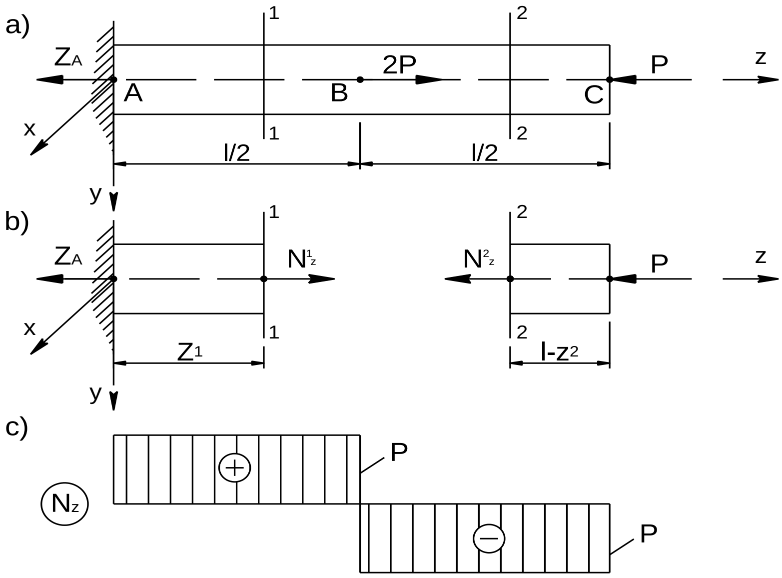


Figure 1.14

Divide the bar into two segments, spilt point B has a concentrated load 2P. AB is called segment I, BC is called segmentII.

Use section 1-1 which is at z_1 distance of point 0 for segment I ($0 \le z_1 \le \frac{l}{2}$) and retain the left part of section 1-1.It can be found thatthere is only one internal force N_z^I on cross-section. It is performed in positive direction which is out the section.Consider the equilibrium of retained part:

$$N_z^I - Z_A = 0$$

$$\Rightarrow N_z^I = Z_A = P$$

In conclusion, longitudinal force is a constant along component I (AB).

Use section 2-2 which is at z_2 distance of point 0 for segment II ($\frac{l}{2} \le z_2 \le l$) and retain the right part of section 2-2. According to equation of static equilibrium:

$$N_z^{II} = -P$$

N_z^{II} is also a constant along component 2-2.

Internal force diagram is drawn as in figure1.14c.

<u>Example 2</u>: Draw the internal force diagram of the bar shown in the figure under the given loads.

Choose co-ordinate system as figure 1.15, origin O is placed at A, axis z goes from left to right.

At point C, there is the reaction moment whose direction is assumed.

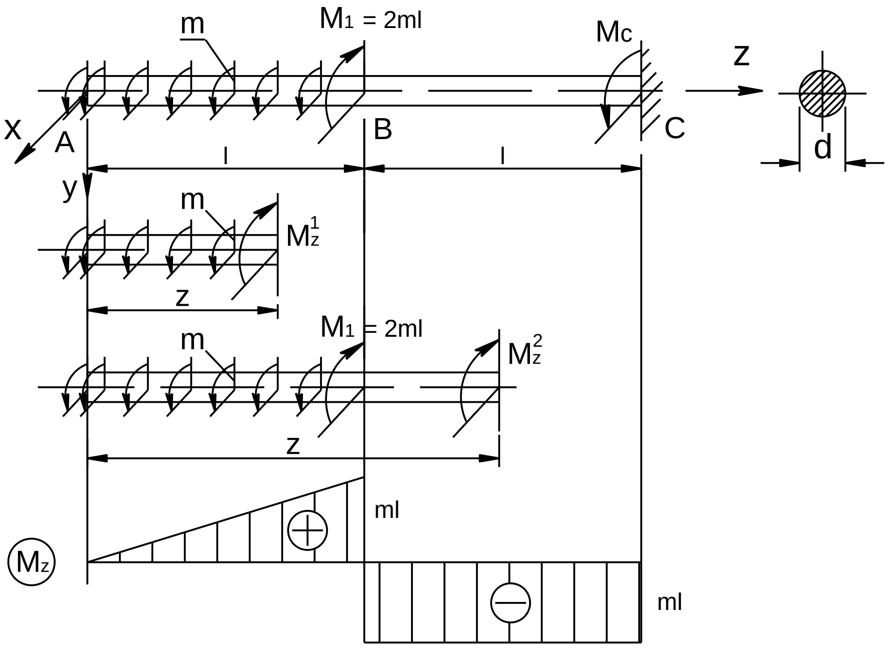


Figure1.15

M_C is determined from equation: $\sum m_z = 0$

$$M_C + m.l - M = 0$$

$$M_C = ml$$

(assumed direction is right.)

However, it is likely to ignore the determination of reaction in this example.

Divide the bar into two segment s

- Consider segment I(AB) :

Use section 1-1 which is at z_1 distance of point 0 for component I ($0 \le z_1 \le l$) and retain the left part of section 1-1. There is only one internal force M_z^I on cross-section. It is performed in the positive direction of convention.

Consider the equilibrium of retained part:

$$\sum_{i=1}^{n} M_z = M_z^I - mz_1 = 0$$

$M_z^I = mz_1$

It is realised that M_z is the linear function of z

- Consider segment I(AB) :

Use section 2-2 which is at z_2 distance of point 0 for segment II ($l \le z_2 \le 2l$) and retain the left part of section 2-2. Internal force on this section is M_z^{II} .

According to the equation of static equilibrium:

$$\sum M_z = M_z^2 + M_1 - ml = 0$$

$$\Rightarrow \quad M_z^2 = -M_1 + ml = -ml$$

M_z^{II} is a constant on section 2-2.Internal force diagram M_z is drawn as in figure 1.15d.

Example 3:

Draw the internal force diagrams of the beam shown in the figure under the given loads.

Co-ordinate system is chosen as the figure.

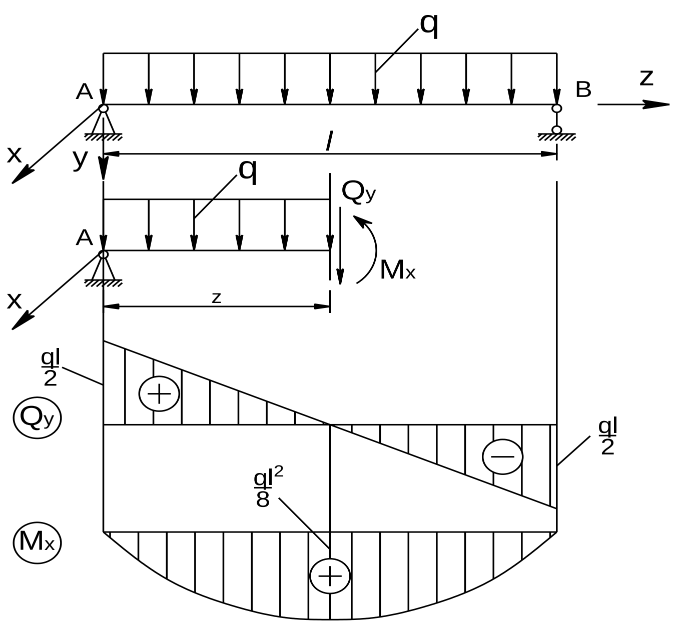


Figure 1.16

At point A, there are two reactions Z_A, Y_A;at point B, there is one reaction Y_B. Apply the equations of static equilibrium:

$$\sum F_y = 0, \sum F_z = 0 \text{ and} \sum M_B = 0$$

Find out:

$Z_A = 0, Y_A = Y_B = ql/2$

(assumed direction is right.) .

Consider beam AB:

Use section 1-1 which is at z distance of point 0 ($0 \le z_1 \le l$) and retain the left part of section 1-1. There is only two internal forces Q_y and M_x on section 1-1. It is performed in the positive direction of convention as shown in the figure 1.16b.

$$\sum F_y = 0 \Leftrightarrow Q_y + qz - Y_A = 0$$

$$\sum M_o(P_i)=0 \Leftrightarrow M_x+qz\frac{z}{2}-Y_A z=0$$

Replace Y $_A$ = ql/2 and solve set of equations:

$$Q_y(z)=-qz+\frac{ql}{2}$$

$$M_x(z)=-\frac{qz^2}{2}+\frac{ql}{2}z$$

Internal force diagrams are drawn as in figure 1.16b,c.

<u>Example4</u>: Draw internal force diagrams of the beam shown in the figure under the given loads.

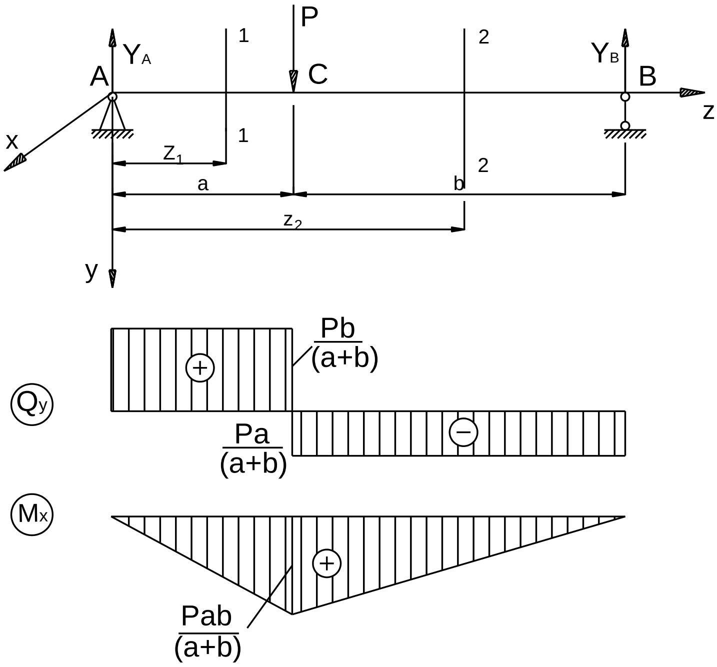


Figure 1.17

- Co-ordinate system is chosen as the figure.
- Determine reactions:

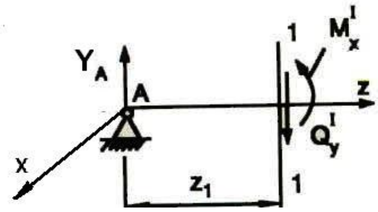
$$\sum M_A = Pa - Y_B(a+b)=0$$

$$\sum F_y = Y_A + Y_B - P = 0$$

Find out:

$$Y_A=\frac{Pb}{(a+b)}$$

$$Y_B=\frac{Pa}{(a+b)}$$



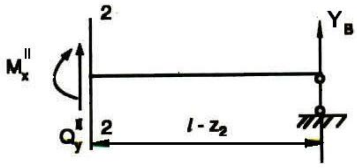
- Divide the bar into two segments
- Consider segment I(AB) :

Use section 1-1 which is at z_1 distance of point 0 $(0\le z_2 \le a)$ and retain the left part of section 1-1.

According to the equation of static equilibrium:Figure 1.18

$$Q_y^1 = Y_A = \frac{Pb}{a+b}$$

$$M_x^1 = Y_A z = \frac{Pb}{a+b} z$$



- Consider segment II(BC) :Use section 2-2 ($a \le z_2 \le l$) and retain the right part of section 2-2.

According to equation of static equilibrium:

$$\sum Y = -Q_y^2 - Y_B = 0$$

Figure 1.19

$$\sum m_{2-2} = M_x^2 - Y_B(l-z) = 0$$

Solve two equations above:

$$Q_y^2 = -Y_B = -\frac{Pa}{a+b}$$

$$M_x^2 = Y_B(l-z) = \frac{Pa}{a+b}(l-z)$$

Internal force diagrams are drawn as in the figure 1.17

<u>Example 5:</u>Draw the internal force diagrams of the beam shown in figure under the given loads.

- It is likely to ignore determination of reaction in this example.

- Co-ordinate system is chosen as the figure.

Use section 1-1 which is at z distance of point 0 ($0 \le z_2 \le l$) and retain the left part of section 1-1.

According to the equation of static equilibrium:

$Q_y(z) = q(l-z)$

$$M_x(z) = -q\frac{(l-z)^2}{2}$$

Internal force diagrams are drawn as in figure 1.20

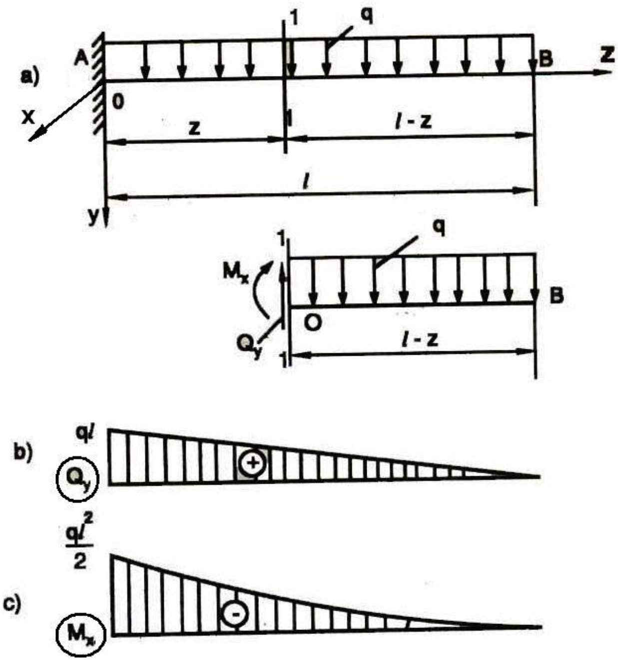


Figure 1.20

1.4.7. Basic relationship between the rate of loading q(z), shear force Q_y and bending mement M_x

Cut a short slice of length dz from a loaded beam at distance ‘z’ of the origin ‘0’ (figure 1.21a)

Distance dz is so small that it is considered that q(z) = constant. Internal forces on the sections of dz are expressed in the figure 1.22b.

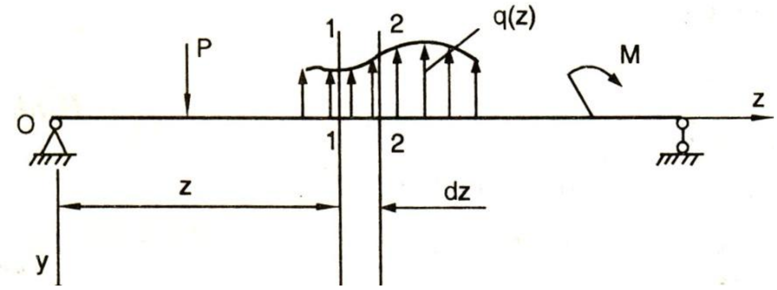


Figure 1.21a

Consider the static equilibrium of the portion above:

$$\sum Y = -Q_y - q(z)dz + (Q_y + dQ_y) = 0$$

$$\sum M_{O_2} = -Q_y dz - M_x + q(z)\frac{dz^2}{z} + (M_x + dM_x) = 0$$

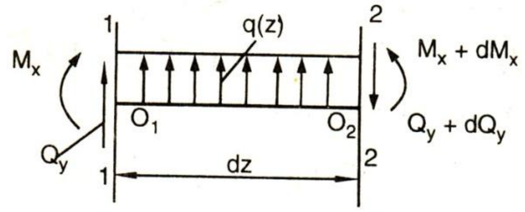


Figure 1.22b

Ignore infinitesimality of high level $q(z)\frac{(dz)^2}{2}$. Thanks toabove equations:

$$\begin{cases} \dfrac{dQ_y}{dz} = q(z) \\ \dfrac{dM_x}{dz} = Q_y \\ \dfrac{d^2M_x}{dz^2} = q(z) \end{cases} \qquad (1\text{-}5)$$

This relationship can be used to draw and check internal force diagram.

1.4.8. Classify the deformation of bar according to its internal force

Thanks tothe existence of different types of internal forces on sections of bar, its deformation cases are classified below:

- Only longitudinal force $N_z \neq 0$: bar is subjected to tensile (compressive) loads.
- Only torque $M_z \neq 0$: bar is subjected to torsion
- Only shear force Q_x or $Q_y \neq 0$: bar is subjected to shear deformation.
- Only bending moment M_x or $M_y \neq 0$: bar is subjected to pure bending.
- Only $Q_y \neq 0$ and $M_x \neq 0$ (or $Q_x \neq 0$ and $M_y \neq 0$): bar is subjected to horizontal plane bending.

If the number of internal forces on section is more than those in above cases, bar is subjected to complicated forces. It will be researched in the next semester.

1.5. Stress

1.5.1. Concept of stress

Consider some cross - sectional areas of the rectangular bar subjected to someloads or forces (in Newtons)

Imagine that the same rectangular bar is assumed to be cut into two halves atsection XX. The each portion of this rectangular bar is in equilibrium under the action ofload P and the internal forces acting at the section XX has been shown.

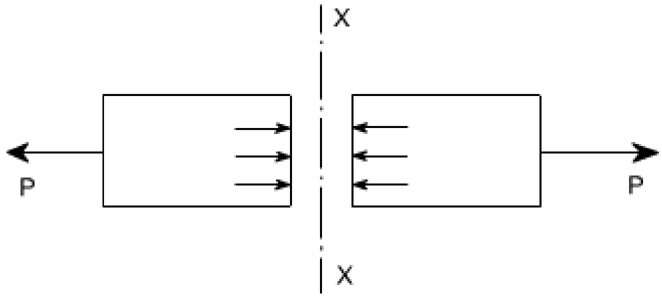


Figure 1.23

Now stress is defined as the force intensity or force per unit area. Here we use a symbol σ to represent the stress.

Here we are using an assumption that the total force or total load carried by therectangular bar is uniformly distributed over its cross - section.

But the stress distributions may not be in uniform, with the local regions of high stressknown as stress concentrations.

If the force carried by a component is not uniformly distributed over its cross – sectionalarea, A, we must consider a small area, ‘dF' which carries a small load dP, of the totalforce ‘P'.

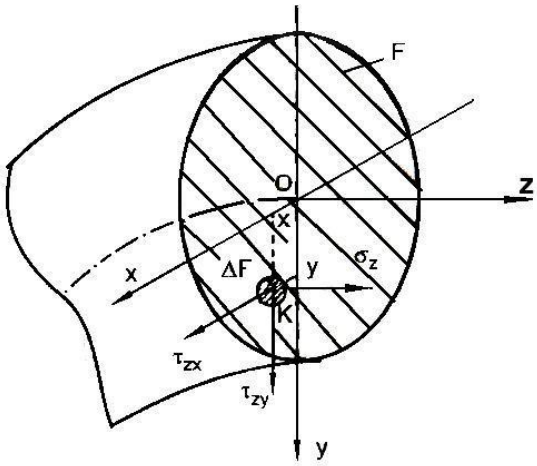


Figure 1.24

Then the definition of stress is $\vec{p} = \dfrac{\overrightarrow{dP}}{dF}$.

As a particular stress generally holds true only at a point, therefore it is definedmathematically as

$$\vec{p} = \lim_{dF \to 0} \frac{\overrightarrow{dP}}{dF}$$

Units:The basic units of stress are N/mm^2, kN/cm^2, MN/m^2.

Analyse $\vec{p}$ into components along three directions of co-ordinate system. The component which is along the direction of normal 0z is called normal stress and signed $\vec{\sigma}$, two components which are along directions of axes x, y are called shear stress and signed $\overrightarrow{\tau_{zx}}$, $\overrightarrow{\tau_{zy}}$.

1.5.2. *Sign convention of stress*

$\vec{\sigma}$: normal stress has sign convention like longitudinal force N_z.

$\overrightarrow{\tau_{zx}}$: shear stress has sign convention like shear force Q_x.

$\overrightarrow{\tau_{zy}}$: shear stress has sign convention like shear force Q_y.

1.5.3. *Relationship between stresses and internal forces*

On a small element of material, we have area dF, relationship between stresses and internal forces are shown below:

$$\int_F \sigma_z dF = N_z \quad \int_F \sigma_z y dF = M_x$$

$$\int_F \tau_{zx} dF = Q_x \quad \int_F \sigma_z x dF = M_y \qquad (1\text{-}6)$$

$$\int_F \tau_{zy} dF = Q_y \quad \int_F (\tau_{zx} y - \tau_{zy} x) dF = M_z$$

This relationship is used to determine stress in calculating.

1.5.4. Relationship between stress and deformation

Normal stress causes longitudinal deformation while shear stress causes angular deformation.

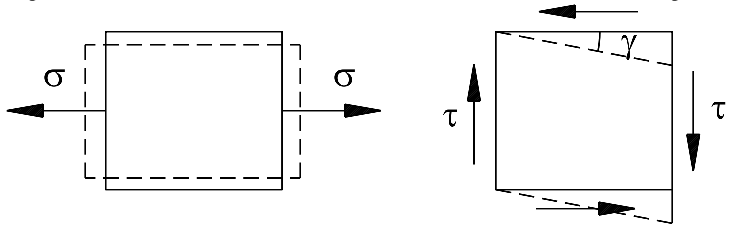


Figure 1.25

Theoretical questions

1.1. Raise objective and the researched object of the subject.

1.2. Which requirements do structures need to ensure in order to work safely? Take illustrative examples.

1.3. Raise the basic assumptions of the subject. How are those assumptions necessary in research of strength of materials? Take illustrative examples.

1.4. Distinguish between load and reaction. Raise popular supports and the way to determine reactions.

1.5. Define internal forces and raise the way to determine them. Take illustrative examples.

1.6. How many internal forces are there on cross-section. Raise their signs, names, sign conventions and determinations. Draw illustrative pictures. In horizontal plane problem, which internal forces exist on cross-section. Take example.

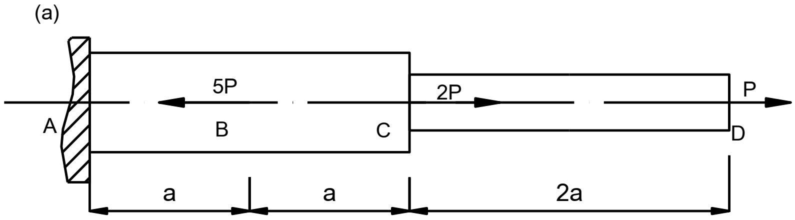
1.7. The concept of internal force diagram and the procedure of drawing internal force diagram. Raise the conclusions which help to draw internal force diagram quickly and check drawn diagram.

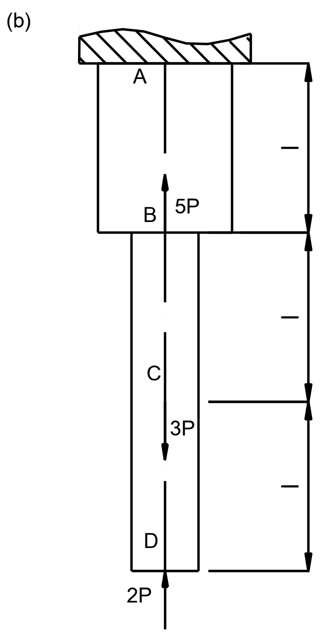
1.8. Distinguish between stresses and internal forces on cross-section. Which distributed force is stress? Raise unit, sign convention and relationship between stress and deformation.

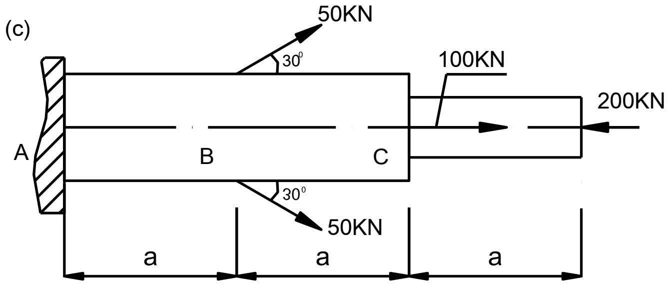
1.9. The concept of deformation and strain. Raise the characteristics of deformation.

Numerical problems

<u>Exercise 1</u>: Draw the internal force diagrams of the bars below.

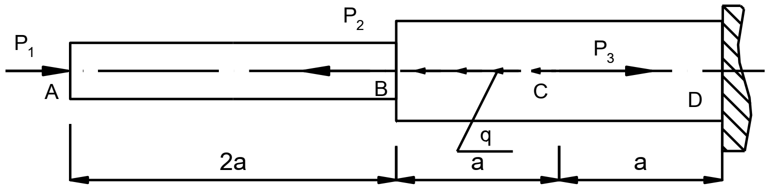


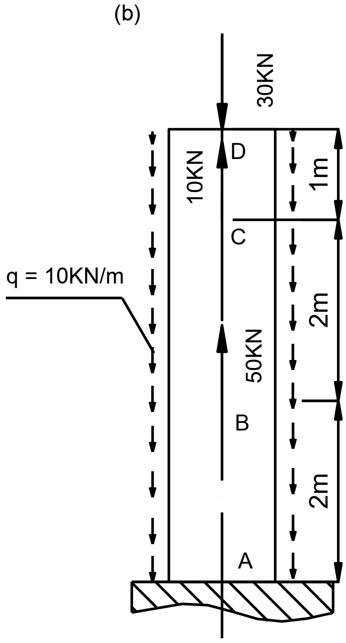




Exercise 2: Draw the internal force diagrams of the bars below.

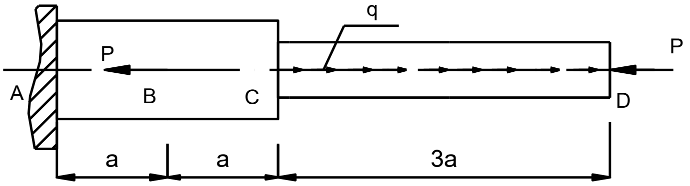
a, Know P_1 = 20 kN, P_2 = 15 kN
P_3 = 10 kN, q = 20 kN/m,
a = 1m





b,

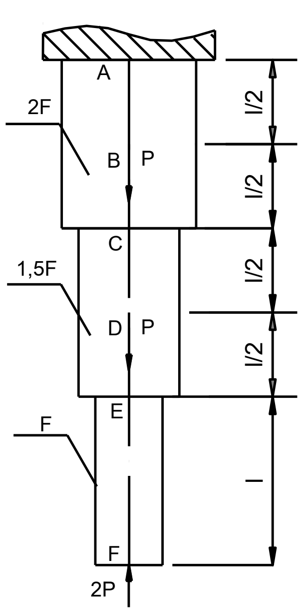
c, Know P = 50 kN, q = 20 kN/m, a = 1m

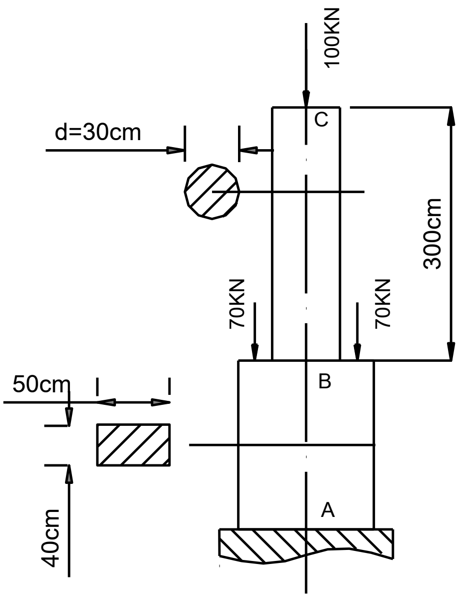


Exercise 3:

Draw the internal force diagrams of the following bars and include the own gravity of bar. Know its specific gravity is γ.

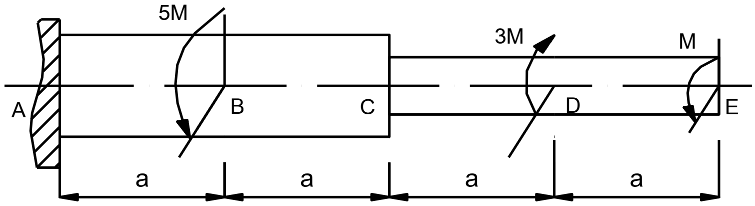
a, Know $P = \gamma$ Fl   b, Know $\gamma = 80$ kN/m^3



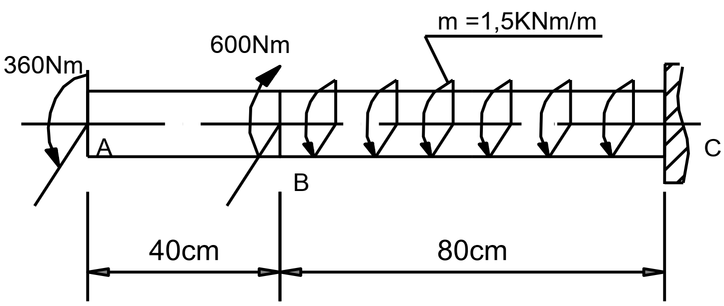


Exercise 4: Draw the internal force diagrams of the following shafts.

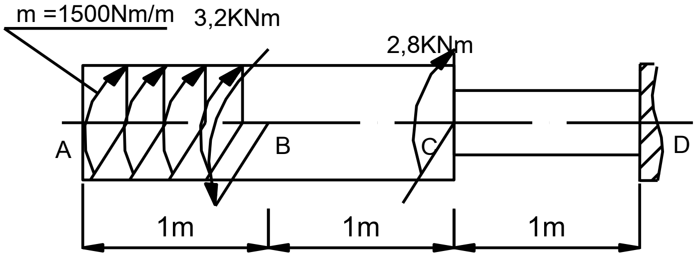
a,



b,



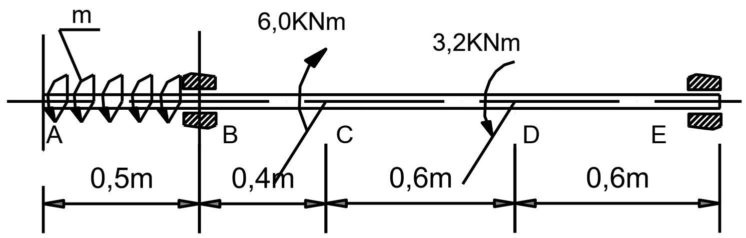
c,



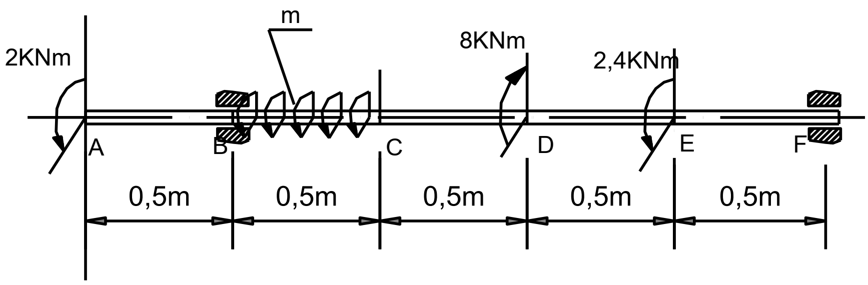
Exercise 5:

- Determine m to equilibrate shaft.
- Draw the internal force diagram of the shaft with determined value m.

a,



b,



Exercise 6: Draw the internal force diagram of the shaft. Know n = 840 revolutions/minute

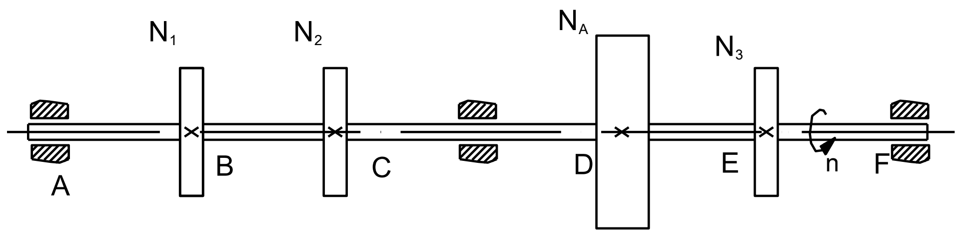
A isactive gear. Powers of gears are:

N_1 = 20 kW

N_2 = 15 kW

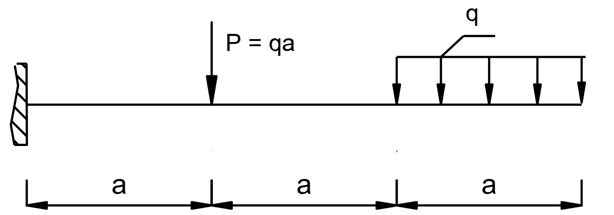
N_3 = 25 kW

N_A = 60 kW

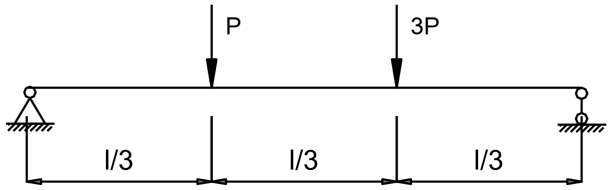


Exercise 7: Draw internal force diagrams of beams below.

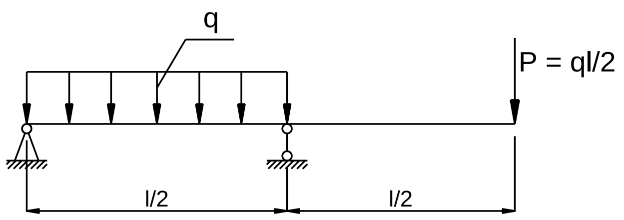
(a)



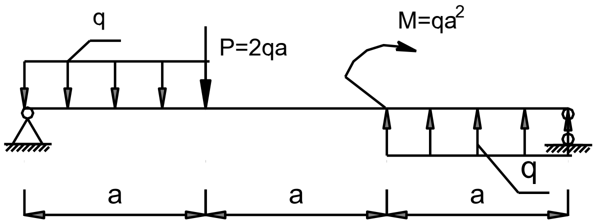
(b)

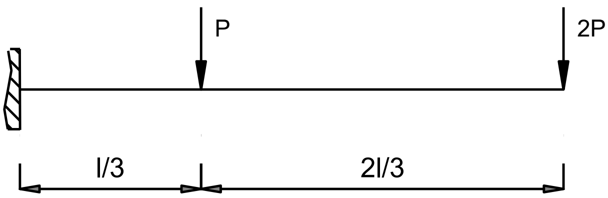


(c)

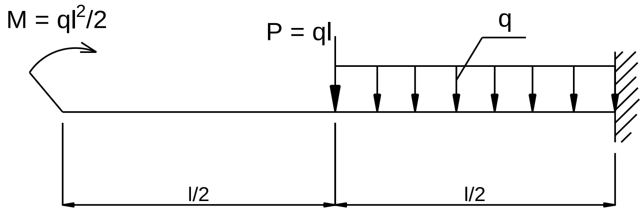


(g)

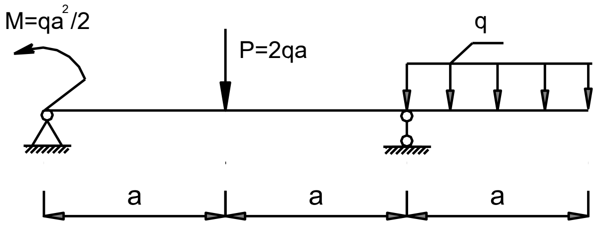




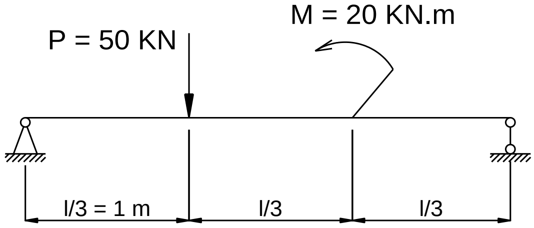
(d)



(e)



(f)



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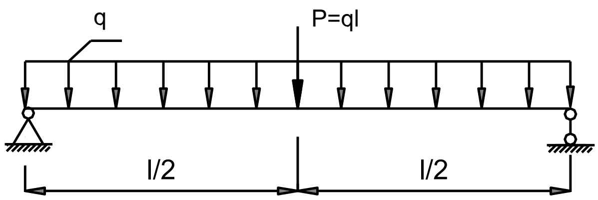
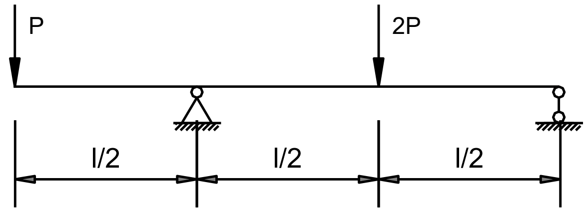
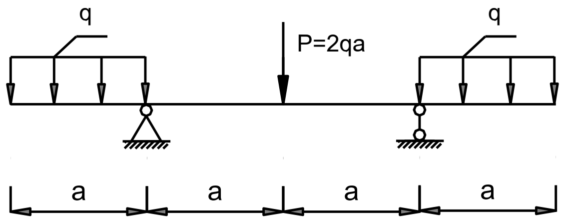






CHAPTER II - AXIALLY LOADED BAR

2.1. Concept

A bar is called pulled or pushed one along its axis if there is only a type of internal force which is longitudinal force N_z on every its cross-section.

If $N_z > 0$, bar is stretched.

If $N_z < 0$, bar is shortened.

Examples: tow bar is a prismatic member in tension and the landing gear strut is a member in compression. Other examples are the members of a bridge truss, connecting rod in automobile engines, the spokes of bicycle wheels, columns in buildings, and wing struts in small airplanes.

2.2. Stress on cross-section

a. Assumptions about the deformation of bar

Consider the straight bar which its cross-section is constant. Draw on its surface lines which are parallel and perpendicular to axis of bar. They symbolize the longitudinal and horizontal filaments of bar.

Observe it and realise that:

- The lines which are parallel to the axis of bar are still straight and parallel to the axis of bar. They stretch into the same segments.
- The lines which are perpendicular to the axis of bar are straight and perpendicular to the axis of bar.

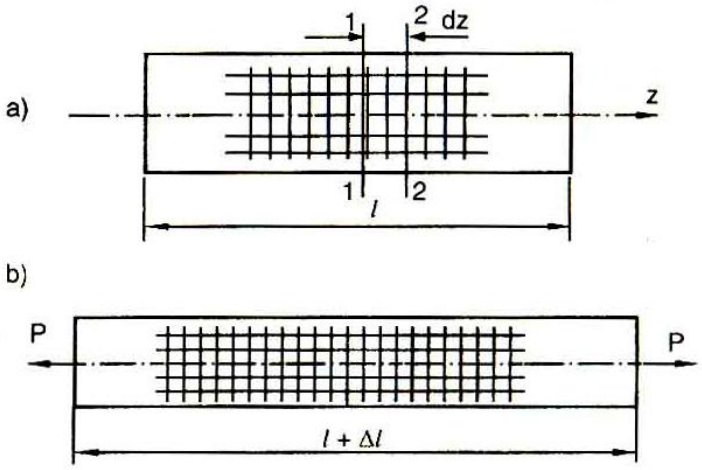


Figure 2.1

Thanks to this observation, there are two given assumptions:

1. Assumption about cross-section (Becnuli assumption)

The cross-sections of a straight bar are always plane and perpendicular to the axis of bar during pulled or pushed process.

2. Assumption about longitudinal filaments

The longitudinal filaments of a bar are not pushed each other during pulled or pushed process.Besides, the material of bar is considered to work in elastic area.

Thanks toabove assumptions, it is concluded that:

- On the cross-section of bar, there is not shear stress. There is only normal stress.
- Normal stress is uniformly distributed over cross-section area.

b. Establish formula to determine stress

Thanks to assumption 1, it is realised that there is not shear stress on cross-section.

Thanks to assumption 2, it is realised that there is only normal stress on cross-section. ($\sigma_x = \sigma_y = 0$).

Therefore, on cross-section, there is only one type of normal stress σ_z. Consider normal stress at a point A on a cross-section. Thanks to (1-4), we have: $\int_F \sigma_z dF = N_z$. When material is in elastic area, according to Hooke's Law, there is : $\sigma_z = E\,\varepsilon_z$

(2-1)

E is modulus of elasticity. It depends on materials and is determined through experiment.

For examples:

Carbon steel: E = (1,8 ÷ 2,1).10^{11} N/m^2

Copper: E = (1 ÷ 1,2) .10^{11} N/m^2

Aluminum: E = (0,7 ÷ 0,8)10^{11} N/m^2

Timber: E = (0,08 ÷ 0,12)10^{11} N/m^2

Accroding to assumption 1, ε_z is constant over cross-section area, so σ_z is also constant over cross-section area.

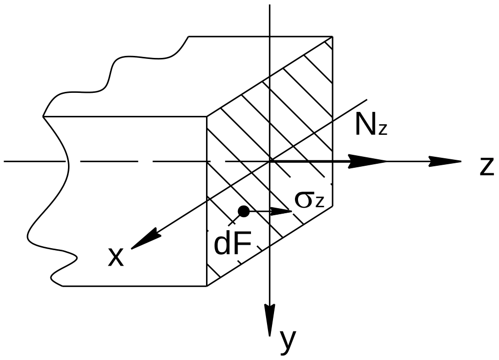


Figure 2.2

We have:

$$N_z = \int_F \sigma_Z dF = \sigma_Z \int_F dF = \sigma_Z .F$$

Or $$\sigma_Z = \frac{N_Z}{F} \qquad (2\text{-}2)$$

F is the cross-section area which contains point needing to determine stress.

Normal stress diagram σ_z : it is drawn along the axis of bar. Normal stresses on every cross-section areexpressed by ordinates on the diagram.

2.3. Strain and deformation of axially loaded bar

2.3.1. Strain: a straight bar will change in length when loaded axially, becoming longer when in tension and shorter when in compression. Therefore, it has only normal strain.

2.3.1.1. Longitudinal strain ε_z :

Consider anelement limited by two sections (1-1) and (2-2) at a distance dz.

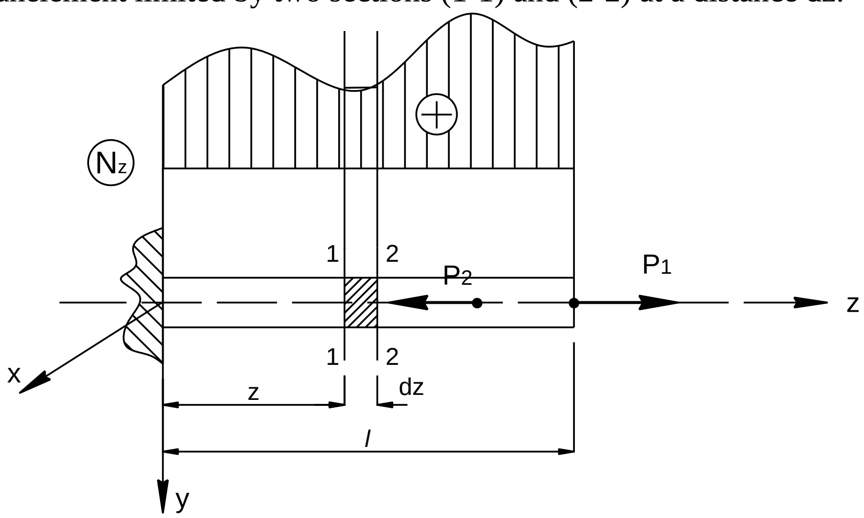


Figure 2.3

According to Hooke's Law: $\sigma_z = E.\varepsilon_z$

(2-1)

We have : $\varepsilon_z = \frac{\sigma_z}{E}$ or $\varepsilon_z = \frac{N_z}{EF}$

(2-3)

Therefore, strain is small when EF is significant. As a result, EF is called the stiffness of axially loaded bar.

2.3.1.2. Lateral strain $\varepsilon_x, \varepsilon_y$

It can be realised that ε_z is opposite in sign to $\varepsilon_x, \varepsilon_y$. Experimental research has proved that the magnitudes of these types of strain have a proportion each other depending on equation:

$\varepsilon_x = \varepsilon_y = -\mu.\varepsilon_z$ (2.4)

The ratio of lateral strain to the longitudinal strain is a constant for a given material when the material is stressed in elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ. The limitation of this ratio is $0 \le \mu \le 0.5$ for all materials.

For examples:

Steel : μ = 0,25 ÷ 0,33

Copper: μ = 0,31 ÷ 0,34

Concrete: μ = 0,08 ÷ 0,18

Rubber: μ = 0,47

2.3.2. Elasticity of bar (Δl ***)***

A straight bar will change in length when loaded axially, becoming longer when in tension and shorter when in compression.The elongation or shrotening of a bar is signed Δl .

According to equation: $\varepsilon_Z = \frac{\Delta dz}{dz}$ we have $\Delta dz = \varepsilon_Z.dz$ or $\Delta dz = \frac{N_Z}{E} dz$ (2.5)

- In general case:when a bar has many segments and on every segment, longitudinal force N_z and stiffness EF vary continuously:

$$\Delta l = \sum_{i=1}^{n} \int_0^{l_i} \left(\frac{N_z}{EF} \right)_i dz \quad (2.6)$$

n is the number of segments of bar.

$l_i$is the length of segment i.

- In particular case: when a bar has many segments and on every segment, longitudinal force N_z and stiffness EF are constant:

$$\Delta l = \sum_{i=1}^{n} \left(\frac{N_Z l}{EF} \right)_i$$

(2-7)

2.3.3. The displacement of cross-section

When loaded axially, the axis of bar is still straight, only the positions of cross-sections change along the axis. At cross-section with co-ordinate z, its displacement is signedΔ_z.

In some numerical problems, after deformation is determined, displacement can be determined simply by using formula to calculate Δl and geometric relation on figure.

Example 1: Draw longitudinal force diagram, normal stress diagram, longitudinal strain diagram and determine the elasticity of bar. (figure 2.4)

Because the bar has a freedom end, it is not necessary to determine reactions.

To determine N_z and σ_z, the bar is divided into three segments AB, BC, CD and each segment is researched in turn.

- Segment AB (i=1) and $0 \leq z \leq a$

$$N_z^1 = 8P - 3P = 5P$$

$$\sigma_z^1 = \frac{N_z^1}{F_1} = \frac{5P}{2F}; \quad \varepsilon_z^1 = \frac{\sigma_z^1}{E} = \frac{5P}{2EF}$$

- Segment BC (i=2), $a \leq z \leq 2a$

$$N_z^2 = -3P$$

$$\sigma_z^2 = -\frac{3P}{2F}; \quad \varepsilon_z^2 = \frac{-3P}{2EF}$$

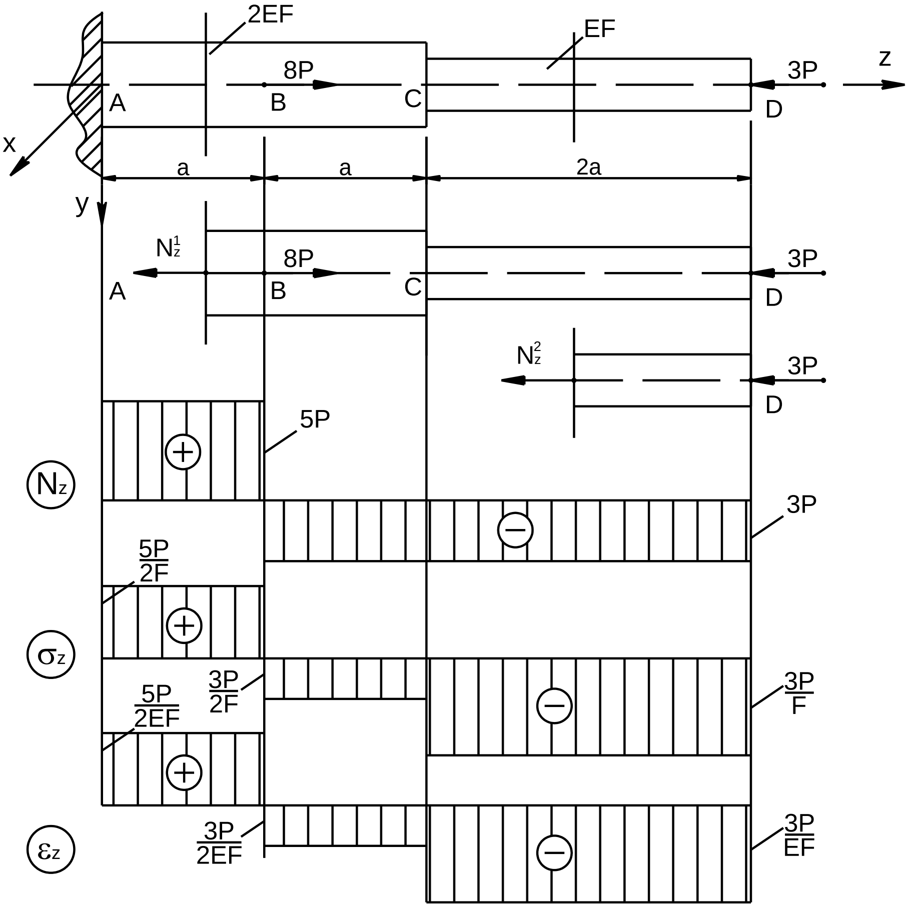


Figure 2.4

- Segment CD (i=3), $2a \leq z \leq 4a$

$$N_z^3 = -3P$$

$$\sigma_z^3 = -\frac{3P}{F}; \quad \varepsilon_z^2 = \frac{-3P}{2EF}$$

Strain diagram is drawn thanks to the formula determining Δl:

$$\Delta l = \sum_{i=1}^{3}\left(\frac{N_z.l}{EF}\right)_i = \left(\frac{N_z.l}{EF}\right)^{AB} + \left(\frac{N_z.l}{EF}\right)^{BC} + \left(\frac{N_z.l}{EF}\right)^{CD}$$

$$= \frac{5Pa}{2EF} + \frac{-3Pa}{2EF} + \frac{-3P.2a}{EF}$$

$$= -\frac{10Pa}{2EF} = -\frac{5Pa}{EF}$$ (minus sign proves that the length of bar is shortened.)

The diagrams are drawn as figure 2.4.

2.4. Mechanical properties of materials

The design of machines and structures so that they will function properly requires that we understand the mechanical behavior of the materials being used. Ordinarily, the only way to determine how materials behave when they are subjected to loads is to perform experiments in the laboratory. The usual procedure is to place the small specimens of the material in testing machines, apply the loads, and then measure the resulting deformations (such as changes in length and changes in diameter). Most materials-testing laboratories are equipped with machines capable of loading specimens in a variety of ways, including both static and dynamic loading in tension and compression.

Based on the behaviour, the materials may be classified as ductile or brittle materials.

- Ductile materials:the capacity of materials to allow the large deformations or large extensionswithout failure is termed as ductility. The materials with high ductility are termed asductile materials.
- Brittle materials:a brittle material is the one which exhibits a relatively small extensions or deformations tofracture, so that the partially plastic region of the tensile test graph is much reduced.

Each of materials will be researched.

2.4.1. Tensile test

2.4.1.1 Specimen: The standard tension specimen is a cylinder. It has an initial length l_0 and inital diameter d_0 so cross-section area is $F_0 = \frac{\pi d_0^2}{4}$. In order that test results will be comparable, the dimensions of test specimens and the methods of applying loads must be conformed to Vietnamese standard.

2.4.1.2. Experiment

The test specimen is installed into machine and pulled. Loads are gradually increased from 0 to P. Deformation Δl which corresponds with each load P will be read from a dial.

2.4.1.3. Tensile diagram: shows relationship between loads and deformations.

a. Specimen made from ductile material:

The tensile diagram of ductile material is shown on the figure 2.5. The load-resistant procedure of material is divided into three stages:

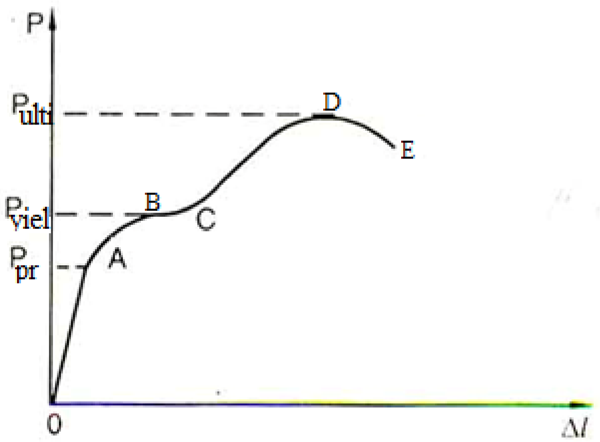


Figure 2.5

- Proportional sttlage (elastic stage): The diagram begins with a straight line from the origin O to point A, which means that the relationship between load P and deformation Δl in this initial region is not only linear but also proportional. Beyond point A, the proportionality between stress and strain no longer exists; hence the force at A is called the proportional force P_{pr}. The slope of the straight line from O to A is called the modulus of elasticity. The corresponding stress is known as the proportional stress of the material $\sigma_{pr} = \frac{P_{pr}}{F_0}$.

- Yielding stage: With an increase in load beyond the proportional limit, the deformation begins to increase more rapidly for each increment in load. Consequently,the load-deformation curve has a smaller and smaller slope, until, atpoint B, the curve becomes horizontal. Beginning at thispoint, the considerable elongation of the test specimen occurs with no noticeable increase in the tensile force (from B to C). This phenomenonis known as the yielding phenomenonof the material, and point B is called the yieldingpoint. The maximum load in this stage is signed P_{ch}. The corresponding stress is known as the yielding stress of thematerial $\sigma_{yiel} = \frac{P_{yiel}}{F_0}$.In the region from B to C, the material becomes perfectlyplastic, which means that it deforms without an increase in the appliedload. The elongation of a mild-steel specimen in the perfectly plasticregion is typically 10 to 15 times the elongation that occurs in the linearregion (between the onset of loading and the proportional limit). Thepresence of very large strains in the plastic region (and beyond) is thereason for not plotting this diagram to scale.

- Ultimate stage: After undergoing the large deformations that occur during the yielding stage in theregion BC, the material begins to harden. During hardening, thematerial undergoes changes in its crystalline structure, resulting in the increased resistance of the material to further deformation. The elongation of the test specimen in this region requires an increase in the tensile load, and therefore the load-deformation diagram has a positive slope from C to D. The load eventually reaches its maximum value, and the correspondingload (at point D) is called the ultimate load P_b. The corresponding stress is known as the ultimate stress of material $\sigma_{ulti} = \frac{P_{ulti}}{F_0}$. The further stretching of the bar is actually accompanied by a reduction in the load, and fracture finally occurs at a point such as E in Fig. 2.5.

When a test specimen is stretched, lateral contraction occurs. The resulting decrease in cross-sectional area is too small to have a noticeable effect on the calculated values of thestresses up to about point C in Fig 2.5, but beyond that point the reduction in area begins to alter the shape of the curve. In the vicinity of the ultimate stress, the reduction in area of the bar becomes clearly visible and the pronounced necking of the bar occurs (see Figs 2.6).

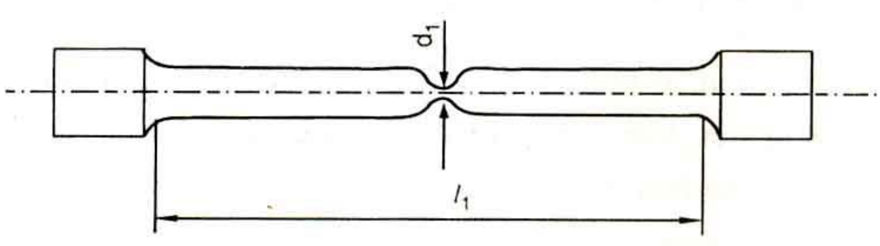


Figure 2.6

* Characteristics for the ductility of materials: σ_{pr}, σ_{yiel},σ_{ulti}.

* Characteristics for the plasticity of materials: δ, ψ

In which:

δ is the extension in the length of the specimen after fracture to its initial gauge length and is defined as follows $\delta = \frac{l_1 - l_0}{l_0}.100\%$ in which $l_1$is the gauge length of specimen after fracture.

$\psi$is the percent reduction in area measuring the amount of necking thatoccurs and is defined as follows: $\psi = \frac{F_0 - F_1}{F_0}.100\%$ in which $F_1$is the final area at the fracture section.

Thanks to diagram P - Δl, we have σ_z - $\varepsilon_z$diagram shown in figure 2.7. P_{tl}, P_{ch}, $P_b$is replaced byσ_{tl}, σ_{ch},σ_b respectively.

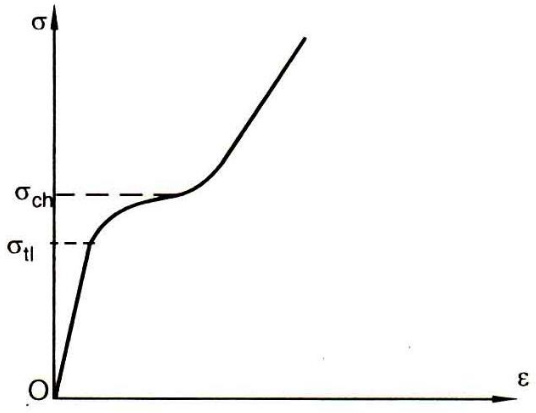


Figure 2.7

b. Specimen made from brittle materials

Materials that fail in tension at the relatively low values of strain areclassified as brittle materials. Examples are concrete, stone, cast iron, glass,ceramics, and a variety of metallic alloys. Brittle materials fail with onlylittle elongation after the proportional limit (the stress at point A inFig. 2.8) is exceeded. Furthermore, the reduction in area is insignificant,and so the nominal fracture stress (point B) is the same as the trueultimate stress. Therefore, brittle materials do not have proportional limit and yielding limit. They only have ultimate limit.

$$\sigma_{ulti} = \frac{P_{ulti}}{F_0} \quad (2\text{-}8)$$

The values$\sigma_{ulti}$of brittle materials in tension are very small in comparison with the values$\sigma_{ulti}$of ductile materials.

Although brittle materials do not have elastic limit, it can be considered a period as elastic period. For example, OA is on the diagram. At that time, the maximum stress here is proportional limit and OA is regarded as a line.

The mechanical property of brittle materials in tension is σ_{ulti}^{comp}.

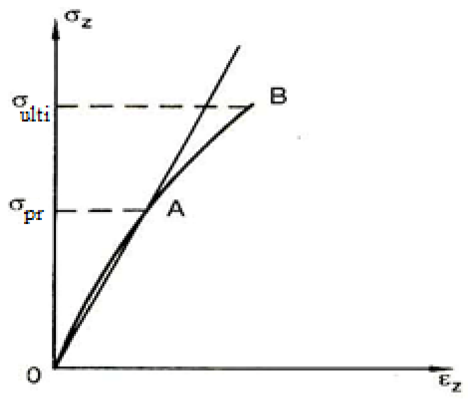


Figure 2.8

2.4.2 Compression test

2.4.2.1.Specimen: It is in the form of cylinder with h = (1 ÷ 2)d for specimen made from metals and certain plastics such as cast-iron and iron.It is in the form of cube for specimen made from building materials such as concrete.

2.4.2.2. Experiment:The test specimen is installed into machine and pushed.

2.4.2.3. Compression diagram:

a. Ductile materials: Stress-strain curves for materials in compression differ from those in tension. Ductile metals such as steel, aluminum, and copper have proportional limits in compression very close to those in tension, and the initial regions of their compressive and tensile stress-strain diagrams are about the same. However, after yielding stage begins, the behavior is quite different. In a tension test, the specimen is stretched, necking may occur, and fracture ultimately takes place. When the material is compressed, it bulges outward on the sides and becomes barrel shaped, because friction between the specimen and the end plates prevents lateral expansion. With increasing load, the specimen is flattened out and offers greatly increased resistance to further shortening (which means that the stress-strain curve becomes very steep). Since the actual cross-sectional area of a specimen tested in compression is larger than the initial area, the true stress in a compression test is smaller than the nominal stress.

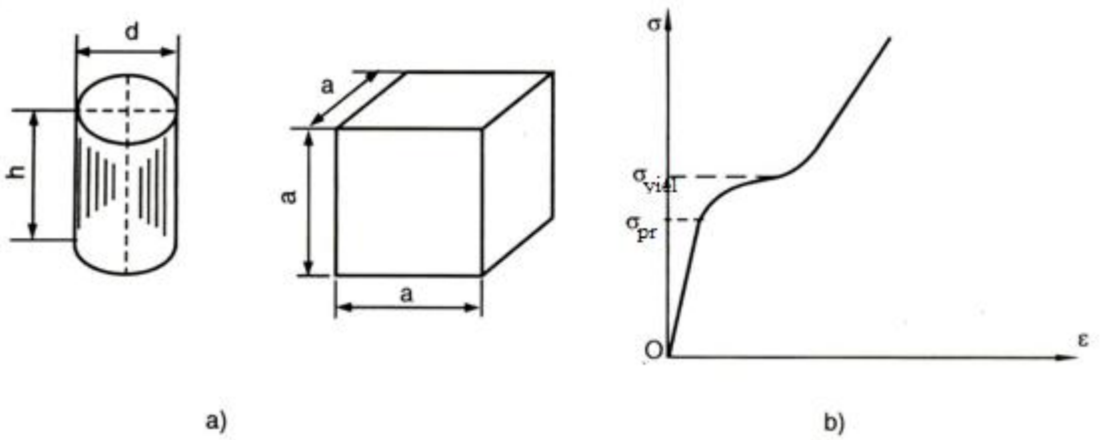


Figure 2.9

The mechanical properties of ductile materials areσ_{pr}andσ_{yiel}. It can be said that ability in the tension and compression of ductile materials is the same.

b. Brittle materials

Brittle materials loaded in compression typically have an initial linear region followed by the region in which the shortening increases at a slightly higher rate than does the load. The stress-strain curves for compression and tension often have similar shapes, but the ultimate stresses in compression are much higher than those in tension. Also, unlike ductile materials, which flatten out when compressed, brittle materials actually break at the maximum load:

$$\sigma_{ulti} = \frac{P_{ulti}}{F_0} \qquad (2\text{-}9)$$

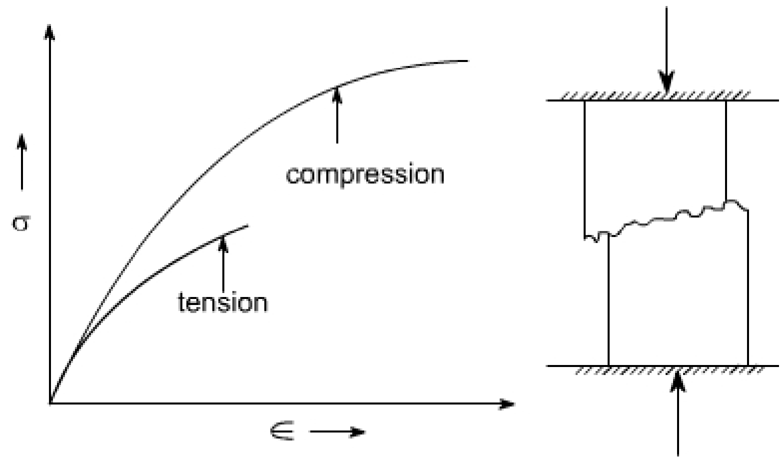


Figure 2.10

The mechanical property of brittle materials isσ_{ulti}^{comp}. Brittle materials suffer from compression better than tension.

2.5. Compute axially loaded bar

2.5.1. Dangerous stress-Allowable stress

2.5.1.1.Dangerous stress: σ_O

Dangerous stress is the value at which material is destroyed.

For brittle material: $\sigma_0 = \sigma_{ulti}$

For ductile material: $\sigma_0 = \sigma_{yiel}$

2.5.1.2. Factor of safety: n

If structural failure is to be avoided, the load that a structure is capable of supporting must be greater than the load it will be subjected to when in service. Since strengthis the ability of a structure to resist loads, the preceding criterion can be restated as follows: The actual strength of astructure must exceed the required strength. The ratio of the actual strength to the required strength is called the factor of safety n:

n = actual strength / required strength

Of course, the factor of safety must be greater than 1.0 if failure is to be avoided. Depending upon the circumstances, factors of safety from slightly above 1.0 to as much as 10 are used.

The incorporation of factors of safety into design is not a simple matter, because both strength and failure have many different meanings. Strength may be measured by the load-carrying capacity of a structure, or it may be measured by the stress in the material. Failure may mean the fracture and complete collapse of a structure, or it may mean that the deformations have become so large that the structure can no longer perform its intended functions. The latter kind of failure may occur at loads much smaller than those that cause actual collapse. The determination of a factor of safety must also take into account such matters as the following: the probability of accidental overloading of the structure by loads that exceed the design loads; the types of loads (static or dynamic); whether the loads are applied once or are repeated; how accurately the loads are known; possibilities for fatigue failure; inaccuracies in construction; variability in the quality of workmanship; variations in the properties of materials; deterioration thanks to corrosion or other environmental effects; the accuracy of the methods of analysis; whether failure is gradual (ample warning) or sudden (no warning); the consequences of failure (minor damage or major catastrophe); and other such considerations. If the factor of safety is toolow, the likelihood of failure will be high and the structure will be unacceptable; if the factor is too large, the structure will be wasteful of materials and perhaps unsuitable for its function (for instance, it may be too heavy).

$$n = n_1.n_2.n_3.n_4\ldots$$

Because of these complexities and uncertainties, factors of safety must be determined on a probabilistic basis. They usually are established by the groups of experienced engineers who write the codes and specifications used by other designers, and sometimes they are even enacted into law. The provisions of codes and specifications are intended to provide reasonable levels of safety without unreasonable costs.

2.5.1.3. Allowable stress: [σ]

Factors of safety are defined and implemented in various ways. Formany structures, it is important that the material remain within thelinearly elastic range in order to avoid permanent deformations when theloads are removed. Under these conditions, the factor of safety isestablished with respect to yielding limit of the structure. Yielding stage beginswhen the yield stress is reached at any point within the structure. Therefore,by applying a factor of safety with respect to the yield stress (oryield strength), we obtain an allowable stress (or working stress) thatmust not be exceeded anywhere in the structure. Thus,

allowable stress = yield strength / factor of safety

In building design, a typical factor ofsafety with respect to yielding in tension is 1.67; thus, a mild steelhaving a yield stress of 36 ksi has an allowable stress of 21.6 ksi.Sometimes the factor of safety is applied to the ultimate stressinstead of the yield stress. This method is suitable for brittle materials,such as concrete and some plastics, and for materials without a clearlydefined yield stress, such as wood and high-strength steels. In these cases the allowable stresses in tension and shear are:

allowable stress = ultimate strength / factor of safety

Factors of safety with respect to the ultimate strength of a material areusually larger than those based upon yield strength. In the case of mildsteel, a factor of safety of 1.67 with respect to yielding corresponds toa factor of approximately 2.8 with respect to the ultimate strength.

Because n > 1, we have $[\sigma] < \sigma_0$

2.5.2. Strength of axially loaded bar

Stress in bar is not exceeded allowable stress to ensure that bar operates safely.

- Maximum stress in bar:

$$\sigma_{z\max}^{tens} = \left(\frac{N_z}{F}\right)_{\max}^{+} \quad \text{(Maximum tensile stress)}$$

$$\sigma_{z\max}^{comp} = \left(\frac{N_z}{F}\right)_{\max}^{-} \quad \text{(Maximum compressive stress)}$$

- Allowable stress:

Brittle material: $[\sigma]_{tens} = \frac{\sigma_0^{tens}}{n} = \frac{\sigma_{ulti}^{tens}}{n};[\sigma]_{comp} = \frac{\sigma_0^{comp}}{n} = \frac{\sigma_{ulti}^{comp}}{n}$

Ductile material: $[\sigma]_{tens} = [\sigma]_{comp} = [\sigma] = \frac{\sigma_0}{n} = \frac{\sigma_{yeil}}{n}$

Thus, expression of strength is:

Brittle material: $\begin{cases} \sigma_{z\max}^{tens} \le [\sigma]_{tens} \\ \left|\sigma_{z\max}^{comp}\right| \le [\sigma]_{comp} \end{cases}$

Ductile material: $\left|\sigma_Z\right|_{max} \le [\sigma]$

According to the condition of strength, there are three basic problems.

2.5.3. Stiffnesss

The strain and deformation which happen in bar are not exceeded allowable values to ensure that bar operates safely. The expression of stiffness is shown as one of the following forms:

According to strain:

$$|\varepsilon_Z|_{\max} = \left|\left(\frac{N_z}{EF}\right)\right|_{\max} \leq [\varepsilon] \quad ([\varepsilon]:\ \text{allowably longitudinal strain})$$

According to the elasticity of bar:

$$|\Delta l| \leq [\Delta l] \qquad ([\Delta l]:\ \text{allowable elasticity})$$

According to the displacement of cross-section:

$$\Delta_z^K \leq [\Delta] \qquad ([\Delta]:\ \text{allowable transposition})$$

According tothe conditions ofstiffness, there are three basic problems.

2.5.4. Three basic problems used to compute axially loaded bar thanks tothe conditions ofstrength and stiffnesss

a. Test problem: it is asked to check whether bar has enough strength and stiffness or not.

Procedure:

- Draw internal force diagram N_z, stress diagramσ_z.

- According to stress diagram$\sigma_z \rightarrow$determine the dangerous cross-section which suffers from maximum stress.

- Replace the value in the condition of strength, compare and conclude strength.

- Compute the maximum strain (orΔl, orstrain).

- Replace the value inthe condition of stiffness, compare and conclude strength.

b. The problem of determining allowable load

It is asked to determine the maximum value of allowable load put on structure to ensure that bar still ensures strength and stiffness.

Procedure:

- Step1: Determine and draw longitudinal force diagram, stress diagram and strain diagram through the parameters of loads.

- Step2: According to diagrams, determine dangerous cross-section which suffers from maximum stress and maximum strain.

- Step 3: Determine allowable load thanks tothe condition of strength and stiffness.

- Step 4: Select smaller value to conclude the general result for problem.

c. The problem of selecting dimensions of cross-section

It is asked to determine the necessary area of cross-section so that bar meets requirements about strength, stiffness but still saves materials.

Procedure:

- Step 1: Determine and draw internal force diagram, stress diagram and deformation diagram. Thus, dangerous cross-section is determined.

- Step 2: The determine dimensions of cross-section thanks tothe condition of strength:

$$\sigma_{z\max} = \left(\frac{N_z}{F}\right)_{\max} \leq [\sigma]$$

Thus, we have: $F \geq \left(\frac{N_z}{[\sigma]}\right)$

Through area F found, the dimensions of cross-section are determined.

- Step 3: Determine the dimensions of cross-section thanks tothe condition ofstiffness.

$$\varepsilon_{z\max} = \left(\frac{N_z}{EF}\right)_{\max} \leq [\varepsilon]$$

We have: $F \geq \left(\frac{N_z}{E[\varepsilon]}\right)$

Through F, the dimensions of cross-section are determined.

Example 2: Check strength and stiffnessfor the bar shown in the figure 2.11. Know that P = 5kN; F = 1cm^2; [σ] = 16kN/cm^2, [ε]= 10^{-2} và E = 2.10^{4}kN/cm^2.

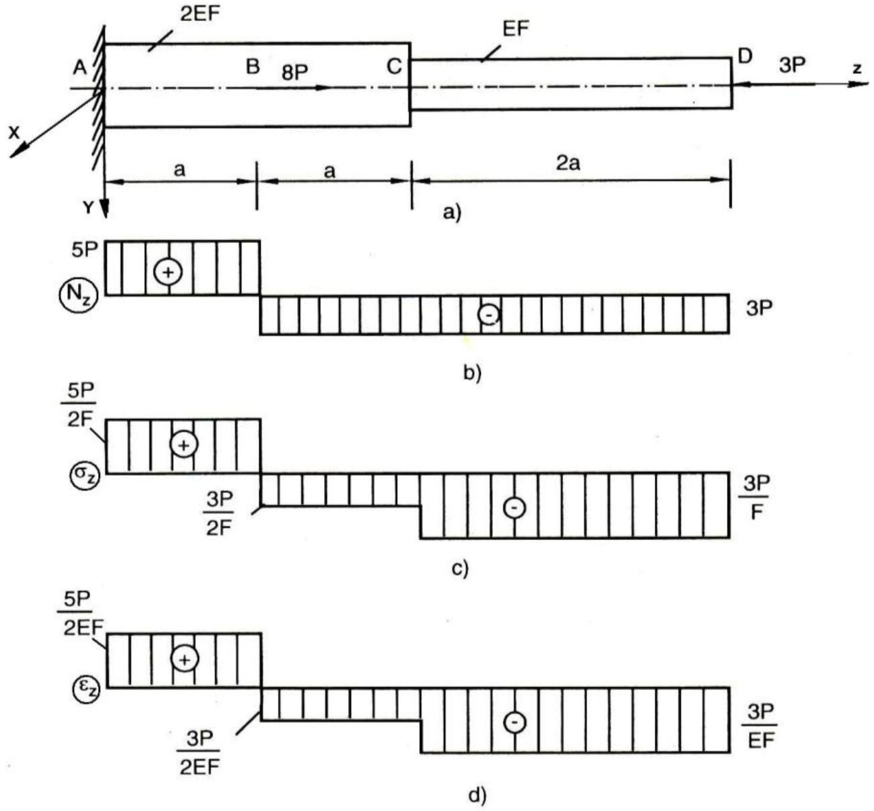


Figure 2.11

- Step 1: Draw N_z diagram (b), stress diagram (c) and deformation diagram (d)
- Step 2: According to diagrams, dangerous cross-section belongs to CD with:

$$|\sigma_z|_{max} = \frac{3P}{F}$$

$$|\varepsilon_z|_{max} = \frac{3P}{EF}$$

- Step 3: Check strength

$$|\sigma_z|_{max} = \frac{3P}{F} = \frac{3.5}{1} = 15kN/cm^2 < [\sigma]$$

In conclusion, bar is enough strength.

- Step 4: Check stiffness

$$|\varepsilon_z|_{max} = \frac{3P}{EF} = \frac{3.5}{2.10^4.1} = 7,5.10^{-4} < [\varepsilon]$$

In conclusion, bar is enough stiffness.

Example3: The bar is subjected as shown in the figure 2.11. Determine allowable load [P] if we know that F = 2cm^2; [σ] = 16kN/cm^2; [ε] = 10^{-3} và E = 2.10^4 kN/cm^2.

- Step 1, 2: Draw diagrams. Dangerous cross-section is CD.

- Step 3: According to the condition of strength, we have:

$$|\sigma_z|_{\max} = \frac{3P}{F} \le [\sigma] \rightarrow [P] = \frac{F[\sigma]}{3} = \frac{2.16}{3} = \frac{32}{3} kN$$

According to stiffness, we have:

$$|\varepsilon_z|_{\max} = \frac{3P}{EF} \le [\varepsilon] \rightarrow [P] = \frac{EF[\varepsilon]}{3} = \frac{2.10^4.2.10^{-3}}{3} = \frac{40}{3} kN$$

- Step 4: Select [P] $= \frac{32}{3} = 10{,}67$kN .

Example 4: The structure is subjected to load as shown in the figure 2.11. The rod BC is made from circular steel. Determine its diameter. Know that P = 400N; [σ] = 14kN/cm^2; [ε] = 10^{-2}; α = 30^0.

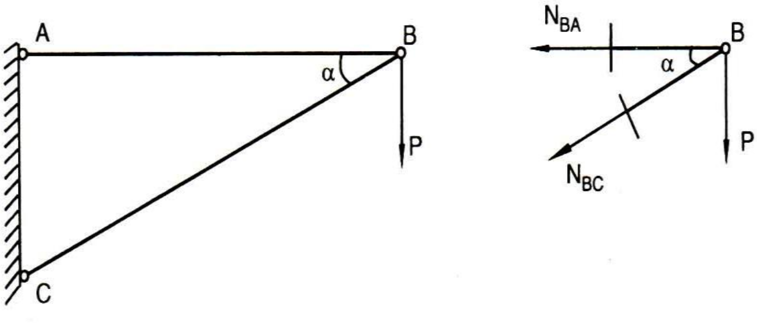


Figure 2.12

Solution: Split joint B by the cross-sections of rods. Longitudinal forces areexpressed in the figure 2.12.

Static equation is established thanks to summing forces in the vertical direction:

$$N_{BC}.\sin\alpha + P = 0$$

Thus:

$$N_{BC} = -\frac{P}{\sin\alpha} = -\frac{400}{\sin 30^0} = -800 = -0{,}8kN$$

Minus sign (-) shows that the rod BC is compressed.

According to the condition of strength, we have:

$$\sigma_Z = \frac{N_{BC}}{F} = \frac{N_{BC}}{\frac{\pi d^2}{4}} \le [\sigma]$$

or $d = \sqrt{\dfrac{4N_{BC}}{\pi[\sigma]}} = \sqrt{\dfrac{4.0,8}{3,14.14}} = 0,27cm$

According to the condition of stiffness, we have:

$$\varepsilon_Z = \frac{N_{BC}}{EF} = \frac{N_{BC}}{E\dfrac{\pi d^2}{4}} = \frac{4N_{BC}}{E\pi d^2} \le [\varepsilon]$$

or $d \ge \sqrt{\dfrac{4N_{BC}}{\pi E[\varepsilon]}} = \sqrt{\dfrac{4.0,8}{3,12.2.10^4.10^{-2}}} = 0,05cm$

with $E = 2.10^4 kN/cm^2$

Therefore, we select d = 0,27cm.

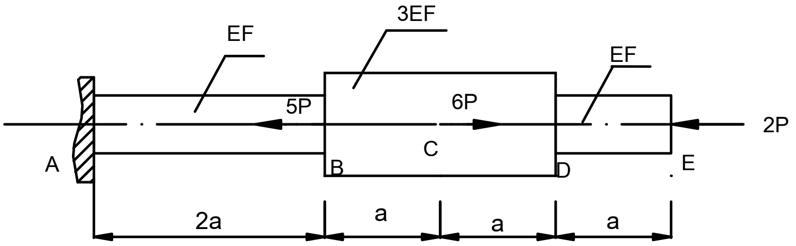
Theoretical questions

1. Raise examples about axially loaded structures. Why can it said that axial tension or compression is one reasonable load-resistant form of bar?
2. What kind of stress exists on the cross-section of axially loaded bar? What is its expression? Take illustrative pictures about the distribution of stress on some sections?
3. What is the longitudinal strain, lateral strain and deformation of cross-section? What is the expression of longitudinal strain, lateral strain and elasticity of bar?
4. Raise the mechanical properties of ductile materials and brittle materials. Do you have any ideas about their load-resistant ability.
5. Raise the condition of strength and stiffness of axially loaded bar. Raise the way to solve three basic problems.

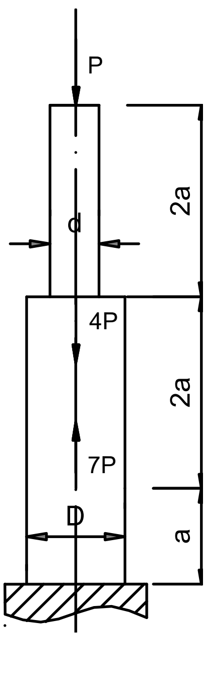
Numerical problems

Exercise 1:

a, Draw the internal force diagram, stress diagram, strain diagram of the following bar. (through P, a, E, F)



b, Determine cross-section area. Know that P = 120 kN, $[\sigma] = 150 MN/m^2$.



Exercise 2:

A circular bar has D = 2d. Know that: P = 100 kN, d = 5 cm, $[\sigma]_{tens} = 120 MN/m^2$, $[\sigma]_{comp} = 360 MN/m^2$, $[\varepsilon_z] = 10^{-4}$, a = 2 m, $E = 2.10^5 MN/m^2$

- Check strength and stiffness.
- Determine allowable load.
- Compute the elasticity of bar.

Exercise 3:

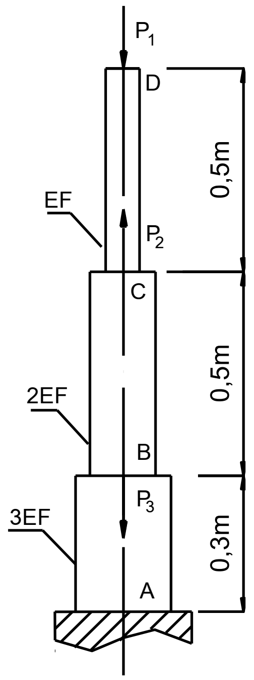
Know that: P_1 = 20 kN, P_2 = 60 kN, P_3 = 120 kN, E = 2.10^{11} N/m^2, F = 2 cm^2.

a, Draw internal force diagram, stress diagram, longitudinal strain diagram, deformation diagram.

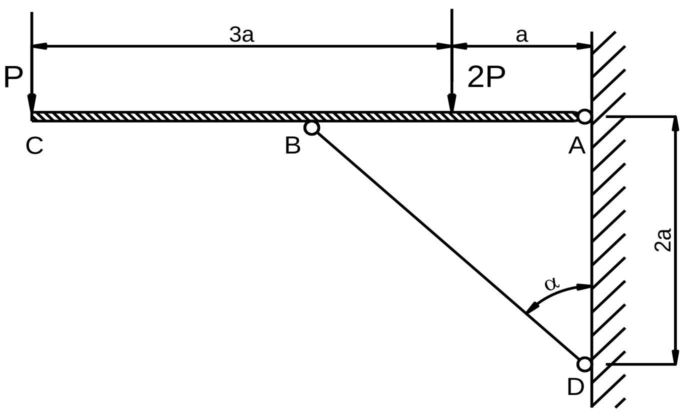
- ComputeΔl.

- Check strength and stiffness. Know that:[σ] = 140 MN/m^2 , [Δ^D_z] = 0,2 cm.

b, Be still above requirements but include the gravity of bar (know thatγ = 80 kN/m^3).



Exercise 4:



Know that P = 200 kN, a = 0,6 m

α = 45^0, E = 2.10^4kN/cm^2

[σ] = 140 MN/m^2

[ε_z] = 3.10^{-4}

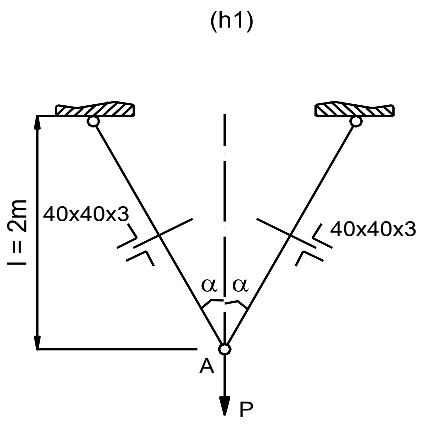
- Select sign number for the I-steel of the bar BD

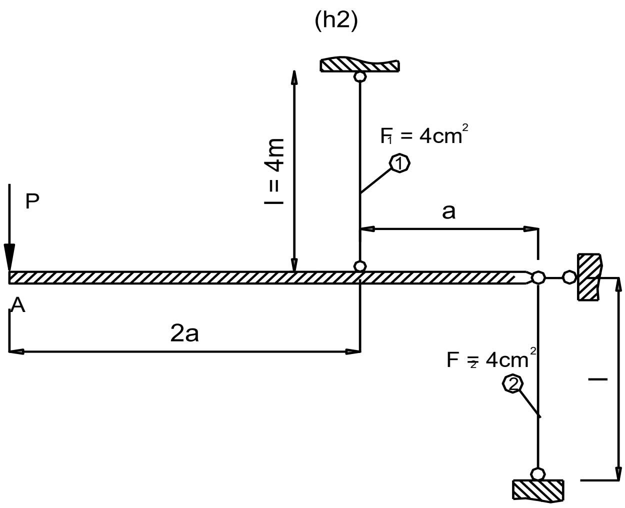
- According to selected sign number, compute the vertical deflection of point C.

Exercise 5:

The two-bar truss is shown in the figure. (N^{0}1 and N^{0}2). Determine [P].

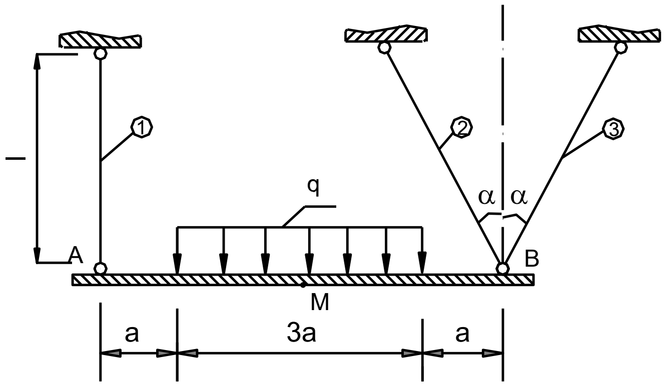
Know that: [σ] = 140 MN/m^2 , E = 2.10^4kN/cm^2 , α = 30^o. The deflection of point A is not exceeded 1,5 mm.





Exercise 6:

a,

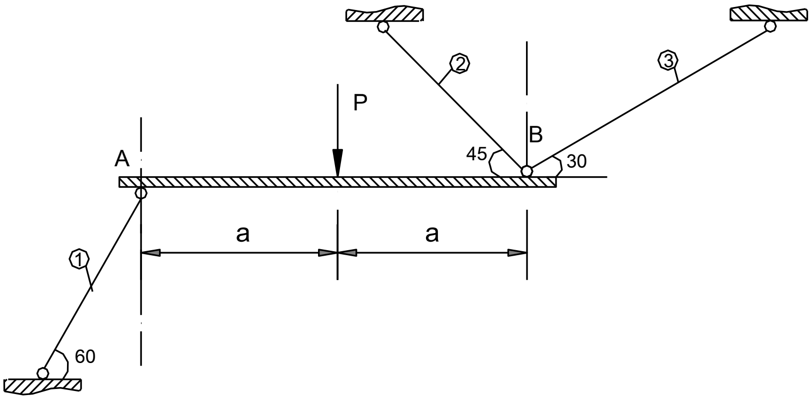


- Determine the cross-section areas of bars.
- According to selected areas, compute the deflection of point M of the bar AB.

q = 500 kN/m , a = 0,8 m

[σ] = 150 MN/m^2, l = 3 m, E = 2.10^5 MN/m^2, $\alpha = 30^0$.

b,



Know that P = 200 kN

[σ] = 150 MN/m^2.

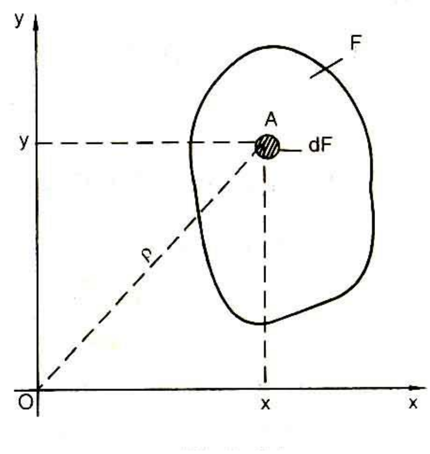
Compute the diameters of bars.

CHAPTER III - PROPERTIES OF AREAS

The cross-sectional areas of beams, shafts and columns have seven properties that we need in order to calculate stresses, deflections, angles of twist, and buckling resistance in these structures. These properties are dimensions, area, centroid, centroidal x-x and y-y axes, moment of inertia, radius of gyration, and polar moment of inertia.

3.1. Properties of areas

Assume that there is a cross-section which has an area of F as shown in the figure 3.1. Determine a co-ordinate system x0y in section and x,y are called the co-ordinates of point A in that area F. Consider a small area dF surrounding A.



3.1.1 Static moment

The static moments of area F about axis x or axis y are following integration expressions:

$$S_x = \int_F y dF \qquad S_y = \int_F x dF \qquad (3\text{-}1)$$

Figure 3.1

Because x, y can be negative or positive, static moment can also be negative or positive. Through the expressions above, the unit of static moment is [(length)^3] and is measured by the units of cm^3, m^3 vv...

Whenthe static moment of area F aboutan axis equals 0, that axis is called centroidial axis. The node of two centroidial axes is called the centroid of section.

It is obvious that the axis which goes through centroid is also centroidial axis. It is explained that becausethe co-ordinate of centroid about that axis equals 0, the static moment of area about that axis also equals 0.

3.1.2. Moment of inertia to an axis

Moments of inertia of area F about axis x or y are following integration expressions:

$$J_x = \int_F y^2 dF \qquad J_y = \int_F x^2 dF \qquad (3\text{-}2)$$

The unit of moment of inertia is [(length)^4]. It is measured by units of cm^4, m^4... The magnitude of moment of inertia is always positive.

Moment of inertia (it is also known as the second moment of area) is used in the study of mechanics of fluids and mechanics of solids.

3.1.3. Polar moment of inertia (moment of inertia about a point)

Polar moment of inertia of area F about origin 0 is following integration expression:

$$J_P = \int_F \rho^2 dF \qquad (3\text{-}3)$$

ρ is distance from point A(x, y) to origin 0. Because we have the relation

$\rho^2 = x^2 + y^2$

It can be realised that

$J_p = J_x + J_y$

Like moment of inertia, polar moment of inertia is always positive.

3.1.4. Product of inertia of area F (centrifugal moment of inertia)

Centrifugal moment of inertia of area F about axes x0y is integration expression:

$$J_{xy} = \int_F xydF \qquad (3\text{-}4)$$

Because x, y can be opposite in sign, centrifugal moment of inertia can be negative or positive.

When centrifugal moment of inertia of area F about axes equals 0, those axes are called principal axesof inertia.

We can determine principal axes of inertiaat any point of section.

If cross-section has a symmetric axis, any axis which is perpendicular to that symmetric axis also creates principal axesof inertia.

On the other hand, the centroid C of cross-section is on the symmetric axis; therefore, if a line is drawn to be perpendicular to axis y through C, it will createcentroidally principal axesof inertia.

According to expression (3-4), it can be proved that if cross-section has a symmetric axis, that axis is also an axis of centroidally principal axesof inertia.

3.1.5. Radius of gyration

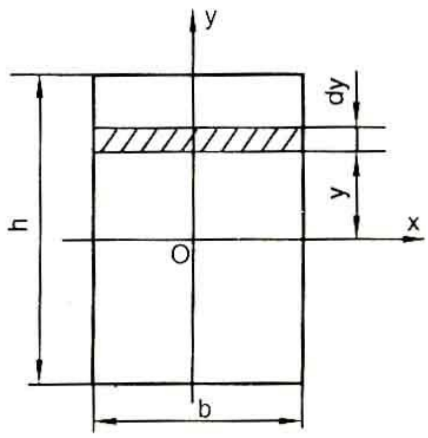
Radius of gyration of area F about axes x0y is the following expressions:

$$i_x = \sqrt{\frac{J_x}{F}}$$

$$i_y = \sqrt{\frac{J_y}{F}}$$

3.2. Moment of inertia of some popular cross-sections

3.2.1. Rectangular section



Assume that we have a rectangular section which has awidth b and a depth h. Let 0x is horizontal axis and 0y is the vertical axis passing through center of gravity of rectangular section.

Because x0y is centroidally principal axesof inertia, $S_x = S_y = J_{xy} = 0$

To determine J_x, we consider a rectangular elementary stripof thickness dy at a distance y of axis 0y.

Area of strip: dF = b.dy Figure 3.2

Therefore, moment of inertia of the area of the strip about 0x axis = area of strip x $y^2 = y^2.dF$.

Moment of inertia of the whole section will be obtained by integrating the above equation between the limits $-\frac{h}{2}$ to $\frac{h}{2}$.

$$J_x = \int_F y^2 dF = \int_{-h/2}^{h/2} y^2 bdy = b\frac{y^3}{3}\Bigg|_{-h/2}^{h/2} = \frac{bh^3}{12} \qquad (3\text{-}5)$$

Similarly, moment of inertia of the rectangular section about 0y axis passing through center of gravity of the section is given by:

$$J_y = \frac{hb^3}{12} \qquad (3\text{-}6)$$

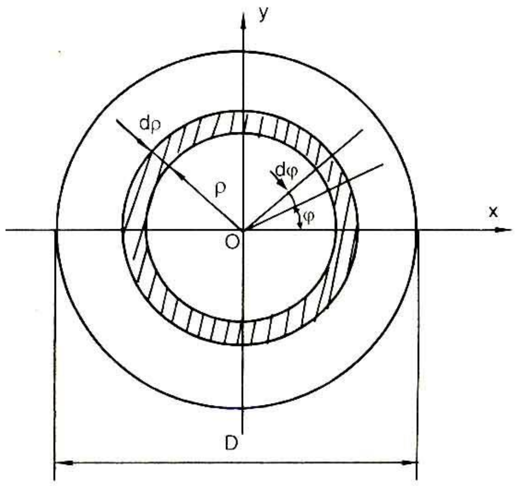
3.2.2. Circular section

Assume that we have a circular section which has a diameter D = 2R.

Co-ordinate system is selected as in the figure.

Because x0y is centroidally principal axes, $S_x = S_y = J_{xy} = 0$

Split a small strip which is limited by two circles having the same center and two rays. The first circle has radius ρ and the second circle has radius ρ + dρ. The first ray associates with axis 0x to create an angle φ. The second ray associates with the first ray to create an angle dφ. The area of element is dF = ρ.dφ.dρ.



At that moment,

$$J_P = \int_F \rho^2 dF = \int_0^{2\pi}\int_0^R \rho^3 d\rho d\varphi = 2\pi \frac{\rho^4}{4}\Big|_0^R = \frac{\pi R^4}{2}$$

Figure 3.3

If replace D = 2R or $R = \frac{D}{2}$ in the above expression,

$$J_P = \frac{\pi D^4}{32} \qquad (3\text{-}7)$$

Because section is circular,

$$J_x = J_y = \frac{J_P}{2} = \frac{\pi D^4}{64} \qquad (3\text{-}8)$$

If section is hollow circular section which has external diameter D, internal diameter d,

$$J_P = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \frac{\pi D^4}{32}\left[1 - \left(\frac{d}{D}\right)^4\right] \qquad (3\text{-}9)$$

The ratio $\frac{d}{D} = \alpha$ is called hollow coefficient. We have

$$J_P = \frac{\pi D^4}{32}\left(1 - \alpha^4\right)$$

and $J_x = J_y = \frac{\pi D^4}{64}\left(1 - \alpha^4\right)$ (3-10)

3.2.3. The section of shaped steel

In parctice, people tend to use the steel bars which has shaped sections I ; [; L. The properties of these sections has already been calculated and tabulated. These tables are printed in technical handbooks and documents.

3.3. Parallel axes theorem for static moment and moment of inertia

Some compound cross-sections are made of components which do not share the same centroidal axis. As long as the centroidally principal axes are parallel, we can use the formula of transfering axes to find the moment of inertia of the sections.

Assumne that moment of inertia of area F about axes x0y is known. We must calculate moment of inertia of area F about axes XO_1Y. Consider axes XO_1Y created by equally advancing axis 0x at a distance b and axis 0y at a distance a. (figure 3.4)

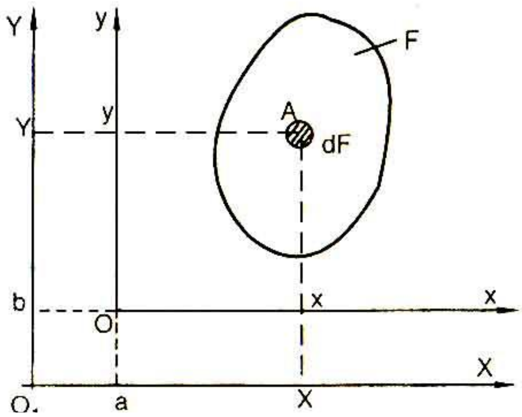


Figure 3.4

In this new axes, point A (X,Y) is determined by:

X = x + a (a)

Y = y + b

According to concept (3-1):

$$S_X = \int_F YdF = \int_F (y+b)dF = \int_F ydF + b\int_F dF = S_x + bF$$

$$S_Y = \int_F XdF == \int_F (x+a)dF = \int_F xdF + a\int_F dF = S_y + aF$$

(3-11)

$$J_X = \int_F Y^2dF = \int_F (y+b)^2 dF = \int_F y^2dF + 2b\int_F ybF + b^2\int_F dFJ_X = J_x + 2bS_x + b^2F$$

$$J_Y = \int_F X^2dF = \int_F (x+a)^2 dF = \int_F x^2dF + 2a\int_F xF + a^2\int_F dF = J_y + 2aSy + a^2F$$

$$J_{XY} = \int_F XYdF = \int_F (x+a)(y+b)dF = \int_F xydF + a\int_F ydF + b\int_F xdF + ab\int_F dF = J_{xy} + aS_x + bS_y + abF$$

In case axes x0y are centroidally principal axes, $S_x = S_y = 0$ and the formulas (3-11) are written as below:

$S_X = Y_c.F$

$S_Y = X_c.F$

$J_X = J_x + b^2F$ (3-12)

$J_y = J_y + a^2F$

$J_{XY} = J_{xy} + abF$

Therefore, formulas to determine center of gravity of the section can be inferred:

$$X_C = \frac{S_Y}{F}$$

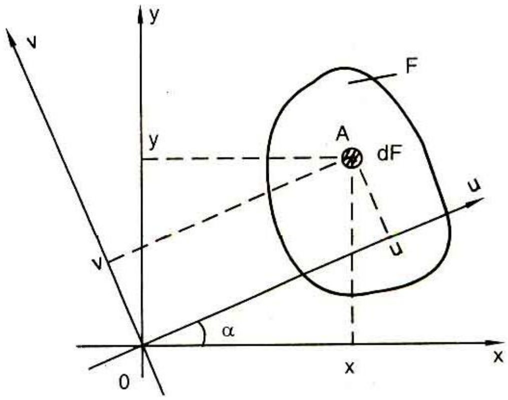
(3-13)

$$Y_C = \frac{Sx}{F}$$

3.4. The formulas of rotating axes of moment of inertia - principal axes of inertia

3.4.1. The formulas of rotating axes

Figure 3.5 shows a body of area F with respect to old axes (x, y) and new axes (u,v). The new axes u and v have been rotated through an angle α in anticlockwise direction. Consider a small area dF. The co-ordinates of the small area with respect to old axes are (x, y) whereas with respect to new axes are (u ,v). The new co-ordinates (u,v) are expressed in terms of old co-ordinates (x, y) and angle α:



$$u = x\cos\alpha + y\sin\alpha$$

$$v = y\cos\alpha - x\sin\alpha \qquad \text{(a)}$$

Moment of inertia and product of inertia of area F with respect to new axes are:

$$J_u = \int_F v^2 dF$$

$$J_v = \int_F u^2 dF$$

$$J_{uv} = \int_F uvdF$$

Subsitute the values of u, v from equations (a), we get:

Figure 3.5

$$J_u = J_x\cos^2\alpha - 2J_{xy}\sin\alpha\cos\alpha + J_y\sin^2\alpha$$

$$J_v = J_x\sin^2\alpha + 2J_{xy}\sin\alpha\cos\alpha + J_y\cos^2\alpha$$

$$J_{uv} = J_x\sin\alpha\cos\alpha + J_{xy}\left(\cos^2\alpha - \sin^2\alpha\right) - J_y\sin\alpha\cos\alpha$$

Use following trigonometry alteration:

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

We get:

$$J_u = \frac{J_x + J_y}{2} + \frac{J_x - J_y}{2}\cos 2\alpha - J_{xy}\sin 2\alpha$$

$$J_v = \frac{J_x + J_y}{2} - \frac{J_x - J_y}{2}\cos 2\alpha + J_{xy}\sin 2\alpha \qquad \text{(3-14)}$$

$$J_{uv} = \frac{J_x - J_y}{2}\sin 2\alpha + J_{xy}\cos 2\alpha$$

Adding two sides of the first two equations, we get:

$$J_u + J_v = J_x + J_y = 0$$

The equation shows that the sum of moments of inertia about old axes (x, y) and new axes (u, v) are the same. Hence, the sum of moments of inertia of area F is independent of orientation of axes.

3.4.2. Centroidally principal axes of inertia and centroidally principal moment of inertia

We have already defined the principal axesof inertia. Principal axes of inertiaare the axes about which product of inertia is zero. Now, the new axes (u, v) will become principal axes of inertiaif product of inertia given by equation (3-14) is zero:

$$J_{uv} = \frac{J_x - J_y}{2}\sin 2\alpha + J_{xy}\cos 2\alpha = 0$$

Hence:

$$tg2\alpha = -\frac{2J_{xy}}{J_x - J_y} \qquad (3\text{-}15)$$

Use following trigonometry alteration:

$$\sin 2\alpha = \pm \frac{tg2\alpha}{\sqrt{1 + tg^2 2\alpha}}$$

$$\cos 2\alpha = \pm \frac{1}{\sqrt{1 + tg^2 2\alpha}}$$

Substitute the value tg2α in equation (3-15) and continue to substitute in the first equations (3-14), the values of moments of inertia can be obtained.

$$J_{max} = \frac{J_x + J_y}{2} + \sqrt{\left(\frac{J_x - J_y}{2}\right)^2 + J_{xy}^2}$$

$$J_{min} = \frac{J_x + J_y}{2} - \sqrt{\left(\frac{J_x - J_y}{2}\right)^2 + J_{xy}^2} \qquad (3\text{-}16)$$

The values of moments of inertia are maximum because the derivation of J_u about angle α is:

$$\frac{dJ_u}{d\alpha} = -2\left[\frac{J_x - J_y}{2}\sin 2\alpha + J_{xy}\cos 2\alpha\right] = -2J_{uv}$$

In principal axes, $J_{uv} = 0$; so the above derivation is zero.

In terms of mathematics, the relationship between J_u, J_{uv} and J_x , J_y, J_{xy} expressed by equation (3-14) is similar to the relationship between σ_u, τ_{uv} and σ_x, σ_y, τ_{xy} which was established in the previous chapter. Therefore, it can be realized that if we use a co-ordinate system in which horizontal axis expresses values of J_u and vertical axis expresses values of J_{uv} , the relationship between J_u and J_{uv} is expressed through a circle. This circle is called Morh's circle of inertia.

Procedure to determine this circle is similar to that to determine Morh's circle of stress.

- Thecenter C of circle is the middle point of the points which have abscissas J_x and J_y.
- Origin point D has co-ordinates J_x, J_{xy}.
- Polar point P has co-ordinates J_y, J_{xy}.

If we draw the line PM which associates with PD to create an angle α, the co-ordinates of M will express values of moments of inertia of area F about axes x0y. (the location of axes x0y was rotated an angle α)

Thedirections of principal axes are the lines PA and PB.

According to figure, the following equations are determined:

$$tg\alpha_1 = \frac{J_{xy}}{J_x - J_{max}}$$

$$tg\alpha_2 = \frac{J_{xy}}{J_y - J_{min}}$$

α_1, α_2 are the angles which are created by the main direction and PD. The equations of J_{max} và J_{min} are also determined as equations (3-16).

3.5. Determine moment of inertia of compound sections

To determine centroidally principal axesof inertia and centroidally principal moment of inertia of compound sections, we use the following steps:

- Step 1: Pick a co-ordinate system x0y. Determine theco-ordinates of centroid C of section in this co-ordinate system:

$$X_C = \frac{\sum_i S_y^i}{\sum_i F_i};\ y_C = \frac{\sum_i S_x^i}{\sum_i F_i} \qquad \text{(a)}$$

S^i_x; S^i_y are thestatic moments of components about axex 0x and 0y; F_i is thearea of component i.

- Step 2: Draw a centroidal co-ordinate system XCY. Thecentroidal axes of components are $x_iO_iy_i$. Use thetheorem of equally advancing axes, we get:

$$J_X = \sum_i \left(J_{xi}^i + b_i^2 F_i\right)$$

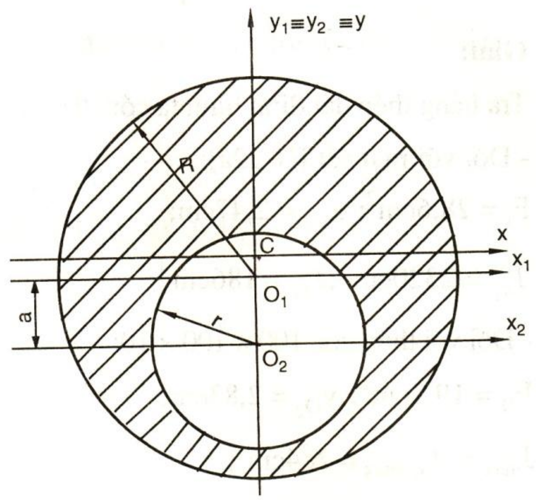
$$J_Y = \sum_i \left(J_{yi}^i + a_i^2 F_i\right)$$

$$J_{XY} = \sum_i \left(J_{x_i y_i}^i + a_i b_i F_i\right)$$

- Step 3: Rotate co-ordinate system XCY an angle α_0, we get centroidally principal axesof inertia 1C2.

- Step 4: Use theorem of rotating axes, we determine centroidally principal moment of inertia.

$$J_{1,2} = \frac{J_X + J_Y}{2} \pm \frac{1}{2}\sqrt{\left(J_x - J_y\right)^2 + 4J_{xy}^2}$$



Example 1:A circular section has radius $R = \frac{D}{2} = 20cm$ and is cut a circular hole which has radius $r = \frac{d}{2} = 10cm$. The centers of two circles are at a distance of a = 5 cm.

Determine centroidally principal axesof inertia and centroidally principal moment of inertia.

Solution: Select the basic axes are $x_1O_1y_1$ which are thecentroidal axes of big circle. The centroidal axes of small circle are $x_2O_2y_2$. It is easy to realize that axis O_1y_1 coincides with symmetric aixs 0y.Figure 3.6

According to consequence, symmetric axis is an axis of centroidally principal axesof inertia. Thecentroid C of section will lie on this symmetric axis.

In the basic axes $x_1O_1y_1$, centroid C has co-ordinates:

$$y_c = \frac{\sum_i S_{x_i}^i}{\sum_i F_i} = \frac{0 - F_2\left(-O_1O_2\right)}{F_1 - F_2}$$

$$= \frac{\frac{\pi d^2}{4}.a}{\frac{\pi D^2 - \pi d^2}{4}} = \frac{d^2 a}{D^2 - d^2} = \frac{\left(20\right)^2.5}{\left(40\right)^2 - \left(20\right)^2} = 1,66cm$$

Centroidally principal axesof inertia are xCy (figure 3.6). In this system, the centroids of the circles has co-ordinates $O_1(0, 1, 66)$; $O_2(0, 6, 66)$.

Now, we determine centroidally principal moment of inertia:

$J_x = J^1_x - J^2_x$

In which:

$$J^1_x = J^1_{x_1} + \overline{O_1C}^2 F^1$$

$$J^2_x = J^2_{x_2} + \overline{O_1C}^2 F^2$$

Replace the numerical values, we get:

$J_x \approx 107.10^3 cm^4$

and: $J_y = J^1_y - J^2_y = J^1_{y1} - J^2_{y2}$

Replace the numerical values, we get:

$J_y \approx 117.10^3 cm^4$

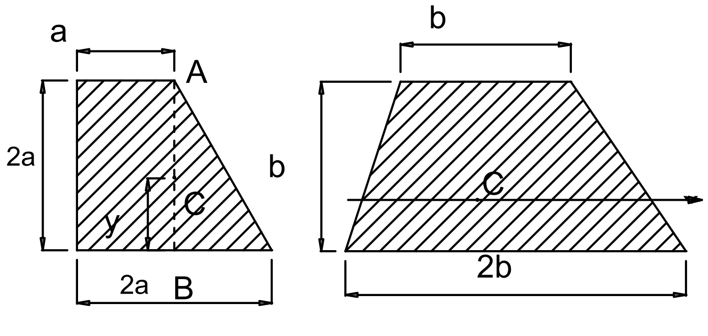
Theoretical questions

1. Define the properties of cross-sections? Raise units and the ways to determine.
2. Define centroidal axes. How to determine thecentroid of a compound section which is created from simple sections.
3. What are the principal axesof inertia, centroidally principal axesof inertia? Point the location of these axes in the rectangular section, circular section, equilaterally triangular section and section having a symmetric axis.
4. Write formulas to compute centroidally principal moment of inertia of rectangle, square, circle, hollow circle, equilateral triangle, isosceles triangle.
5. Write the formulas of transfering axes parallel to the initial axes of principal moment of inertia.
6. Write theformulas to determine moment of inertia when rotating axes, formulas to determine extreme moment of inertia, formulas to determine principal axesof inertia?
7. How to determine centroidally principal axesof inertia and centroidally principal moment of inertia of compound section which has at least a symmetric axis and the compound section which does not have symmetric axis.

Numerical problems

Exercise 1:

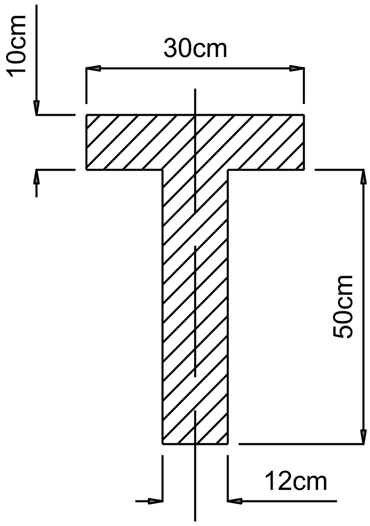
Determine centroid y_c and moment of inertia about centroidal axis which is parallel to the base side of trapezium.

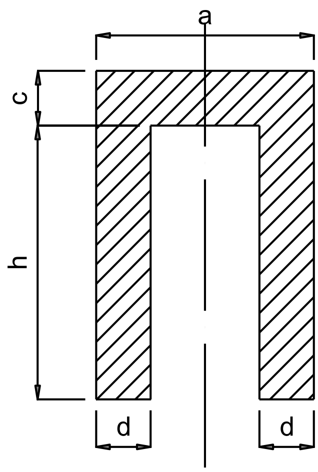


(Figure a) (Figure b)

Exercise 2:

Determine centroidally principal axesof inertia and centroidally principal moment of inertia of thefollowing sections:





a =12 cm

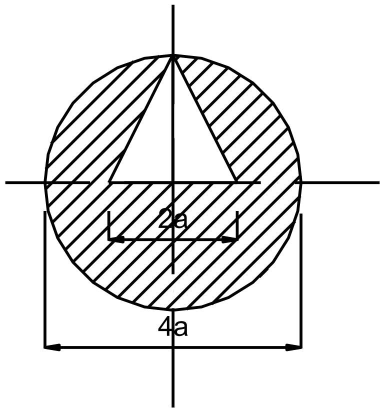
d = 3 cm

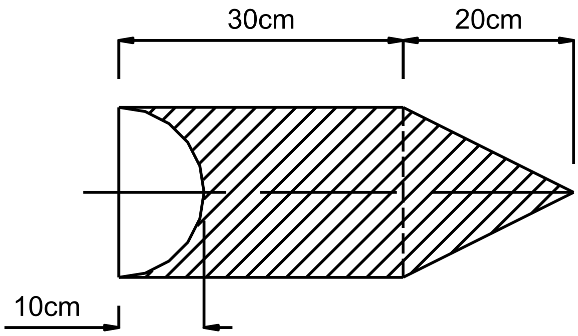
c = 4 cm

(Figure a) (Figure b)

<u>Exercise 3:</u>

Determine centroidally principal axes of inertia and centroidally principal moment of inertia of the following sections:



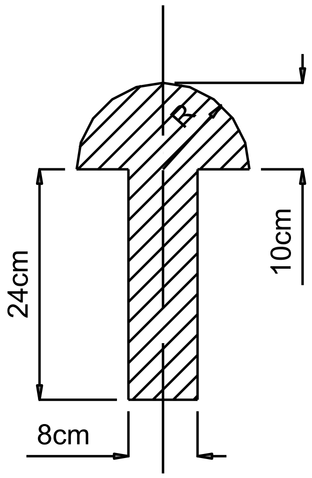


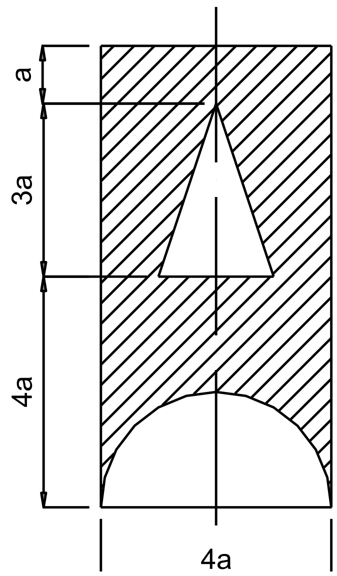
(Figure b)

(Figure a) (Figure b)

<u>Exercise 4:</u>

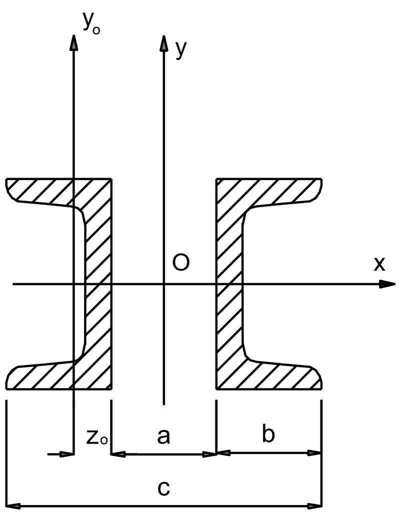
Determine centroidally principal axes of inertia and centroidally principal moment of inertia of the following sections:

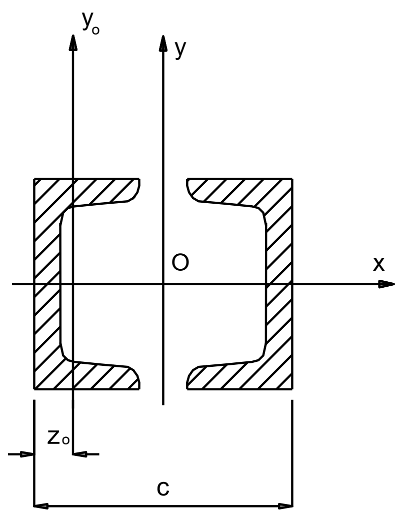




Exercise 5:

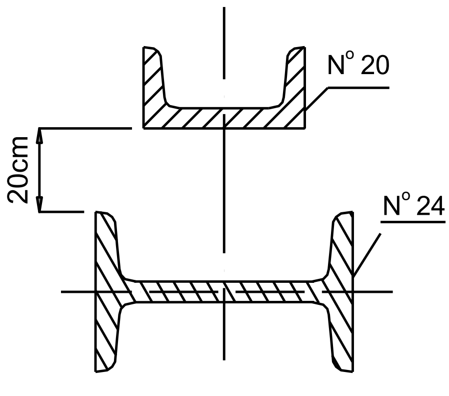
Determine the distance c of the compound section which is created by [$N^0$30 - steel and is ordered as in the figure so that $J_x = J_y$



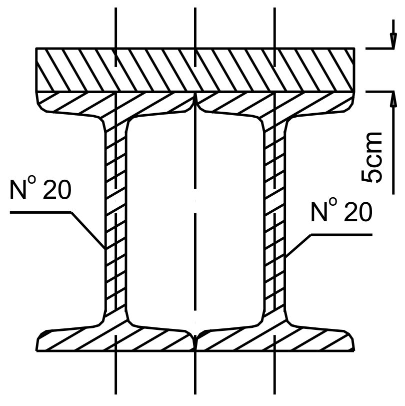


Exercise 6:

Determine centroidally principal axesof inertia and centroidally principal moment of inertia of the following sections:



(Figure a)



(Figure b)

CHAPTER IV: TORSION IN ROUND SHAFTS

4.1. Concepts

A shaft is said to be in torsion when there is only one internal force which is called torque M_z on its cross-section.

The sign convention of torque M_z is shown as below:

If we look at cross-section of remaining shaft and realise that the direction of torque M_z is clockwise, it will be positive.The external forces which cause torsion are usually moments, force couple in the plane which is perpendicular to the axis of shaft. Axle shaft, coil spring… are the bodies subjected to torsion.

4.2. Stress on cross-section

a. Experiment

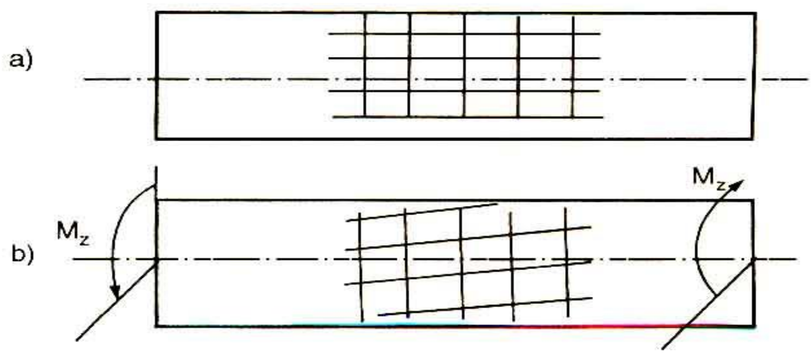


Figure 4.1

Assume that there is a shaft. Before shaft is subjected to torsion, draw lines parallel as well as perpendicular to its axis on its surface. (Figure 4.1)

After shaft is deformed by torque M_z, we realize that lines parallel to theaxis of shaft become spirals around axis. The lines perpendicular to axis are still perpendicular to axis and distances among them are constant.

b. Hypotheses

According to the above comments, the following hypotheses are given to calculate the shaft subjected to torsion.

* Hypothesis about cross-section: thecross-sections of a shaft in process of torsion are always plane and perpendicular to theaxis of shaft. Distances between cross-sections are constant.

* Hypothesis about radius: the radiuses of a shaft in theprocess of torsion are always straight and their lengths are constant. These radiuses only rotate an angle around the axis of shaft.

Besides two above hypotheses, materials are considered to work in elastic region.

Through the above hypotheses, some conclusions can be given:

- Thecross-sections ofround shaft in torsion do not have normal stresses because there is not longitudinal strain.

- Thedirection of shear stress of a point on cross-section is perpendicular to radius at that point and conforms to thedirection of torque. (figure 4.2)

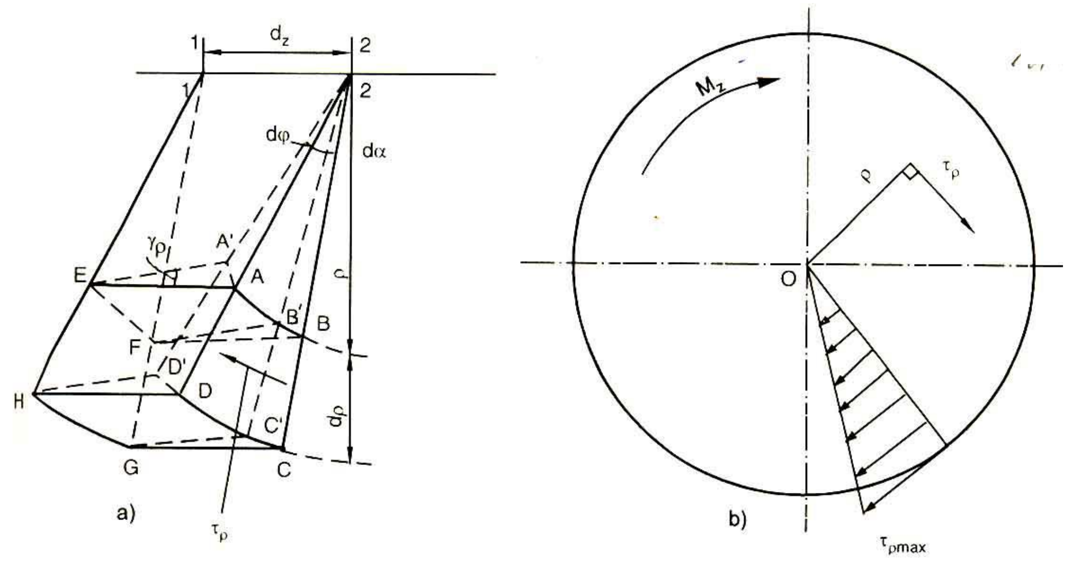


Figure 4.2

c. Formula to determine shear stress

To establish theformula of shear stress on cross-section, we split a small element by six sides: two cross-sections at a distance dz; two planes going through axis and forming an angle dα; two surfaces of cylinders having radiuses ρ and ρ + dρ. That element is expressed in the figure 4.2a. It has eight vertices A, B, C, D, E, F, G, H. Assume that the first cross-section is fixed while the second cross-section will rotate an angle dφ as shown in the figure 4.2.

Because distance between cross-section 1-1 and 2-2 is constant, the element does not have longitudinal strain. It only has the angular strain caused by shear stress τ_ρ. Therefore, this element is in pure slide.

Angle of twist dφ is called relative angle of twist between cross-section 2-2 and 1-1.

As the figure 4.2, we realise that:

$$\gamma_\rho \approx tg\gamma_\rho = \frac{AA'}{EA} = \frac{\rho d\varphi}{dz} \qquad \text{(a)}$$

On the other hand, according to Hook’s Law about slide, we get:

$$\gamma_\rho = \frac{\tau_\rho}{G} \qquad \text{(b)}$$

Compare two expressions (a) and (b), we get:

$$\tau_\rho = \frac{Gd\varphi}{dz}\rho \qquad \text{(c)}$$

If there is the small element of area dF surrounding A, we have the relationship:

$$\int_F \tau_\rho.\rho.dF = M_z \qquad \text{(d)}$$

Substitue (c) for (d), we get:

$$G\frac{d\varphi}{dz}\int_F \rho^2 dF = M_z \qquad \text{(e)}$$

Expression $\int_F \rho^2 dF = J_p$. Consider the ratio $\frac{d\varphi}{dz} = \theta$, we get:

$$\theta = \frac{d\varphi}{dz} = \frac{M_z}{GJ_p} \qquad \text{(4-1)}$$

Here θ is called relative angle of twist, which specilises in torsion strain. Product GJ_p is called torsion-resistant stiffness in cross-section.

Substitute (5-1) for (c), we have:

$$\tau_\rho = \frac{M_z}{J_p}\rho \qquad (4\text{-}2)$$

The equation (4-2) is the expression of shear stress on thecross-section of round shaft in torsion.

Shear stress diagram on cross-section

According to equation (4-2), shear stress is the linear function of ρ. Thedistribution of shear stress is expressed by the diagram called shear stress diagram on each cross-section. (figure 4.2b)

The maximum shear stress is stress at the points lying on theperimeter of section. Its magnitude is calculated thanks to the following expression:

$$\tau_{\rho\max} = \frac{M_z}{J_p}\rho_{\max} = \frac{M_z}{W_p} \qquad (4\text{-}3)$$

with $W_p = \frac{J_p}{R}$ called thetorsion-resistant moment of cross-section.

- If cross-section is circular and has diameter D,

$$W_p = \frac{J_p}{D/2} = \frac{\pi D^3}{16} \qquad (4\text{-}4)$$

- If cross-section is hollow circular and has diameter D and hollow coefficient η,

$$W_p = \frac{\pi D^3}{16}\left(1-\eta^4\right) \qquad (4\text{-}5)$$

4.3. The strain and displacement of cross-section

4.3.1. Strain

Consider theround shaft subjected to torsion as shown in the figure 4.3

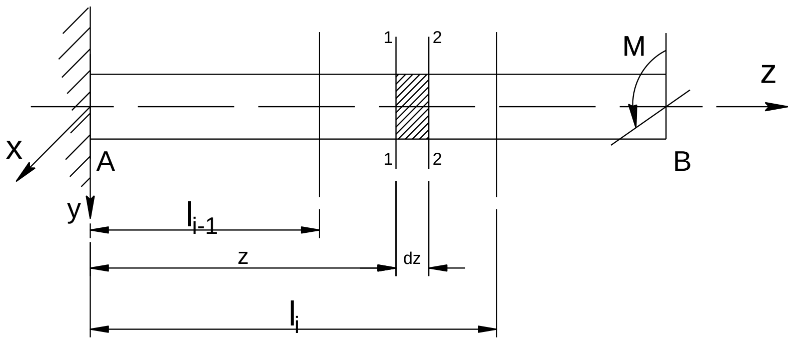


Figure 4.3

The strain of the round shaft subjected to torsion is expressed by relative angle of twist θ

$\theta = \frac{d\varphi}{dz}$ (dφis the relative angle of twist among two cross-sections at a distance dz)

According to the figure (4.1), we have: $\theta = \frac{Mz}{GJ_p}$

Product GJ_p is called torsion-resistant stiffness.

4.3.2. The relative angle of twist among two ends of shaft φ

From equation $\theta = \frac{d\varphi}{dz} = \frac{Mz}{GJ_p}$

Infer $d\varphi = \frac{Mz}{GJ_p} dz$

If the length of shaft is l, the relative angle of twist among two ends of shaft will be

$$\varphi = \int_0^l d\varphi = \int_0^l \frac{Mz}{GJ_p} dz$$

- In general case: shaft has many segments, M_z, GJ_p continuously vary on each segment

$$\varphi = \sum_{i=1}^{n} \int_0^{li} \left(\frac{M_z}{GJ_p} \right)_i dz$$

- In particular case: shaft has many segments, M_z, GJ_p are constant on each segment

$$\varphi = \sum_{i=1}^{n} \left(\frac{M_z . l}{GJ_p} \right)_i$$

4.3.3. The displacement of cross-section

When a shaft is subjected to torsion, its cross-sections will rotate an angle around its axis. TheThe displacement of cross-section is its absolute angle of twist φ(z).

To calculate displacementφ(z), we can rely on the formula calculating the relative angle of twist among two ends of shaft or geometric relation between deformation and displacement in each particular case.

4.4. Compute the spring helical, cylindrical, having small pitch

Spring is the part widely used in practice. They are theparts of damping structures, theprotective equipments of high-pressured machines. The spring prevailingly used is cylindrically helical spring which has circular spring wire. Therefore, we only research this kind of spring.

4.4.1. The properties of spring

The figure 4.4 expresses a cylindrically helical spring. This spring has the following properties:

- D: the diameter of spring coil
- d: the diameter of spring wire
- h: the pitch of helical spring
- n: the number of coils

Here, we only consider a spring with small pitch. It means that h ≤ 2d.

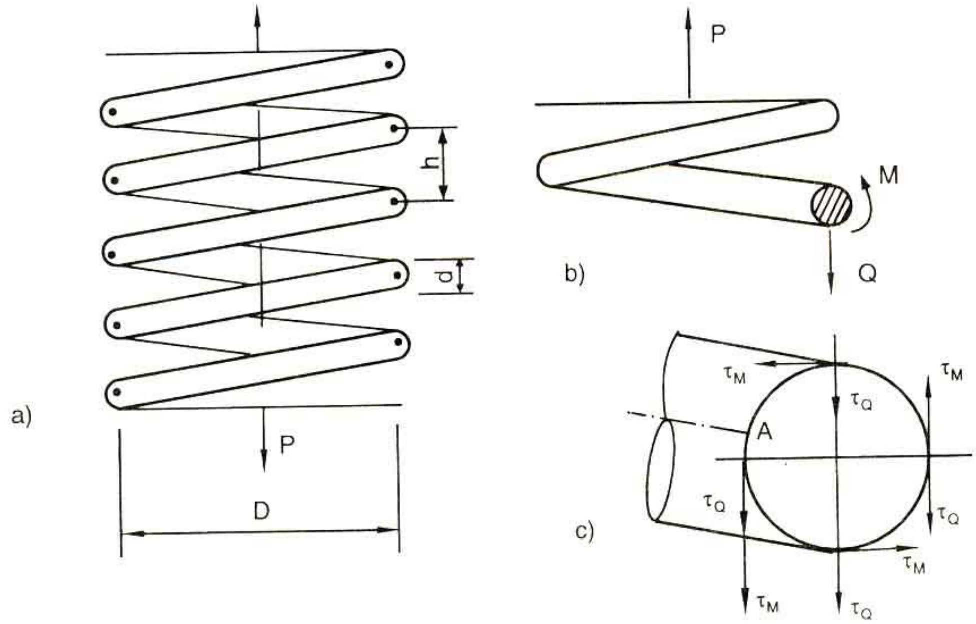


Figure 4.4

4.4.2. Stress on cross-section of spring wire

Imagine that we will cut spring by the plane containing its axis. Divide spring into two segments. Consider the equilibrium of upper segment (figure 4.4). It can be realized that there are two internal forces on the cross-section of spring wire: shear force Q and torque M.

To research spring with small pitch, I assume that the cross-section of spring wire is circular.According to the static equilibrium of upper segment, we get:

$$Q = P; \; M = \frac{D}{2} P \qquad \text{(a)}$$

The shear stress caused by shear force Q is distributed equally and follows downward direction (the same direction as Q). Themagnitude of this stress is determined as the following formula:

$$\tau_Q = \frac{Q}{F} = \frac{4P}{\pi d^2} \qquad \text{(b)}$$

The shear stress induced by torque on cross-section is distributed in accordance with thelinear function of radius ρand achieves maximum value at the points in the perimeter of cross-section.

$$\tau_M^{max} = \frac{M}{W_p} = \frac{8PD}{\pi d^3} \qquad \text{(c)}$$

The direction of shear stress is always perpendicular to the radius of cross-section. (figure 4.4)

Hence, maximum shear stress arises from point A in the interior corner of cross-section because both two stresses here have the same direction and their magniutde is:

$$\tau_{max} = \tau_Q + \tau_M^{max} = \frac{4P}{\pi d^2} + \frac{8PD}{\pi d^3} = \frac{8PD}{\pi d^3}\left(1 + \frac{d}{2D}\right) \qquad \text{(d)}$$

Because diameter d is very small in comparison with the diameter of spring coil D, we can ignore value $\frac{d}{2D}$ in expression (d). Hence:

$$\tau_{max} = \frac{8PD}{\pi d^3} \qquad \text{(4-6)}$$

According to this expression, we realize that theprimary deflection of spring is twisting deflection while shear deflection is insignificant.

When establishing the expression (4.6), we ignored theflexure of spring wire. To calculate more accurately, we need to refer to theflexure and declination of spring wire. Hence, the formula τ_{max} will be:

$$\tau_{max} = k\frac{8PD}{\pi d^3} \qquad (4\text{-}7)$$

k is the adjustable coefficient calculated by the formula:

$$k = \frac{\frac{D}{d} + 0,25}{\frac{D}{d} - 1} \qquad (4\text{-}8)$$

4.4.3. The deflection of spring

Assume that there is the spring subjected to force P. At that moment, the spring is stretched (if the force induces tension) or shortened (if the force induces compression) and a value λ called the deflection of the spring.

To determine thedeflection of spring, we use thelaw of conservation of energy:

A is thework of force P to creat motionλ, we get:

$$A = \frac{1}{2}P\lambda \qquad (e)$$

In terms of magnitude, this work is equal to the elastic strain energy stored in spring wire when it is deformed. Specifically,

$$U = \frac{1}{2}.\frac{M_z^2}{G.J_p}l = \frac{1}{2}.\frac{P^2D^2}{4}.\frac{\pi Dn}{G\frac{\pi d^4}{32}} = \frac{1}{2}.\frac{8P^2D^3n}{Gd^4} \qquad (f)$$

Hence $\lambda = \frac{8PD^3n}{Gd^4}$ (4-9)

Themagnitude of the force acting and making spring stretched or shortened aunit of length is called modulus of rigidity of spring and signed C:

$$C = \frac{P}{\lambda} = \frac{Gd^4}{8D^3n} \qquad (4\text{-}10)$$

Example 1: Check thestrength of the spring having following properties:

D = 10cm; d = 0,6cm; n = 16 coils; G = 8.10^3kN/cm^2; [τ] =6kN/cm^2; P = 50N. Determine thedeflection of spring. Know that P is tensile force.

Solution:The maximum stress of spring is:

$$\tau_{max} = \frac{8PD}{\pi d^3} = \frac{8.50.10^{-3}.10}{3,14.(0,6)^3} \approx 5,89 kN/cm^2$$

Compare$\tau_{max}$with [τ] we realize thatτ_{max}< [τ], hence, the spring has enough strength.

Thedeflection of the spring is determined according to the expression (4.9):

$$\lambda = \frac{8PD^3n}{Gd^4} = \frac{8.50.10^{-3}.10^3.16}{8.10^3.(0,6)^4} \approx 6,17cm$$

4.5. Compute the shaft subjected to torsion

4.5.1. Strength

The shaft subjected to torsion is considered enough strength when maximum shear stress in shaft is not exceeded allowable shear stress.

The condition of strength: $\max\left|\tau_{\rho\max}\right| = \left|\frac{Mz}{W_p}\right|_{\max} \leq [\tau]$ (4-11)

Maximum shear stress in shaft $\max\left|\tau_{\rho\max}\right|$ is determined by comparing maximum shear stress in each cross-section $\left|\tau_{\rho\max}\right|$.

Allowable shear stress [τ] has two methods to determine:

* According to experiment:

$[\tau] = \frac{\tau_0}{n}$ with τ_0 is the dangerous shear stress determined from experiment.

* According to reliability theory: we split the dangerous segment having $\tau_{\rho\max}$. This segment is in pure slide.

This segment has:

$\sigma_1 = \tau_{\rho\max}$

$\sigma_2 = 0$

$\sigma_3 = -\tau_{\rho\max}$

According to the reliability theory $N^0 3$: $[\tau]_{t3} = \frac{[\sigma]}{2}$

According to the reliability theory $N^0 4$: $[\tau]_{t4} = \frac{[\sigma]}{\sqrt{3}}$

According to the reliability theory Morh:

$$[\tau]_{tMO} = \frac{[\sigma]_k}{1+\alpha}$$

in which $\alpha = \frac{[\sigma]_k}{[\sigma]_n}$

4.5.2. Stiffness

Besides strength, the shaft subjected to torsion has to ensure stiffness. Stiffness is as important as strength in the shaft subjected to torsion. According to each particular condition, stiffness can be one of three following conditions:

$$\left|\theta_{\max}\right| = \left(\frac{M_z}{GJ_p}\right)_{\max} \leq [\theta]$$

(4-12)

$$\left|\varphi_{AB}\right| = \sum_{i=1}^{n}\int_0^{li}\left(\frac{M_z}{GJ_p}\right)_i dz \leq [\varphi_{AB}]$$

$$\left|\varphi^{k}_{(z)}\right| \le [\varphi]$$

In which $\varphi^{k}_{(z)}$ is the displacement of cross-section K.

4.5.3. Three basic problems to compute the shaft subjected to torsion thanks tostrength and stiffness

When computing the shaft subjected to torsion, we also have to solve three basic problems whose contents are similar to those in axially loaded bar.

a. Test problem

We have to check strengththanks to expression (4-11) and stiffness thanks to formula (4-12).

b. The problem of determing allowable load

We select allowable load thanks tothecondition of strength and follow the formula (4-11)

$M_z \le W_p[\tau]$ (a)

And thanks tothecondition of stiffness (4-12)

$M_z \le GJ_p[\theta]$ (b)

After that, compare and select smaller result to be allowable load acting on axis.

c. The problem of determing diameter of cross-section

According to thecondition of strength, we have:

$$\max\left|\tau_{p\max}\right| = \left|\frac{M_z}{W_p}\right|_{\max} \le [\tau] \text{ Infer } W_p \ge \frac{M_z}{[\tau]} \quad \text{(c)}$$

Hence, we determine the diameter D of cross-section

According to thecondition of stiffness:

Thanks to θ_z we have: $$\left|\theta_z\right|_{\max} = \left|\frac{M_z}{GJ_p}\right|_{\max} \le [\theta_z] \text{ Infer } J_p \ge \frac{M_z}{G[\theta_z]} \quad \text{(d)}$$

After that, we compute D thanks tothe formula relating to J_p.

Finally, compare and select bigger results to be the diameter of axis.

In two expressions (c) and (d), hollow coefficient η isthehollow coefficient of cross-section. If cross-section is circular, $\eta = 0$.

<u>Example 2</u>: The shaft subjected to torsion as shown in the figure 4.5. Thecross-section of shaft is hollow and thehollow coefficient of cross-section is $\eta = 0,6$. Know that $M_1 = 4,2$kNm, $M_2 = 1,2$kNm, $[\tau] = 4$kN/cm^2; $[\theta] = 0,25^0$/m. Material has $G = 8.10^3$kN/cm^2.

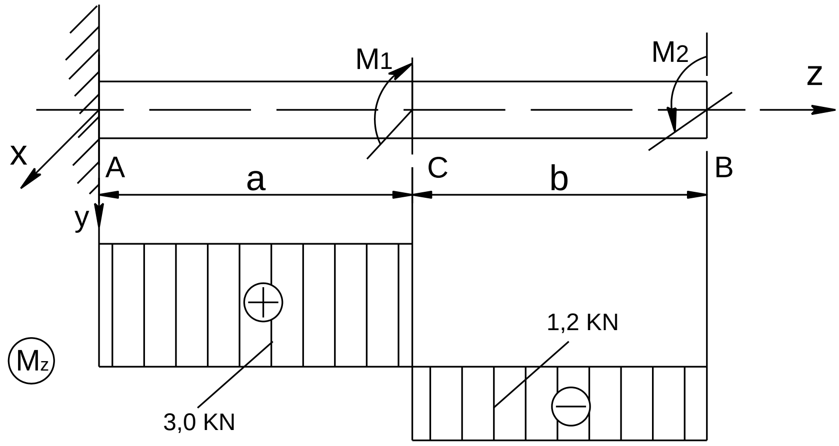


Figure 4.5

Solution: Twisting moment diagram is expressed as shown in the figure 4.5.

According to diagram, we realize that dangerous cross-section belongs to the segment AC which has M_{zmax} = 3,0kNm = 300kNcm.

Thanks tothecondition of strength, we determinethe extermal diameter of shaft:

$$D \geq \sqrt[3]{\frac{300}{0,2.4.(1-0,6^4)}} = 7,55cm$$

Thanks tothecondition of stiffness:

$$[\theta] = 0,25^0 / m = \frac{\pi}{180}.0,25 rad/m = \frac{\pi}{180}.0,25.10^{-2} rad/cm$$

We determine theexternal diameter of shaft:

$$D \geq \sqrt[4]{\frac{300}{0,1.\frac{\pi}{180}.0,25.10^{-2}.8.10^3(1-0,6^4)}} = 14cm$$

In conclusion, we choose external diameter being D = 14 cm and internal diameter d = 0,6D = 0,6.14 = 8,4cm

Theoretical questions

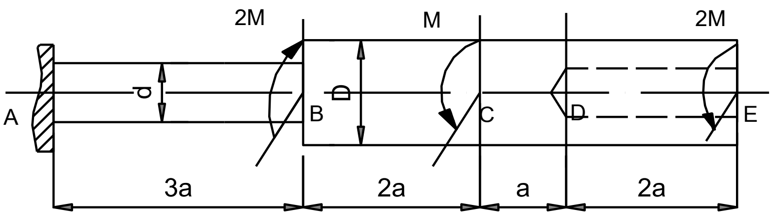
1. Define the shaft subjected to torsion. Raise the kinds of the external forces causingpure torsion. Take practical examples about cases in torsion.
2. What does thecross-section of the shaft subjected to torsion have the kinds of stress? Raise their characteristics and draw illustrative pictures? How is thedistribution of shear stress on cross-section?
3. What quantities specilise for the deformation, the strain of the shaft subjected to torsion. Write formula to calculate stress and explain quantities in the formula. Raise the condition of stiffness.
4. Raise the methods to solve three basic problems to compute the shaft subjected to torsion thanks tothe condition ofstrength and stiffness.
5. Write formula to calculate stress at a point and formula to calculate maximum stress on thecross-section of spring wire helical and cylindrical. Raisethe condition of strength of spring.

Numerical problems

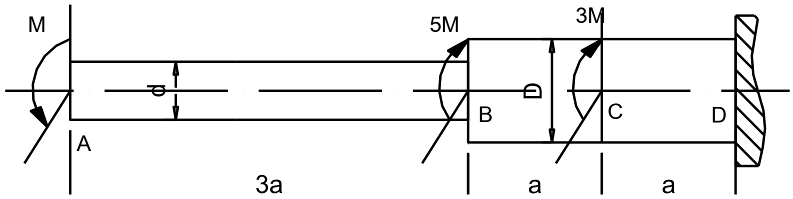
Exercise 1:

Draw internal force diagram M_z, calculate the relative angle of twist among two ends of shaft through M, a, d, G. Know that D = 2d.

a,



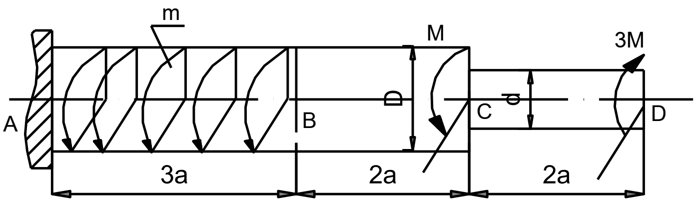
b,



Exercise 2:

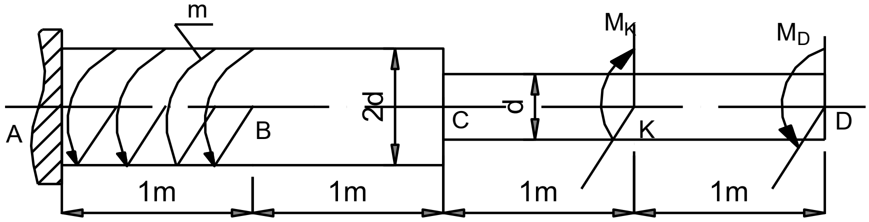
a, - Check strength and stiffness for round shaft: m = 600 Nm/m; M = 0,5 kNm; D = 10 cm; d= 6 cm; $[\tau] = 60$ MN/m^2; $[\theta_z]= 4$ ^{O}/m ; a = 80 cm ; G = 8.10^4 MN/m^2.

- Determine allowable load. Know that: M = ma; $[\tau] = 160$ MN/m^2; $[\varphi_{AD}] = 4{,}6^O$.



b, Theshaft subjected to force is shown in the figure. Know that: M_D = 1,2 kNm; M_K = 2,5 kNm; m = 40Nm/cm; d = 8 cm. Thematerial of shaft is cast-iron having$[\tau] = 50$ MN/m^2; n = 2,5 ; $[\varphi] = 2^O$; G = 5.10^4 MN/m^2.

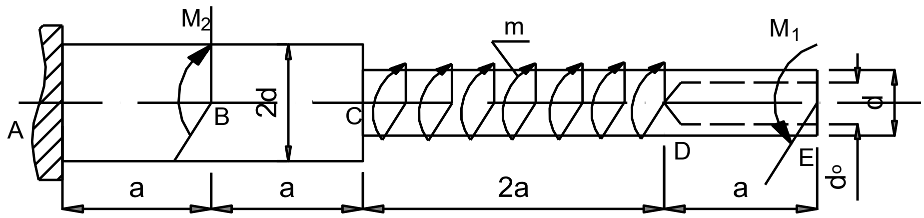
Check strength and stiffness for shaft.



Exercise 3

- Determine the diameter of following shafts: M_1 = 800 Nm; M_2 = 1800 Nm; m = 1000Nm/m; D = 2d; d_o = 0,8d ; $[\tau] = 60$ MN/m^2; $[\theta_z]= 0{,}5$ ^{O}/m ; G = 8.10^6 N/cm^2, a = 1 m.

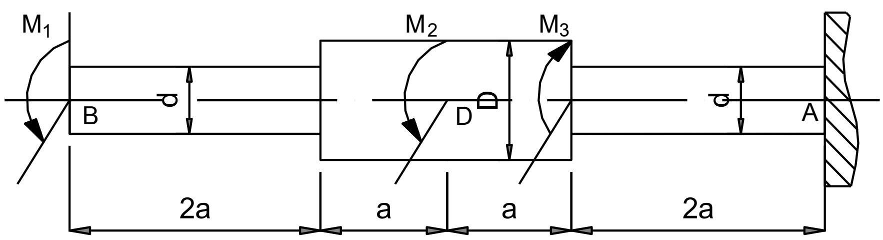
- Thanks to determined diameter, calculate relative angle of twist among two ends of shaft.



Exercise 4:

a,- Draw internal force diagram following M.Know that: M_1 = M; M_2 = 3M; M_3 = 6M; D = 1,5d

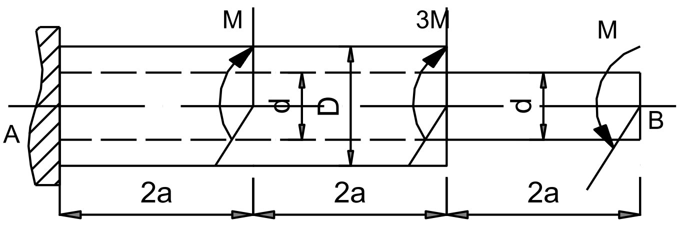
- Determine allowable load. Know that: D = 6 cm; a = 0,6 m; $[\tau] = 80$ MN/m^2 ; $[\varphi_B] = 1{,}6^O$; G = 8.10^4 MN/m^2.



b,- Draw internal force diagram following M. Know that $\eta = \dfrac{d}{D} = 0{,}8$

- Check strength and stiffness. Know that $M = 2{,}4$ kNm; $[\tau] = 60$ MN/m^2 ; D = 10 cm; $[\varphi_{BA}] = 2{,}6^O$; a = 0,5 m; G = 8.10^{3}kN/cm^2.

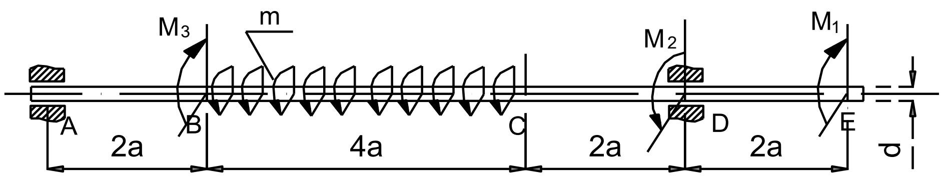
- Determine allowable load.



Exercise 5:

Determine thediameter of shaft. Know that: $M_1 = 800$ Nm; $M_2 = 1000$ Nm; $M_3 = 1500$ Nm; a = 1 m; $[\tau] = 8$ kN/cm^2; $[\theta_z] = 2{,}4$ ^{O}/m ; G = 8.10^4 MN/m^2.

Calculate relative angle of twist among two ends of shaft.

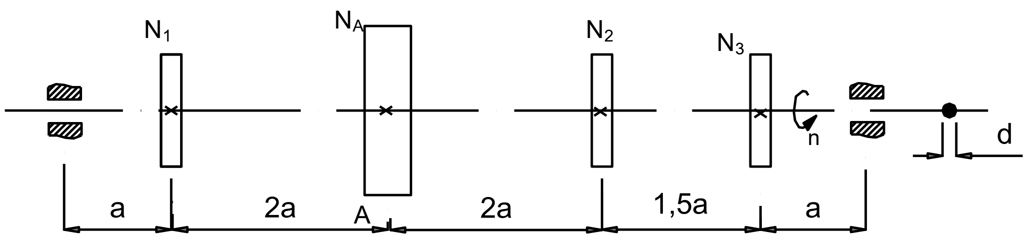


Exercise 6:

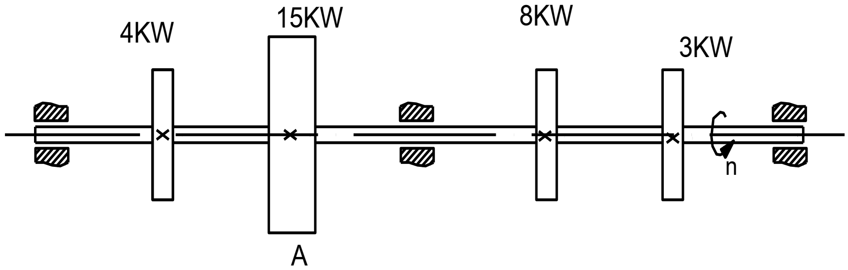
a, A shaft has gear A being active gear. Power of each gear is: $N_1 = 5$ kW; $N_2 = 6$ kW; $N_3 = 3$ kW; $N_A = 14$ kW. The shaft rotates in n = 230 revs per minute; a = 0,5 m. Know that: $[\tau] = 2600$ N/cm^2; $[\theta_z] = 0{,}6$ ^{O}/m ; G = 8.10^4 MN/m^2.

Determine the diameter of shaft.(Use formula $M(\text{Nm}) = \dfrac{N(\text{W})}{\omega(\text{rad/s})}$)

Calculate relative angle of twist among two ends of shaft.



b, Check strength and stiffness for the round shaft having d = 6 cm; $[\tau] = 2000$ N/cm^2; $[\theta_z] = 0{,}4$ ^{O}/m ; G = 8.10^6 N/cm^2. Gear A is active gear, n = 150 revs per minute.



CHAPTER V – FLEXURE OF INITIALLY STRAIGHT BEAM

5.1. Concepts

A bar is called bended if its axis is bended thanks to external forces. This bar is called beam.

Beam is a popular part in structures or machines.

Example: The main beam of a bridge and the axis of striker wheel are shown in the figure 5.1.

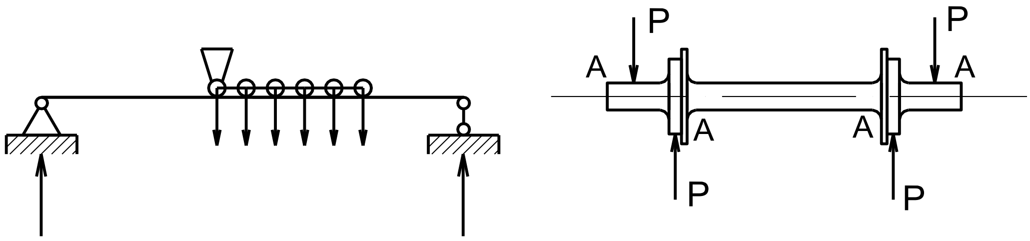


Figure 5.1

The external force causing flexure can be the concentrated force or point load, uniformly distributed load or uniformly varying load which is perpendicular tothe axis of beam. It can be moments in the plane containing theaxis of beam.

We can expose some following concepts:

- If external force is in the plane which containing theaxis of beam, that plane is called loading plane.
- The line of intersection between loading plane and thecross-section of beam is called loading line.
- If after bended, theaxis of beam is a curve in the centroidally principal plane of inertia, this case is called single flexure.

In fact, the beams bended are usually the ones having cross-sections being symmetric through an axis. Hence, to simplify problems, we only consider the beams having the properties as shown above. It means that thecross-sections of beam have at least a symmetric axis. In other words, beams have a symmetric plane going throughthe axes of beams.

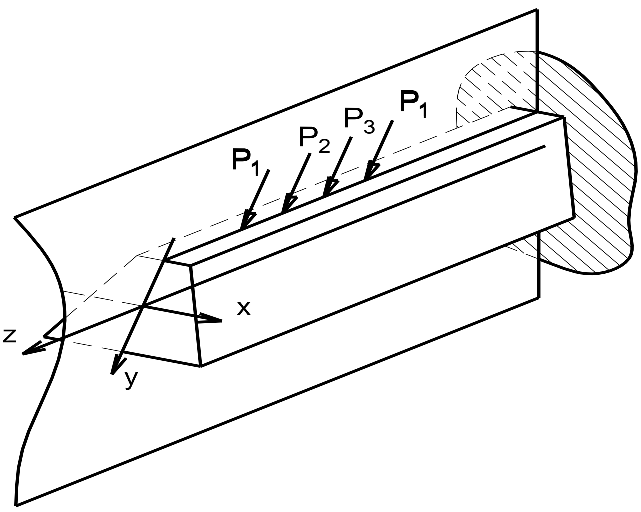


Figure 5.2

We also assume that external force is in that symmetric plane.

If centroidally principal plane of inertia is created by a centroidally principal axis of inertia of beam associated with theaxis of beam, in this case, symmetric plane is not only loading plane but also centroidally principal plane of inertia. Because symmetry, we realise that after bended, theaxis of beam is a curve in that symmetric plane. Thesymmetric axis of cross-section is loading line. Here, we only consider the beam whose width is so small in comparison with height. However, it is not too small to be instable.

We consider two following cases of bending in turn:

- Pure bending or simple bending
- Plane bending

5.2. Pure bending or simple bending

5.2.1. Concept

If the length of a beam is subjected to a constant bending moment and no shear force, then the stresses will be set up in that length of the beam thanks toonly bending moment and that length of the beam is said to be in pure bending or simple bending.

5.2.2. Stress on cross-section

a. Experiment

We assume that there is a prismatic bar. Before it is bended, on its surface, draw lines parallel to its axis to express longitudinal filaments and plane curves perpendicular to its axis to express cross-sections. (figure 5.3)

After the beam is deformed by bending moment M_x (figure 5.3), we realise that the lines parallel to its axis are still parallel to its axis while the plane curves perpendicular to its axis is still plane and perpendicular to its axis.

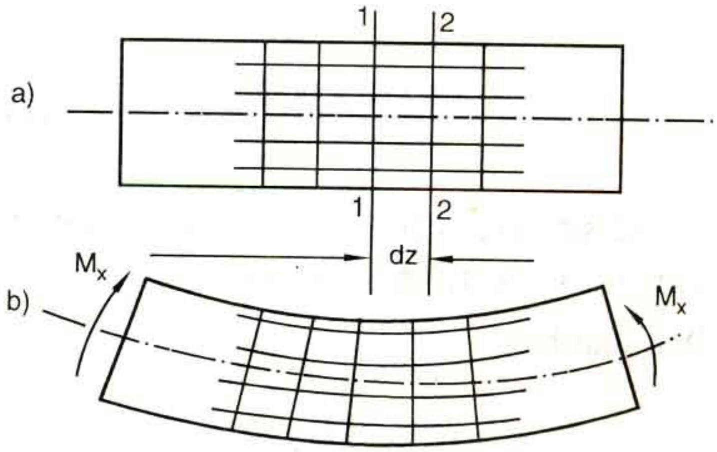


Figure 5.3

b. Assumptions

We can expose some following assumptions thanks to the above comments:

- Assumption about cross-section: thecross-sections of a straight beam are usually plane and perpendicular to theaxis of beam during the process of pure bending.
- Assumption about longitudinal filaments: thelongitudinal filaments of a straight beam do not push and press each other during the process of pure bending.

Besides two above assumptions, we also assume that the material of the beam works in elastic region and conform to Hook's law.

Thanks to the above assumptions, we can have some following conclusions:

- There is no shear stress on the cross-section of beam.
- Longitudinal filaments are elongated or shortened and there is a filament which is neither elongated nor shortened. This layer of filament is called neutral layer. Theline of intersection of neutral layer with cross-section is called neutra axis. Neutral axis is perpendicular to axis Oy.

c. Determine formula to calculate stress on cross-section

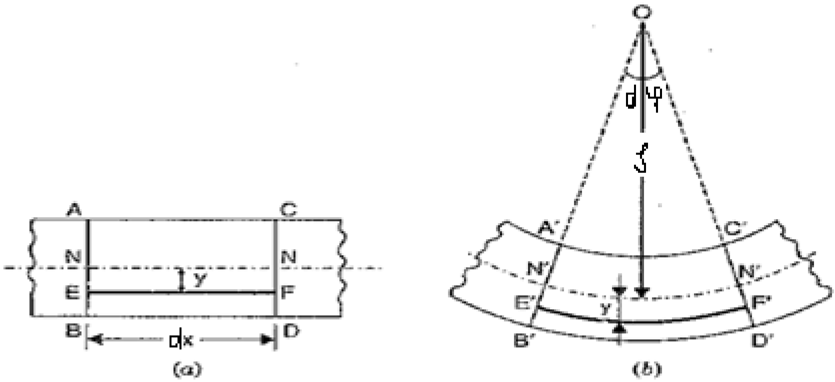


Figure 5.4

Consider a small length dz of the beam subjected to a simple bending. Thanks to the action of bending, the part of length dz will be deformed as shown in the figure 5.4. Let A'B' and C'D' meet at O.

Let ρ = the radius of neutral layer NN'

dφ = the angle created by A'B' and C'D'

Consider a layer EF at a distance y below the neutral layer NN'. After bended, this layer wil be elongated to E'F'.

Theoriginal length of layer: dz = ρdφ

Also thelength of neutral layer: dz = ρdφ

After bended, thelength of neutral layer NN' will remain unchanged. Butthelength of layer EF will increase. Hence: E'F' = dz + Δdz = (ρ + y)dφ (b)

Strain in the layer EF:

$$\varepsilon_z = \frac{\Delta dz}{dz}$$

$$\varepsilon_z = \frac{(\rho + y)d\varphi - \rho d\varphi}{\rho d\varphi} = \frac{y}{\rho} \qquad (c)$$

As ρ is constant, hence, the strain in a layer is proportional to its distance from the neutral axis. The above equation shows the variation of strain along the depth of the beam. The variation of strain is linear.

Assume that stress at point B on cross-section is σ_z, σ_z and ε_z have the relation conforming to Hook's law:

$$\sigma_z = E.\varepsilon_z = \frac{E}{\rho}.y \qquad (d)$$

Thanks to pure bending, the layers above the neutral axis are subjected to compressive stresses whereas the layers below the neutral axis are subjected to tensile stresses. Thanks to these stresses, the forces acting on the layers will be determined. These forces wil have moment about the neutral axis. The total moment of these forces about the neutral axis for a section is known as themoment of the resistance of that section.

The force on the layer at a distance y from neutral axis is given by equation:

$$N_z = \int_F \sigma_z dF = \int_F \frac{E}{\rho} y dF = \frac{E}{\rho}\int_F y dF = \frac{E}{\rho} S_x \qquad (e)$$

It is obvious that as beam is subjected to bend, longitudinal force on cross-section equals 0. Therefore, $N_z = \frac{E}{\rho} S_x = 0$ or $S_x = 0$.

It means that neutral axis is also centroidall principal axis of inertia.

Themoment of this force about neutral axis will be:

$$M_x = \int_F \sigma_z y dF = \int_F \frac{E}{\rho} y^2 dF = \frac{E}{\rho}\int_F y^2 dF = \frac{E}{\rho} J_x \quad \text{(g)}$$

Or $\frac{1}{\rho} = \frac{M_x}{EJ_x}$ (5-1)

From expression (d), we have:

$$\sigma_z = \frac{M_x}{J_x} y \quad \text{(5-2)}$$

In which:

$M_x$is bending moment on cross-section

$J_x$is moment of inertia of cross-section about neutral axis.

y is theorinate of the point calculated stress.

The stress which is positive is called tensile stress while the stress which is negative is called compressive stress.

The diagram of stress distribution across cross-section

a. Unsymmetrical section:

Thanks to expression (5-2), normal stress diagram on cross-section is drawn as in the figure 5.5. On this diagram, we use plus sign (+) to express tensile stress and minus sign to express compressive stress.

There is no stress at the neutral axis. But the stress at a point is directly proportional to its distance from the neutral axis. The maximum stress takes place at the outermost layer. In case of unsymmetrical section, the neutral axis does not pass through the geometrical centre of section. Hence, the value of y for the topmost layer or bottommost layer of the section from neutral axis will not be same. As a result, we will have maximum tensile stress and maximum compressive stress. Their magnitudes are:

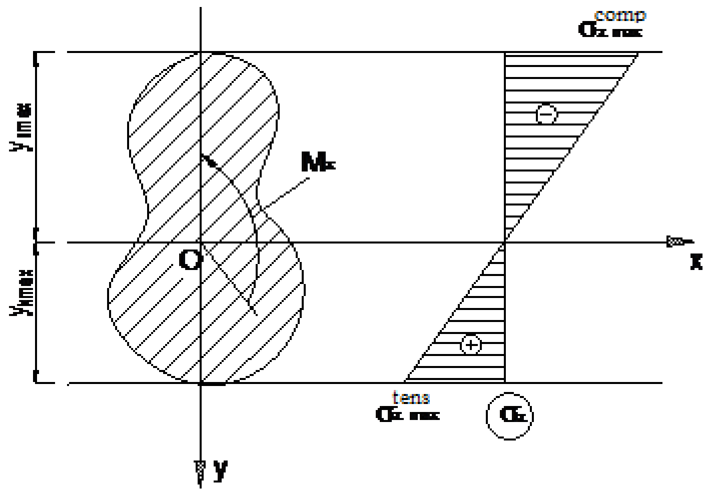


Figure 5.5

$$\sigma_{\max}^{tens} = \frac{M_x}{J_x} y_{\max}^{tens} = +\frac{|M_x|}{W_x^{tens}}$$
$$\sigma_{\max}^{comp} = \frac{M_x}{J_x} y_{\max}^{comp} = -\frac{|M_x|}{W_x^{comp}} \quad (5\text{-}3)$$

In which:

$$W_x^{tens} = \frac{J_x}{\left|y_{\max}^{tens}\right|}; W_x^{comp} = \frac{J_x}{\left|y_{\max}^{comp}\right|} \quad (k)$$

is called the section modulus of cross-section.

b. Symmetrical section:

Symmetrical sections are circle, rectangle, I section…

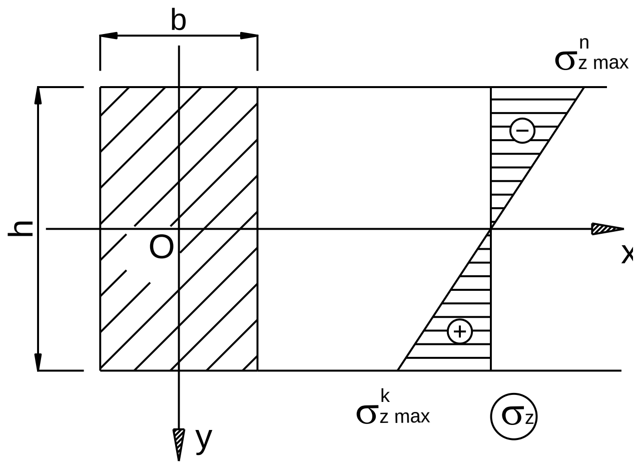


Figure 5.6

$$\left|y_{\max}^{tens}\right| = \left|y_{\max}^{comp}\right| = \frac{h}{2}$$

h is the height of section.

$$W_x^{tens} = W_x^{comp} = W_x = \frac{J_x}{h/2}$$

$$\left|\sigma_{\max}^{tens}\right| = \left|\sigma_{\max}^{comp}\right| = \frac{|M_x|}{W_x}$$

Stress diagram is in the form of symmetry and is expressed as the figure.

The section modulus of some popular sections.

a) Rectangular section

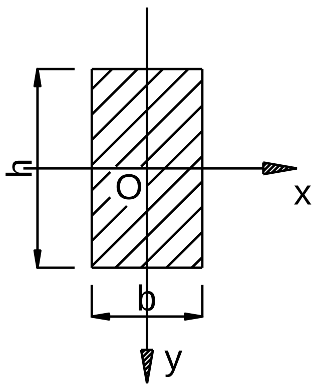


Figure 5.7

Moment of inertia of the rectangular section having dimensions of b x h (figure 5.7) about an axis throught its center of gravity (or through neutral axis) is given by:

$$J_x = \frac{bh^3}{12}$$

The distance of outermost layer from neutral axis is given by:

$$y_{\max}^{tens} = -y_{\max}^{comp} = \frac{h}{2}$$

So section modulus is given by: $W_x^{tens} = -W_x^{comp} = \dfrac{bh^2}{6}$ (5-4)

b) Circular section

In case section is circle having diameter D (figure 5.8):

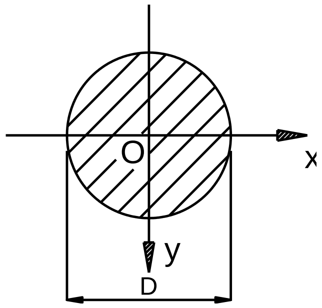


Figure 5.8

$$J_x = \frac{\pi D^4}{64}$$

$$y_{\max}^{tens} = -y_{\max}^{comp} = \frac{D}{2}$$

So $W_x^{tens} = -W_x^{comp} = \dfrac{\pi D^3}{32}$ (5-5)

c) Hollow circular section

In case section is the hollow circle having external diameter D and internal diameter d (figure 5.9). Its hollow coefficient is $\eta = \dfrac{d}{D}$, we get:

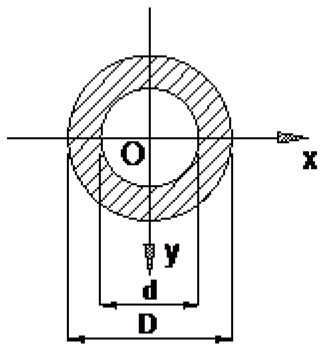


Figure 5.9

$$J_x = \frac{\pi D^4}{64}\left(1-\eta^4\right)$$

$$y_{\max}^{tens} = -y_{\max}^{comp} = \frac{D}{2}$$

So $W_x^{tens} = -W_x^{comp} = \frac{\pi D^3}{32}\left(1-\eta^4\right)$ (5-6)

d) Shaped section

In case of shaped steel, section modulus of cross-section is given in the table of their properties. (see in appendix)

5.2.3 Strength of pure bending

As above analysis, all the points in purely bended beam is subjected to single stress state with normal stress σ_z.

If researched beam is primatic beam, the dangerous section is the one having $\left|M_x\right|_{max}$ and dangerous points are in the topmost layer and bottommost layer of dangerous section. Stresses at dangerous points are $\sigma_{z\max}^{tens}$ and $\sigma_{z\max}^{comp}$.

If beam is made from brittle materials, thecondition of strength is:

$$\begin{cases} \sigma_{z\max}^{tens} \le [\sigma]_{tens} \\ \left|\sigma_{z\max}^{comp}\right| \le [\sigma]_{comp} \end{cases}$$

If beam is made from ductile materials:

Because $[\sigma]_{tens} = [\sigma]_{comp} = [\sigma]$

The condition of strength is: $\max\left|\sigma_z\right| \le [\sigma]$

In which: $\max\left|\sigma_z\right| = \max\left(\sigma_{z\max}^{tens}; \left|\sigma_{z\max}^{comp}\right|\right)$

Thanks tothecondition of strength, three following basic problems are solved:

* Test problem.

* The problem of determing allowable load.

* The problem of desiging cross-section.

5.2.4. The reasonable shape of cross-section

The reasonable shape of cross-section is the shape ensuring thatthe load-resistant capacity of beam is maximum but theuse of material is the least.

When beam is made from brittle material: thecross-section of beam will be the most reasonable if maximum stress satisfiesfollowing conditions:

$$\begin{cases} \sigma_{z\max}^{tens} \le [\sigma]_{tens} \\ \left|\sigma_{z\max}^{comp}\right| \le [\sigma]_{comp} \end{cases}$$

In which: $[\sigma]_{tens}$ is allowable tensile stress and $[\sigma]_{comp}$ allowable compressive stress. Replace the value $\sigma_{z\max}^{tens}$ và $\left|\sigma_{z\max}^{comp}\right|$ calculated by equation (5-3) inabove equations, we get:

$$\frac{|M_x|}{J_x} y_{\max}^{tens} = [\sigma]_{tens}$$

$$\frac{|M_x|}{J_x} y_{\max}^{comp} = [\sigma]_{comp}$$

Divide sides of equations each other, we have:

$$\frac{y_{\max}^{tens}}{y_{\max}^{comp}} = \frac{[\sigma]_{tens}}{[\sigma]_{comp}} \quad \text{(a)}$$

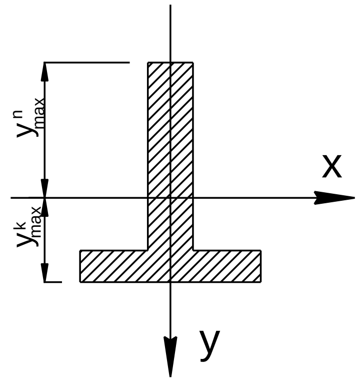
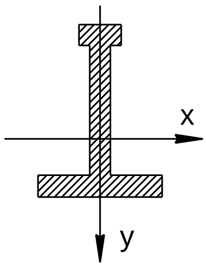


Figure 5.10

We can infer that thereasonable shape of cross-section made from brittle materials has to ensure that neutral axis dividethe height of section by the ratio of $y_{\max}^{tens}$ and $y_{\max}^{comp}$ satisfying equation (a).

In case of ductile material: Because $[\sigma]_{tens} = [\sigma]_{comp}$, the ratio (a) equals 1. It means that $y_{\max}^{tens}$ = $y_{\max}^{comp}$. Hence, neutral axis will be the symmetrical axis of section.

Now, we consider the shape of cross-section to ensure that it can save material most. Thanks to stress diagram as in the figure (5.5) and (5.6), we realize that the points which are nearer to neutral axis have the values of normal stresses smaller. It means that material at those positions works less than those far from neutral axis. Hence, section is designed to ensure that material is not distributed far from neutral axis. As a result, in case of brittle material, cross-section is designed in the form of T letter as in the figure 5.11 and in case of ductile material, cross-section can be in the form of I letter or two letters [associated together as in the figure 5.12.



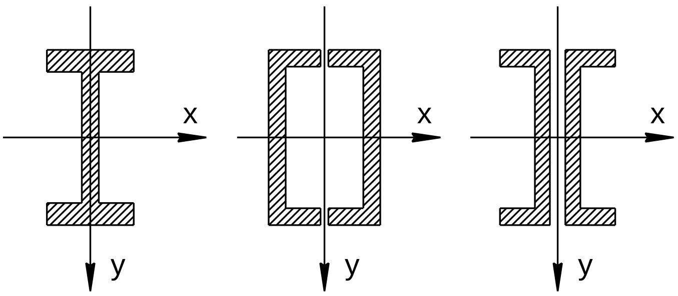


Figure 5.11Figure 5.12

When cross-section has $W_x^{tens} = W_x^{comp}$, to compare theeconomization of using material of different shapes, we can use the ratio $\frac{W_x}{F}$. The section having this ratio big will have ability to resist flexure through a unit of area big. It means that that shape is reasonable. Because the ratio $\frac{W_x}{F}$ has its unit, comparison is not comfortable. Hence, we can use the ratio $\frac{W_x}{\sqrt{F^3}}$ to compare.

Examples:

- Circular section: $\frac{W_x}{\sqrt{F^3}}$ = 0,14
- Rectangular section: $\frac{W_x}{\sqrt{F^3}}$ = 0,167
- Hollow circular section: $\frac{W_x}{\sqrt{F^3}}$ = 0,73 – 0,81

- [letter section: $\frac{W_x}{\sqrt{F^3}}$ = 0.57 – 1,35

- I letter section: $\frac{W_x}{\sqrt{F^3}}$ = 1,02 – 1,51

Thanks to the above values, we realize that I-letter section is the most reasonable.

Sometimes, in case of the beams having circular sections or equilateral triangular section… as in the figure (5.13), if we cut beam a little bit (following the diagonal lines), flexure-resistant capacity will increase.

Simply because: $W_x = \frac{J_x}{y_{max}}$.

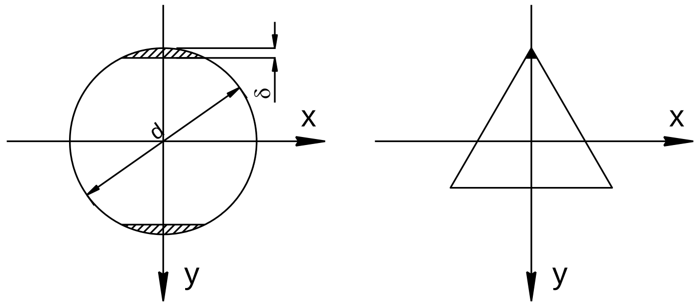


Figure 5.13

When beam is cut, y_{max} will decreaseand simultaneously J_x will also decrease. However, because J_x is the quadratic function of y, so if y is declined to a certain limit, the value of flexure-resistant moment will increase. For example, in case of circular section, if thickness δ = 0,111d, flexure-resistant moment will increase by 0,7%.

5.3. Plane bending

5.3.1. Concept

The beam is planely bended when on its cross-section, besides bending moment M_x , there is also shear force Q_y. These internal forces are in the symmetrical plane of beam.

5.3.2. Stress on cross-section

On the surface of beam, we draw the lines parallel to its axis and the plane curves perpendicular to its axis. After the beam is deformed, the lines parallel to its axis are still parallel to its axis; but they are bended whereas the plane curves perpendicular to its axis are not plane any more. (figure 5.14)

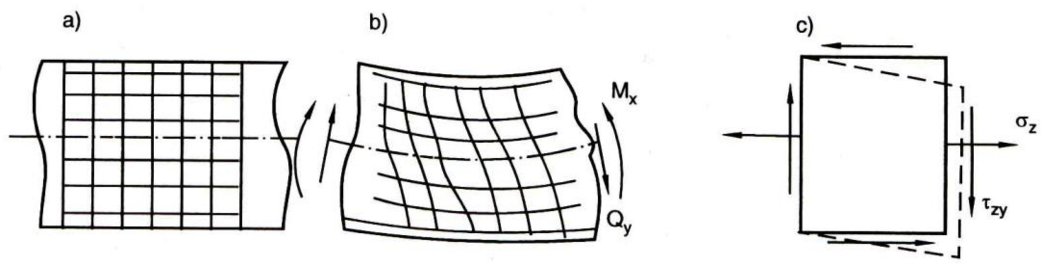


Figure 5.14

If we split a small element on cross-section, we will realise that there are both longitudinal strain and shear strain. (figure 5.14)

Thanks to the above comments, it can be concluded that there are both normal stress and shear stress on thecross-section of bended beam. (figure 5.14)

a. Normal stress on cross-section

It is proved that normal stress onthe cross-section of planely bended beam is caused by bending moment M_x and given by equation (5.2):

$$\sigma_z = \frac{M_x}{J_x} y \qquad (5\text{-}2')$$

The distribution of normal stress on cross-section in this case is similar to that of pure bending.

b. Shear stress on cross-section

Onthe cross-section of planely bended beam, there is a type of shear stress having the same direction as shear force Q_y and constant magnitude along the width of section.

- In case of narrow rectangular section, we can use Duravski's formula:

$$\tau_{zy} = \frac{Q_y.S_x^c}{J_x b^c} \qquad (5\text{-}7)$$

In formual (5-7), S^c_x is the static moment about axis x of the area not containing thecenter of gravity of cross-section and limited by horizontal line AC; b^c is the actual width of small section needing to determine shear stress (line segment AC).

Thanks to the above equation, we can realize that shear stress distribution on cross-section depends on theshape of that section.

*** The diagram of shear stress distribution on cross-section of some popular sections:**

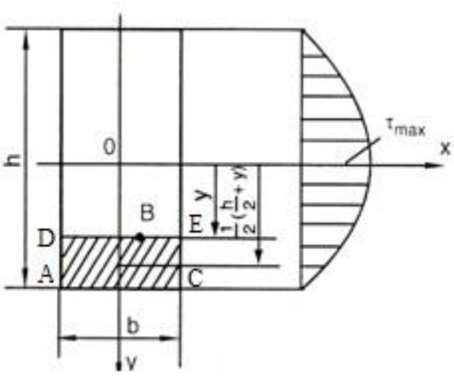
- Rectangular section

A rectangular section of a beam has a width b and a height h. Q_y is the shear force acting on the section. Consider a level DE at a distance y from the neutral axis.

The shear stress at this level is given by expression (5-7) as:

$$\tau_{zy} = \frac{Q_y.S_x^c}{J_x b^c}$$

Where $S_x^c = b\left(\frac{h}{2}-y\right).\frac{1}{2}\left(\frac{h}{2}+y\right) = \frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$



b^c is theactual width of the section at the level DE: b^c = bFigure 5.13

J_x is moment of inertia of the whole section about the neutral axis: $J_x = \frac{bh^3}{12}$

Substitute these values in the above equation, we get:

$$\tau_{zy} = \frac{6Q_y}{bh^3}\left(\frac{h^2}{4}-y^2\right) \qquad (5\text{-}8)$$

From expression (5-8), we see that τ_{zy} increases as y decreases. Also the variation of τ_{zy} with respect to y is a parabola. The figure (5.13) shows the variation of shear stress across the section.

At the top edge, $y = \pm\frac{h}{2}$ and hence, τ_{zy} = 0.

At the neutral axis, y = 0 and hence, $\tau_{zy} = \frac{3Q_y}{2bh}$. This stress is also the maximum shear stress.

- I - section

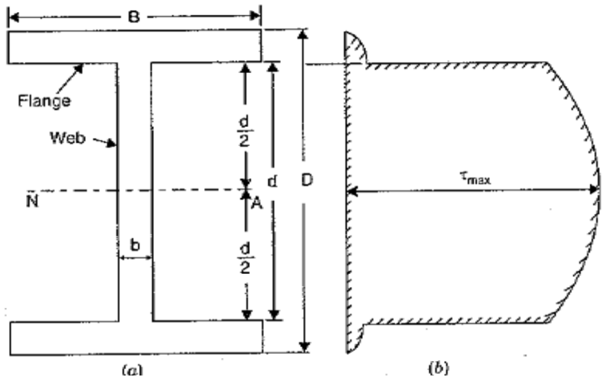


Figure 5.14

To simplify problem, we assume that I-section is created by two flanges and one web. They are both rectangles.

Let B = theoverall width of the section

D=the overall depth of the section

b = thethickness of the web

d = thedepth of the the web

The shear stress at a distance y of the neutral axis is given by equation (5-7) as $\tau_{zy} = \dfrac{Q_y.S_x^c}{J_x b^c}$

In this case, the shear stress distribution in the web and the shear stress in the flange are to be calculated separately.

(i) Shear stress distribution in the flange

Consider a section at a distance y of the neutral axis in the flange as shown in the figure 5.15.

Width of the section = B

The static moment of shaded area of the flange about neutral axis:

The shaded area of flange: $F = B\left(\dfrac{D}{2} - y\right)$

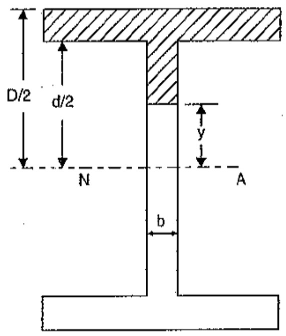


Figure 5.15

The distance of the center of the gravity of the shaded area from neutral axis is given as: $\bar{y} = y + \frac{1}{2}(\frac{D}{2} - y) = \frac{1}{2}\left(\frac{D}{2} + y\right)$

Here shear stress in the flange becomes: $\tau_{zy} = \dfrac{Q_y.B\left(\dfrac{D}{2} - y\right)\dfrac{1}{2}\left(\dfrac{D}{2} + y\right)}{J_x B} = \dfrac{Q_y}{2J_x}\left[\left(\dfrac{D}{2}\right)^2 - y^2\right]$

(5-9)

Hence, the variation of shear stress with respect to y in the flange is parabolic. It is also clear from equation (5-9) that with the increase of y, shear stress decreases.

(a) For the upper edge of the flange: $y = \dfrac{D}{2}$

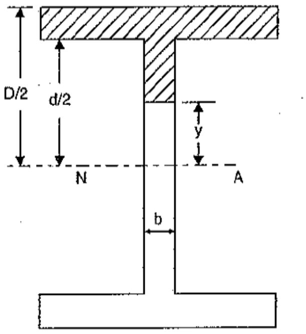
Hence, shear stress $\tau_{zy} = 0$.

(b) For the lower edge of the flange: $y = \dfrac{d}{2}$

Hence, shear stress $\tau_{zy} = \dfrac{Q_y}{2J_x}\left(\dfrac{D^2}{4} - \dfrac{d^2}{4}\right) = \dfrac{Q_y}{8J_x}\left(D^2 - d^2\right)$ (5-10)

(ii) Shear stress distribution on the web

Consider a section at a distance y from the neutral axis in the web as shown in the figure 5.16.



The width of the section = b

Here, S_x^C is made up of two parts, themoment of the flange area about neutral axis plus themoment of the shaded area of the web about neutral axis:

$$S_x^C = B.\left(\frac{D}{2} - \frac{d}{2}\right).\frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) + b.\left(\frac{d}{2} - y\right)\frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$= \frac{B}{8}\left(D^2 - d^2\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Here shear stress in the web becomes as:Figure 5.16

$$\tau_{zy} = \frac{Q_y}{J_x b}\left[\frac{B}{8}\left(D^2 - d^2\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right] \qquad (5\text{-}11)$$

Hence, the variation of shear stress with respect to y in the web is parabolic. It is also clear from expression (5-11) that with the increase of y, shear stress decreases.

At the neutral axis, y = 0 and hence, shear stress is maximum: $\tau_{zy} = \dfrac{Q_y}{J_x b}\left[\dfrac{B}{8}\left(D^2 - d^2\right) + \dfrac{bd^2}{8}\right]$

(5-12)

At the junction of top of the web and bottom of the flange: $y = \dfrac{d}{2}$

Hence, shear stress is given by: $\tau_{zy} = \dfrac{Q_y B(D^2 - d^2)}{8J_x b}$ (5-13)

The shear stress distribution for the web and flange is shown as in the figure 5.14. The shear stress at the junction of the flange and the web changes abruptly. The expression (5-10) gives the stress at the junction of the flange and the web when stress distribution is considered in the flange. But expression (5-13) gives the stress at the junction when stress distribution is considered in the web. From these two equations, it is clear that the stress at the junction changes abruptly from $\dfrac{Q_y}{8J_x}\left(D^2 - d^2\right)$ to $\dfrac{Q_y B(D^2 - d^2)}{8J_x b}$.

- *Circular section:*

The figure 5.17 shows a circular section of a beam. Let R isthe radius of the circular section and Q_y is the shear force acting on the section.

Formula to determine τ_{zy} for circular section is established similarly as in the case of narrow rectangular section.

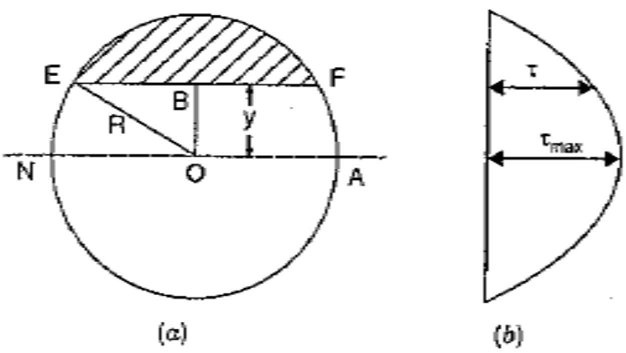


Figure 5.17

$$\tau_{zy} = \frac{Q_y}{3J_x}(R^2 - y^2) \qquad (5\text{-}14)$$

The expression (5-14) shows that shear stress distribution across a circular section is parabolic. It is also clear from equation (5-14) that with the increase of y, shear stress decreases.

At y = ± R, the shear stress $\tau_{zy} = 0$

At the neutral axis, y = 0, the shear stress is maximum: $\tau_{zy\max} = \frac{Q_y}{3J_x}R^2$ *hay* $\tau_{zy\max} = \frac{4Q_y}{3F}$

5.4. The strength of planely bended beam

On thecross-section of planely bended beam, there are three states of stress. Hence, thecondition of strength of planely bended beam is theassociation of both three above conditions. However, if we ignorethe influence of shear force Q,the condition of strength of planely bended beam is similar to that of pure bending.

For brittle material:

$$\begin{cases} \sigma_{z\max}^{tens} = \dfrac{|M_x|_{\max}}{W_x^{tens}} \le [\sigma]_{tens} \\ \left|\sigma_{z\max}^{comp}\right| = \dfrac{|M_x|_{\max}}{W_x^{comp}} \le [\sigma]_{comp} \end{cases}$$

For ductile material:

$$|\sigma_z|_{\max} = \frac{|M_X|_{\max}}{W_x} \le [\sigma]$$

- In case of prismatic beam, dangerous points are in the upper edge and lower edge of section having $|M_X|_{\max}$.

5.4.1. Three basic problems determined by the condition of strength

a) Test problem

To solve test problem, we follow the following procedure:

- Draw shear force diagram and bending moment diagram. After that, we determine the dangerous sections having $M_{x\max}$.

- On dangerous sections, we determine dangerous points and calculate stress at those points.

- Replace the found value inthecondition of strength and conclude.

<u>Example 1</u>: Check thestrength of beam in the figure 5.18. The beam is made from I N^{0}22-steel. Know that q = 10kN/m, l = 4m, [σ] = 10kN/cm^2.

<u>Solution:</u>Draw shear force diagram Q_y and bending moment diagram M_x as shown in the figure 5.18. Thanks to these diagrams, we determine the dangerous section which is in the middle of beam and its magnitude of bending moment is

$$M_{x\max} = \frac{ql^2}{8} = 20kNm$$

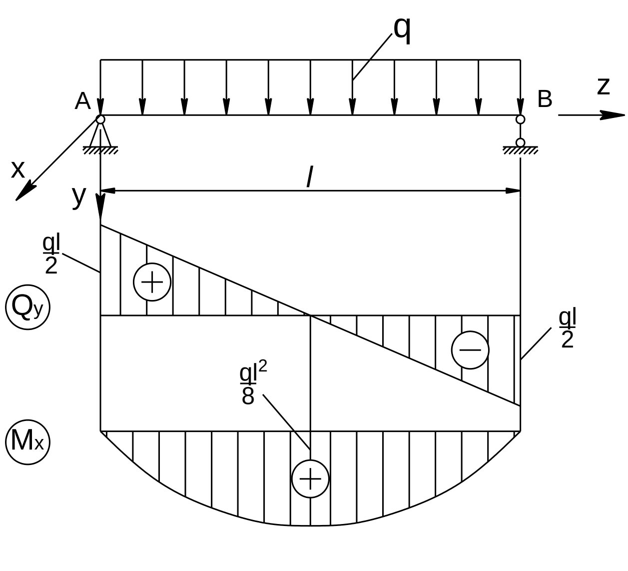


Figure 5.18

Dangerous points are in the upper edge and lower edge of section in the middle of beam. Their normal stressesare:

$$\lfloor \sigma_z \rfloor_{max} = \frac{M_{x\max}}{W_x} = \frac{q.l^2}{8W_x}$$

In case of IN^{0}22-section, we have:

$W_x = 230cm^3$

- Check strength :

$$\lfloor \sigma_z \rfloor_{max} = \frac{M_{x\max}}{W_x} = \frac{q.l^2}{8W_x} = \frac{20.10^2}{230} = 8,65kN/cm^2 < [\sigma]$$

Conclusion: Beam satisfies thecondition of strength.

b) The problem of determining allowable load

Allowable load is the maximum value of load which can act on the beam and still ensures strength. To determine this allowable load, we follow the following procedure:

- Draw shear force diagram and bending moment diagram. After that, we determine the dangerous sections having M_{xmax}.

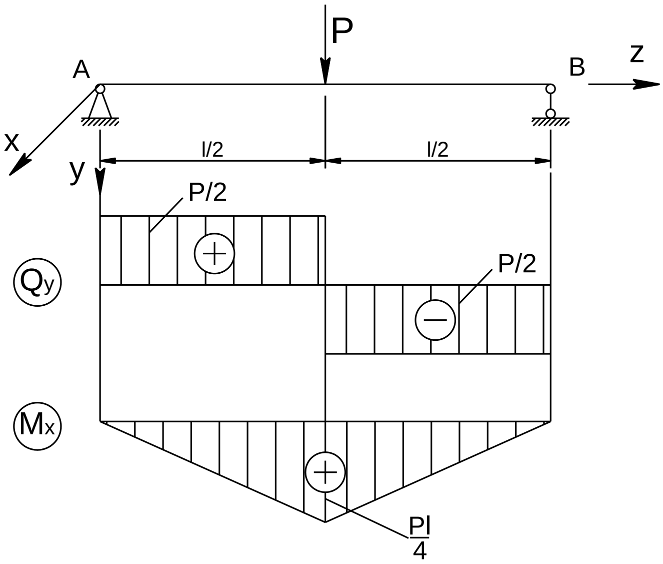
- On dangerous sections, we determine dangerous points and calculate stress at those points.

- Replace the found value inthecondition of strength and infer allowable load.

<u>Example 2</u>: Determine allowable load of beam as shown in the figure 5.10. The beam is made from I N^{0}16-steel. Know that:$[\sigma]$ = 10kN/cm^2, l = 4m

<u>Solution:</u>Shear force diagram and bending moment diagram are expressed in the figure 5.19. According to the figure, we realise that the section having M_{xmax} is in the middle of beam and its magnitudeof bending moment is $M_{x\max} = \frac{Pl}{4}$.

Dangerous points are in the upper edge and lower edge of the section in the middle of beam. Their stresses are $\lfloor \sigma_z \rfloor_{max} = \frac{M_{x\max}}{W_x} = \frac{Pl}{4W_x}$



In case of IN^{0}16-section, we have:Figure 5.19

$W_x = 118cm^3$

According to the condition of strength: $\lfloor \sigma_z \rfloor_{max} = \frac{M_{x\max}}{W_x} = \frac{Pl}{4W_x} \le [\sigma]$

We have:

$$\frac{Pl}{4W_x} \le [\sigma] \to P \le \frac{4W_x[\sigma]}{l} = \frac{4.118.10}{400} = 11,8kN$$

Hence, allowable load is [P] = 11,8kN.

c) The problem of designing

The problem of designing is used to determine thedimensions or minimum sign number of the cross-section of beam in order to ensure that the beam has enough strength. To solve this problem, we follow the following procedure:

- Draw shear force diagram and bending moment diagram. After that, we determine the dangerous sections having M_{xmax}.

- On dangerous sections, we determine dangerous points and calculate stress at those points.

- Replace the found value in condition of strength and infer the necessarydimensions or minimum sign number of cross-section.

Example 3: Determine thesign number of I-section of beam in the figure 5.20. Know that: [σ] = 12kN/cm.

Solution: Shear force diagram and bending moment diagram are expressed in the figure 5.20.

According to bending moment diagram, we realise that dangerous section is at the cantilever and its magnitudeof bending moment is M_{xmax} = Pl = 20kNm.

Dangerous points are in the upper edge and lower edge of the section at the cantilever and their stresses are $\lfloor \sigma_z \rfloor_{max} = \frac{M_{x\max}}{W_x} = \frac{Pl}{W_x}$

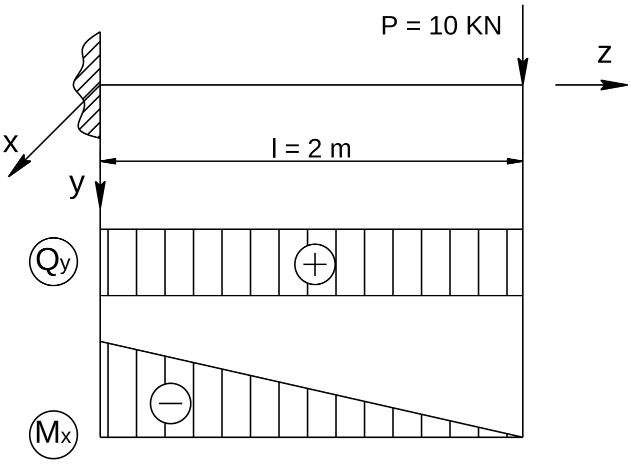


Figure 5.20

Select thesign number of cross-section thanks tothe condition of strength:

$$\sigma_{\max} = \frac{M_{x\max}}{W_x} \le [\sigma]$$

We can infer:

$$W_x \ge \frac{M_{x\max}}{[\sigma]} = \frac{20.10^2}{12} = 166cm^3$$

Consult and choose the I $N^0$20-section having W_x = 184cm^3.

Conclusion : we choose I $N^0$20-section for the beam.

5.5. The deflection of beam

5.5.1. Concept

Consider the beam which is initially straight and horizontal when unloaded. If under the action of loads, the beam is deflected to an other position. In other words, we say that theaxis of beam is bended. The curved axis of beam is called elastic line or deflection curve.

Consider the section at K at a distance z of origin 0. After deformed, K moves to K' and KK' is calledthelongitudinal strain of the section at K.

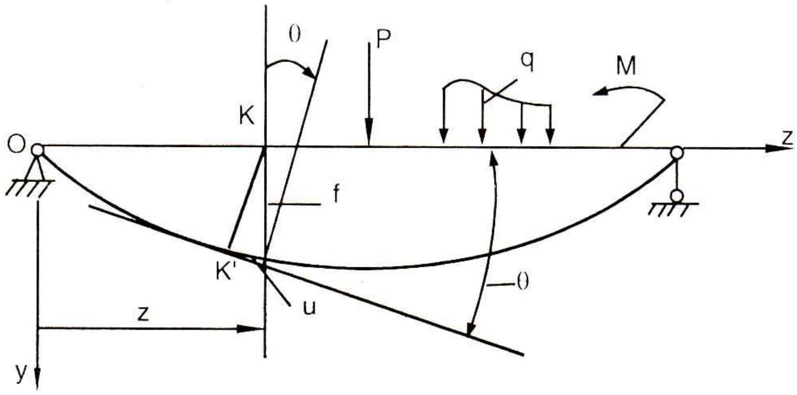


Figure 5.21

We can analyse KK' into two components: u is parallel to theaxis of beam and f is perpendicular to theaxis of beam. Because thedeflection of beam is so small, component u is too small in comparison with f. Hence, we can ignore it. As a result, thedeflection of beam along direction perpendicular to theaxis of beam is called the deflection of section. We realize that deflection changes as section changes. It means that:

$$y(z) = f(z) \tag{a}$$

Besides longitudinal displacement, as beam is deformed, its cross-section has also angular displacement. Angular displacement appears as cross-section rotates around neutral axis because of thedeformation of beam. It is called slope θ. The slopes of different sections are also different. It means that θ(z) is the function of z. Draw the tangent of elastic line at point K'. At that moment, the angle created by this tangent and horizontal direction also equals θ. Hence:

$$\theta \approx tg\theta = \frac{dy}{dz} = y'(z) \tag{b}$$

Hence, thederivative of deflection is the slope of cross-section as beam is deformed.

5.5.2. The differential equation of elastic line

In item 5.2, we established relation between thecurvature of axis of beam and bending moment (see formula 5-1):

$$\frac{1}{\rho} = \frac{M_x}{EJ_x}$$

On the other hand, in terms of mathematics, we also have:

$$\frac{1}{\rho} = \pm \frac{y''}{\left(1 + y'^2\right)^{3/2}}$$

According to (c) and (d), we have equation:

$$\frac{y''}{\left(1 + y'^2\right)^{3/2}} = \pm \frac{M_x}{EJ_x} \tag{e}$$

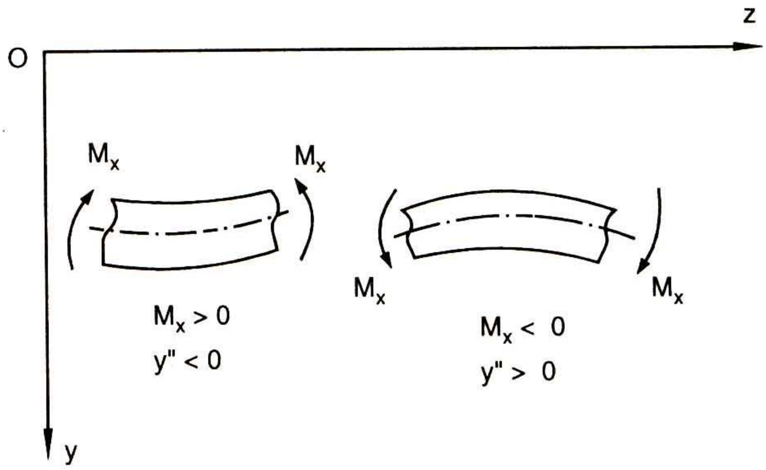


Figure 5.22

This equation is generallythe differential equation of elastic line. We realize that y' is slope and it is usually small in comparison with 1. Hence, we can ignore sign at both two sides of equation and use correlation about sign between y'' and M_x . To choose sign reasonably, we study theelement of bended beam in two cases as shown in the figure 5.22.

According to the figure 5.22, we realize that y'' and M_x always haveopposite signs. Hence,equation (e) will be:

$$y'' = -\frac{M_x}{EJ_x} \qquad (5\text{-}15)$$

Equation (5-15) is called thedifferential equation of elastic line. Product EJ_x is calledtheflexure-resistant stiffness of cross-section.

5.5.3. Methods to determine displacement (deflection, slope)

Now, there are many methods to determine thedisplacement of bended beam. We will study some popular methods.

a. *Direct integration method*

This method's basis is direct integration from differential equation (5-15) as shown below:

$$y'' = -\frac{M_x}{EJ_x} \qquad (5\text{-}15)$$

Integrate first, we get:

$$\theta = y' = \int \left(-\frac{M_x}{EJ_x} \right) dz + C \qquad (5\text{-}16)$$

Integrate second, we get:

$$y = \int \left[\int -\left(\frac{M_x}{EJ_x} \right) dz + C \right] dz + D \qquad (5\text{-}17)$$

Here, C and D are theconstants of integration and determined from boundaryconditions and continuous conditions aboutthe displacement of beam. We will illustrate them by examples.

Example4: Writetheequation of deflection and theslope of beam as shown in the figure 5.23

Solution:Choose co-ordinate system as in the figure 5.23. Bending moment at cross-section having abscissa z is:

$M_x(z) = -P(l-z)$

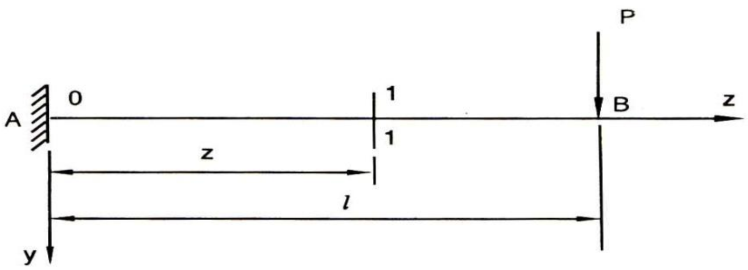


Figure 5.23

Substitute this equation in equation (5-15), we have:

$y'' = \frac{P}{EJ_x}(l - z)$ in which EJ_x is constant as beam is prismatic.

Integrate two sides of above equation in turn, we get:

$$\theta = y' = \frac{P}{EJ_x}\left(lz - \frac{z^2}{2}\right) + C$$

$$f' = y = \frac{P}{EJ_x}\left(l\frac{z^2}{2} - \frac{z^3}{6}\right) + Cz + D$$

In case of given beam, boundary conditions are:

At z = 0, y' = 0 and y = 0

Thanks to these boundary conditions, we have C = D = 0.

Hence, theequations of deflection and slope are:

$$\theta = y' = \frac{P}{EJ_x}\left(lz - \frac{z^2}{2}\right)$$

$$f = y = \frac{P}{EJ_x}\left(\frac{lz^2}{2} - \frac{z^3}{6}\right)$$

As shown in the figure 5.23, the maximum deflection and slope are at the free end of beam. Substitute z = l in the above equation, we get:

$$\theta_{\max} = y'(l) = \frac{Pl^2}{2EJ_x}$$

$$f_{\max} = y(l) = \frac{Pl^3}{3EJ_x}$$

These values are positive. It aproves that cross-section has the slope rotating clockwise and the deflection following positive direction of axis y.

<u>Example 5</u>:Establishthe equation of deflection and theslope of a simply supported beam carrying a point load P as in the figure 5.24.

<u>Solution</u>:Choose co-ordinate system as in the figure 5.24. Here, theequations of bending moment in two segments AC and CB diversely. Hence, theequations of deflection and slope also differ. Call z abscissa of section 1-1 and 2-2. We have theequation of bending moment on these segments:

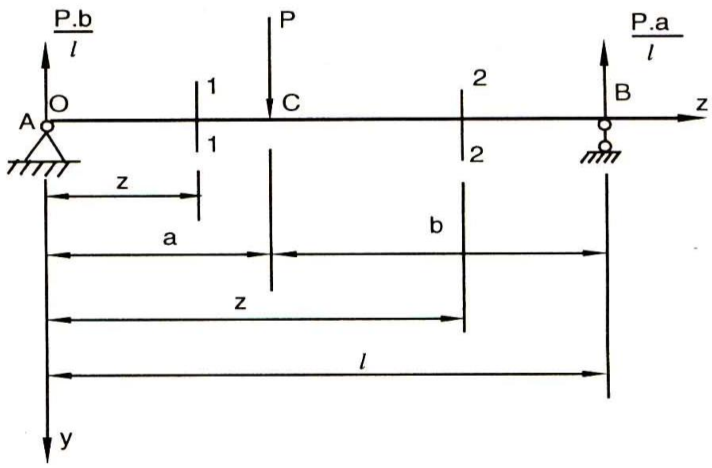


Figure 5.24

$$M_{x1} = \frac{Pb}{l}z \text{ with } 0 \leq x \leq a$$

$$M_{x2} = \frac{Pa(l-z)}{l} \text{ with } a \leq z \leq l$$

The differential equations of elastic line in two segments are:

$$y''_1 = -\frac{Pb}{lEJ_x} z$$

$$y''_2 = -\frac{Pa\left(l-z\right)}{lEJ_x}$$

Continuosly integrate twice the above equations, we get:

$$y'_1 = -\frac{Pb}{lEJ_x} . \frac{z^2}{2} + C_1$$

$$y_1 = -\frac{Pb}{lEJ_x} . \frac{z^3}{6} + C_1 z + D_1$$

$$y'_2 = -\frac{Pa}{lEJ_x}\left(lz - \frac{z^2}{2}\right) + C_2$$

$$y_2 = -\frac{Pa}{lEJ_x}\left(\frac{lz^2}{2} - \frac{z^3}{6}\right) + C_2 z + D_2$$

To determine the constants of integration C_1, D_1, C_2, D_2, we rely on boundary conditions:

At $z = 0 \rightarrow y_1 = 0$

$z = l \rightarrow y_2 = 0$

and continuous conditions about thedisplacement of beam:

$z = a \rightarrow y'_1(a) = y'_2(a)$

$y_1(a) = y_2(a)$

According to these four conditions, we have four equations. Solve this set of equations, we can determine theconstants of integration. Replace them in equation of y'_1, y_1, y'_2, y_2, we get theequations of deflection and slope in each segment.

From this example, we realise that if bending moment varies by many different functions along the length of beam, we have to divide that beam into many segments. In each segment, we have to determine two constants of integration. If there are n segments, we have to determine 2n constants of integration. It means that we have to solve a set of 2n equations. Solving this set of equations consumes a lot of time. This is theweakness of direct integration method. Hence, if beam has to divide into many segments, we seldom use direct integartion method.

b.Morh's integration method associated with multiplying Veresaghin's diagram.

Asume that we have a planely bended beam (figure 5.25) and we need to calculate transposition at the determined section K. This transposition is signed Δ_{km} . Here, theforce-resistant state of beam is called state "m". Hence,Δ_{km} is thetransposition of the section K caused a set of external forces at state "m" (figure 5.25).

Bending momento in state "m" is called M_{xm}. Assume that we ignore all of external forces in state "m" and put a load P_k having the same direction as thedirection of transposition and arbitrary magintude at the section K. This state is called state "k". (figure 5.25)

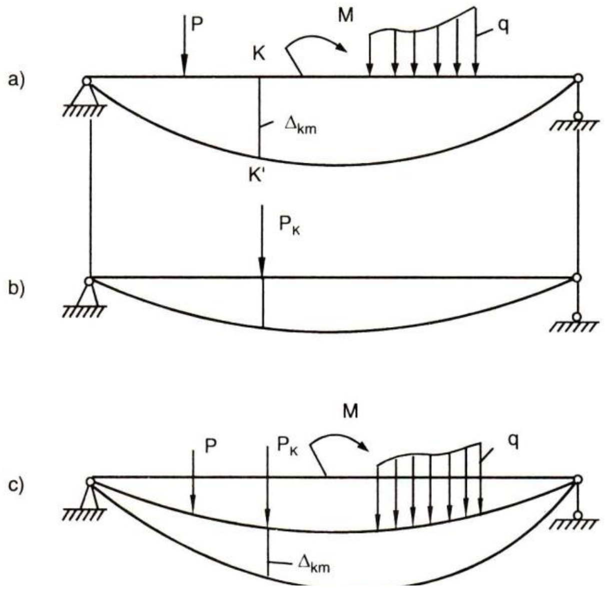


Figure 5.25

Now, put loads at the state “m” on beam at the state “k”. Thanks totheimpacts of these loads, the section K moves a small quantityΔ_{km} and the load P_k also moves a small quantityΔ_{km}. Hence, the load P_k was performed a work P_k. Δ_{km}. If we callthe work of internal forces caused by the above energy A_n, we will have:

$$P_k.\Delta_{km} + A_n = 0 \tag{1}$$

We realize that if A_n is determined, displacementΔ_{km} will be calculated.

If we call elastic strain potential energy corresponding with thework of internal forcesU_{km}, we will get $U_{km} = -A_n$ (1')

Now, we calculate U_{km}.

We call bending moment at state “m” M_{xm} and bending moment at state “k” M_{xk}, bending moment at state “k+m” (figure 5.25) $M_{xm} + M_{xk}$, we will have:

$$U_{k+m} = U_m + U_k + U_{km} \tag{2}$$

Here:

$$U_m = \sum_{i=1}^{n} \int_{l_{i-2}}^{l_i} \frac{M_{xm}^2}{2EJ_x} dz$$

$$U_k = \sum_{i=1}^{n} \int_{l_{i-1}}^{l_i} \frac{M_{xk}^2}{2EJ_x} dz \tag{3}$$

$$U_{k+m} = \sum_{i=1}^{n} \int_{l_{i-1}}^{l_i} \frac{\left(M_{xm} + M_{xk}\right)^2}{2EJ_x} dz$$

According to (2), we infer:

$$U_{km} = U_{k+m} - U_k - U_m \tag{4}$$

Substitute (3) in (4), we get:

$$U_{km} = \sum_{i=1}^{n} \int_{l_{i-1}}^{l_i} \frac{M_{xm}.M_{xk}}{EJ_x} dz \tag{5}$$

Subsitute (5) in (1‘) and put it in (1), we get:

$$P_k \Delta_{km} = \sum_{i=1}^{n} \int_{l_{i-1}}^{l_i} \frac{M_{xm}.M_{xk}}{EJ_x} dz \qquad (6)$$

Because P_k has arbitrary magnitude, for thepurpose of simplification, we choose P_k = 1 and as a result, $M_{xk} = \bar{M}_{xk}$ and theequation of displacement will be:

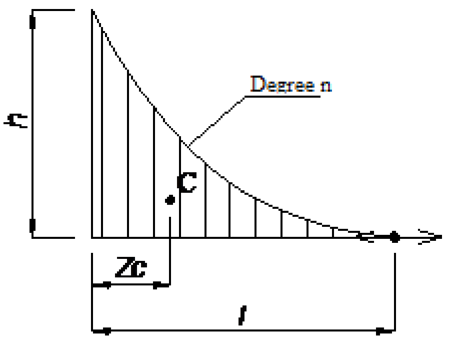
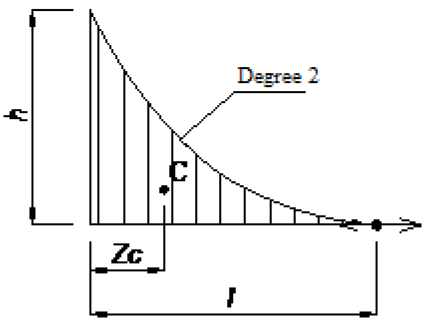
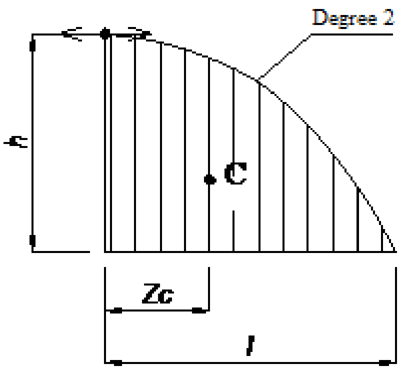
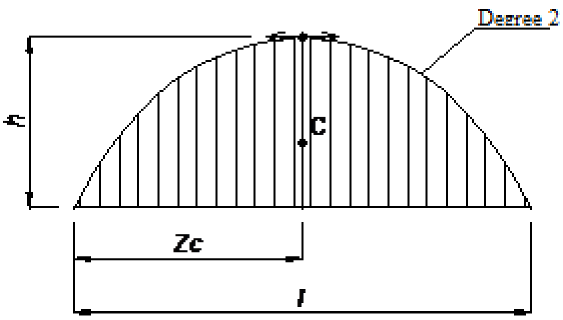
$$\Delta_{km} = \sum_{i=1}^{n} \int_{l_{i-1}}^{l_i} \frac{M_{xm}.\bar{M}_{xk}}{EJ_x} dz \qquad (5\text{-}18)$$

Equation (5-18) is integral equation to calculate transposition. It is named Morh’s integration.

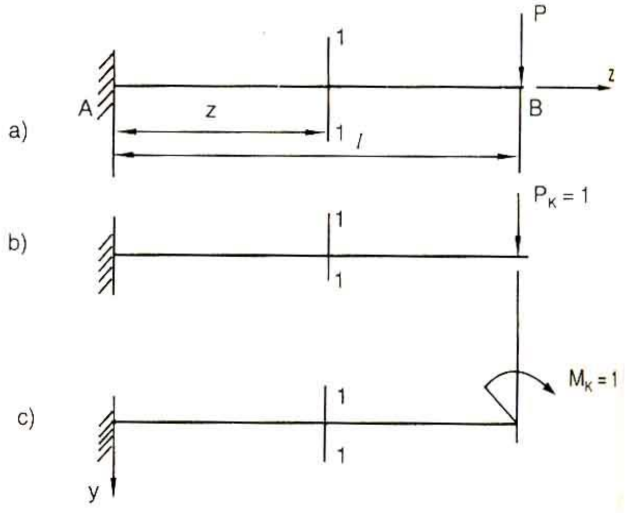
Note: to calculate deflection at the section K, load of unity put at state “k” is the concentrated load whose magnitude is one unit whereas to calculate slope, we put the concentrated moment whose magnitude is one unit.

When we calculate Morh’s integration, if result is positive, thedirection of displacement will concides with the direction of load of unity. If result is negative, thedirection of displacement will be opposite with thedirection of load of unity.

The following tableshows formula to calculate area Ω and theco-ordinate of centroid Z_c of some popular figures.

 $\Omega = \frac{lh}{n+1}; Z_c = \frac{l}{n+2}$	 $\Omega = \frac{lh}{3}; Z_c = \frac{l}{4}$
 $\Omega = \frac{2lh}{3}; Z_c = \frac{3l}{8}$	 $\Omega = \frac{2lh}{3}; Z_c = \frac{l}{2}$

Example 6: Determine deflection and slope at the free end of beam below. (figure 5.26)



Solution: Determine bending moment function on cross-section of beam. Use section 1-1 and consider the right side of beam, we get $M_{xm} = -P(l-z)$.

Establish state "k" to determine deflection at B as shown in the figure 5.26. Bending moment at state "k" is:

$$\bar{M}_{xk} = -l(l-z)$$

Hence, Figure 5.26

$$y_B = \Delta_{km} = \int_0^l \frac{M_{xm}.\bar{M}_{xk}}{EJ_x} dz = \int_0^l \frac{P}{EJ_x}(l-z)^2 dz =$$

$$= \frac{P}{EJ_x}\left(l^2 z - lz^2 + \frac{z^3}{3}\right)\Bigg|_0^l = \frac{Pl^3}{3EJ_x}$$

This positive result means that deflection at B has downward direction which is the same direction as the direction of P_k.

To calculate slope at B, we establish state "k" as in the figure 5.26.

Bending moment at state "k" will be $\bar{M}_{xk} = -1$

Hence,

$$\theta_B = \Delta_{km} = \int_0^l \frac{P}{EJ_x}(l-z)dz = \frac{Pl^2}{2EJ_x}$$

This result is also positive, it means that slope at B has clockwise direction similar to that of M_k .

If stiffness EJ_x in each segment i is constant, Morh's integration (5-18) can be written as below:

$$\Delta_{km} = \sum_{i=1}^{n}\left(\frac{1}{EJ_x}\int_{l_{i-1}}^{l_i} M_{xm}\bar{M}_{xk}dz\right)$$

We can calculate the above integration by multiplying Veresaghin's diagram whose content is:

Consider beam i. Bending moment diagram caused by set of loads "m" and bending moment diagram caused by load of unity are drawn as in the figure 5.27.

We call thearea of M_{xm} diagram Ω_I, theequation of diagram $\bar{M}_{xk}$ has the maximum degree being the first and written as:

$$\bar{M}_{xk} = az + b (7)$$

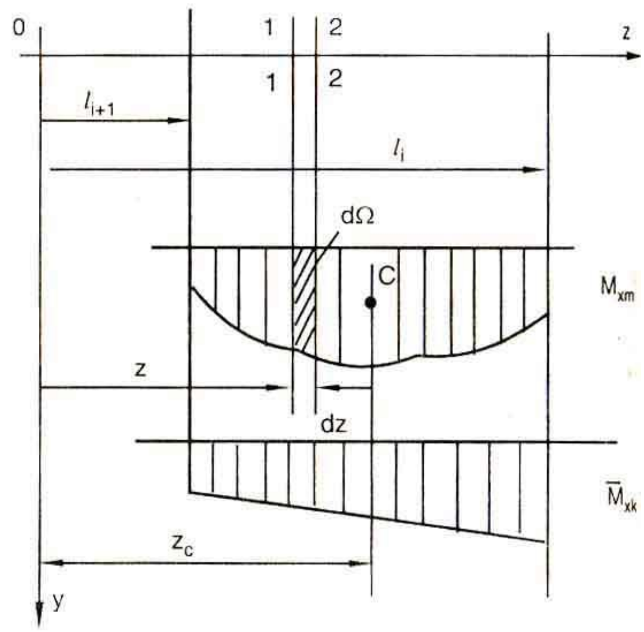


Figure 5.27

Split into the beam i a small element dz by two sections 1-1 and 2-2. We realize that thearea of diagram M_{xm} of element dz will be:

$d\Omega = M_{xm}dz$ (8)

Substitute (7) in (8) and (6-21), we get:

$$\Delta_{km} = \sum_{i=1}^{n} \frac{1}{EJ_x} \int_{\Omega_i} (az+b)\, d\Omega \tag{9}$$

Consider integration:

$$I = \int_{\Omega_i} (az+b) d\Omega = \int_{\Omega_i} azd\Omega + b\int_{\Omega_i} d\Omega \tag{10}$$

we realize that $a\int_{\Omega_i} zd\Omega = aS_y^{\Omega_i}$ (5-19)

$S_y^{\Omega_i}$ is thestatic moment of area of diagram M_{xm} about axis y and can be calculated by the formula: $S_y^{\Omega_i} = \Omega_i . z_c$ (11)

Here z_c is theabscissa of centroid C of diagram M_{xm} (figure 5.27)

And integration $b\int_{\Omega_i} b\Omega = b\Omega_i$ (12)

Substitute (11) in (12) and (10), we get:

$I = (az_c + b)\Omega_i$ (13)

But $az_c + b = \bar{M}_{xk}(z_c)$ (14)

Put (13) in (9), we get:

$$\Delta_{xm} = \sum_{i=1}^{n} \left(\frac{1}{EJ_x} \Omega_i \bar{M}_{xk}(z_c) \right) \tag{5-20}$$

The multiplication of diagram (5-20) is called multiplication of Veresaghin's diagram.

Note:

- As we multiply two diagrams having the same sign, the result is positive whereas if two diagrams have opposite signs, the result is negative.

- If diagram M_{xm} has complicated shape, we can divide it into the small parts having simpler shapes and easy to determine the position of centroid.

- We can use formulas in the table at item b to determine area and theposition of centroid of diagram M_{xm}.

Example7: Determine deflection and slope at thefree end of beam as in the figure 5.28.

Solution: Draw the bending moment diagram M_{xm} caused by force P. (figure 5.28)

Establish state "k" to calculate deflection at B by ignoring P and substitute it by a force $P_k = 1$.

Bending moment diagram is in the figure 5.28.

Multiply diargram (b) and (c), we get: $y_B = \frac{1}{EJ_x} . \frac{1}{2} Pl.l \frac{2}{3} l = \frac{Pl^3}{3EJ_x}$

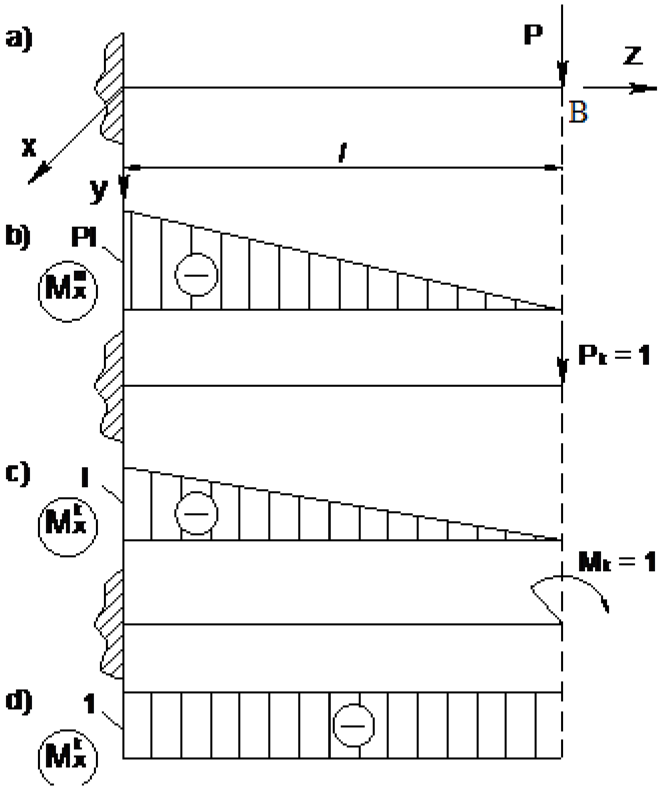


Figure 5.28

Establish state "k" to calculate slope at B by ignoring P and substitute it by a concentrated momentM$_k$ = 1.

Bending moment diagram $\bar{M}_k$ is in the figure 5.28.

Multiply diargram (b) and (d), we get $\theta_B = \frac{1}{EJ_x}.\frac{1}{2}Pl.l.1 = \frac{Pl^2}{2EJ_x}$

<u>Example8</u>: Determine deflection at the section in the middle of the beam and slope at the left end A of the beam as shown in the figure 5.29.

<u>Solution</u>: Bending moment M_{xm} is shown as in the figure 5.29.

To determine deflection at the section in the middle of the beam (section c), we establish state "k": ignore load q, put at the section c a point load P_k = 1. Bending moment diagram $\bar{M}_{xk}$ is shown as in the figure.

Multiply diagram (b) and (c), we get deflection at the section in the middle of the beam:

$$y_c = \frac{1}{EJ_x}.\frac{2}{3}.\frac{ql^2}{8}.\frac{l}{2}.\frac{5}{8}.\frac{l}{4}.2 = \frac{5Pl^4}{384EJ_x}$$

To determine slope at A, we establish state "k": ignore load q, put at the section A a concentrated moment M_k = 1. Bending moment diagram $\bar{M}_{xk}$ is shown as in the figure.

Multiply diagram (b) and (d), we get slope at end A:

$$\theta_A = \frac{1}{EJ_x}.\frac{2}{3}.\frac{ql^2}{8}.l.\frac{1}{2} = \frac{ql^3}{24EJ_x}$$

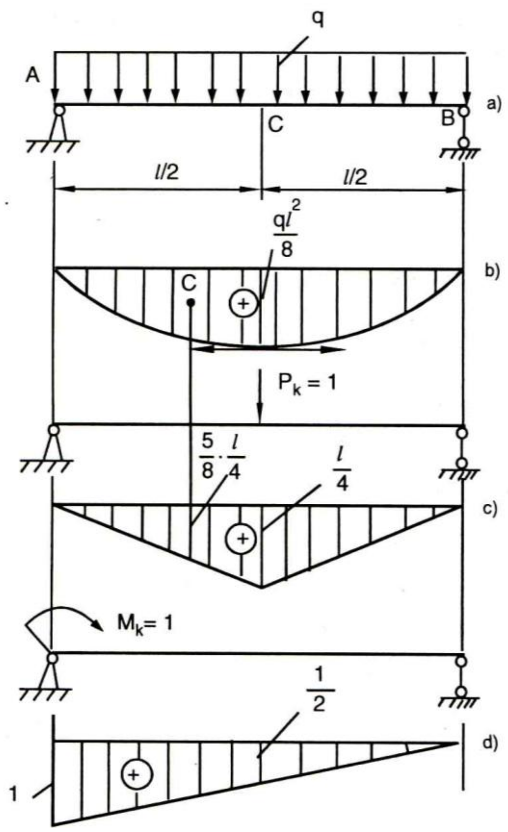


Figure 5.29

Example9: Determine deflection at the section C of the beam. (figure 5.30)

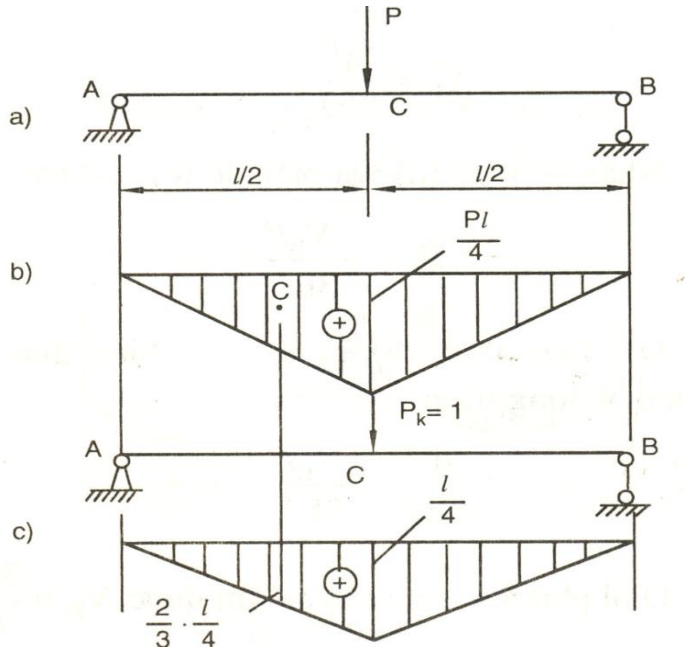


Figure 5.30

Solution: Draw the bending moment M_{xm} caused by load P. (figure 5.30)

To determine deflection at the section C, we establish state "k": ignore load P, put at the section C a concentrated load $P_k = 1$. Bending moment diagram $\bar{M}_{xk}$ is shown as in the figure 5.30.

Multiply diagram (b) and (c), we get deflection at the section C:

$$y_c = \frac{1}{EJ_x}.\frac{1}{2}.\frac{Pl}{4}.\frac{l}{2}.\frac{2}{3}.\frac{l}{4}.2 = \frac{Pl^3}{48EJ_x}$$

5.6. Compute planely bended beam thanks tothe condition of stiffness

5.6.1. The condition of stiffness

As a beam is subjected to a plane flexure, its condition of stiffness is:

$$\begin{cases} |Y|_{max} \le [f] \\ |\varphi|_{max} \le [\varphi] \end{cases}$$

Or

$$\begin{cases} \frac{|Y|_{max}}{l} \le \left[\frac{f}{l}\right] \\ |\varphi|_{max} \le [\varphi] \end{cases}$$

In which:

l: the length of a span of beam. In technique, $\left[\frac{f}{l}\right] = \frac{1}{100} \div \frac{1}{1000}$

5.6.2. Basic problems

Thanks totheabove condition of stiffness, we also have threefollowing basic problems.

a) Test problem

To solve test problem, we follow the followingprocedure:

- Determine the position of the section having maximum deflection and maximum slope.
- Calculate maximum deflection, maximum slope by one of the above methods.
- Substitute in thecondition of stiffness, compare and conclude.

Example 10: Check thestiffness of beam in the figure 5.29. The beam is made from I $N^0$22-steel. Know that q = 10kN/m, l = 4m, $\left[\frac{f}{l}\right] = \frac{1}{400}$, E_{th} = 2.10^4kN/cm^2.

Solution:

- According to the beam, we realize that the section having maximum deflection is the section at C.
- Calculate deflection at C by Morh's integration method associated with multiplying Veresaghin's diagram.
- Draw shear force diagram Q_y and bending moment diagram $M_x$at state "m" as in the figure.
- Draw bending moment diagram M_x at state "k" as in the figure.

Multiply diagrams as in the above examples, we get:

$$y_{\max} = y\left(\frac{l}{2}\right) = \frac{5ql^4}{384EJ_x} = \frac{5.10.10^{-2}.4^4.10^8}{384.2.10^4.2530} = 0,6cm$$

(In case of I$N^0$22-section, we have: J_x = 2530cm^4)

- The condition of stiffness: $\frac{|Y|_{max}}{l} \le \left[\frac{f}{l}\right]$

$$\frac{|Y|_{max}}{l} = \frac{0,6}{400} < \left[\frac{f}{l}\right] = \frac{1}{400}$$

Compare and realize that stiffness is satisfied.

b) The problem of determining allowable load

Allowable load is the maximum value of load which can act on the beam and still ensures stiffness.

To determine this allowable load, we follow the followingprocedure:

- Determine theposition of the section having maximum deflection and maximum slope.
- Calculate maximum deflection, maximum slope by one of the above methods.
- Thanks tothecondition of stiffness, determine allowable load.

Example 11: Determine theallowable load of beam in the figure 5.30. The beam is made from I $N^0$16-steel. Know that[f] = 1cm, l = 4m, E = 2.10^4kN/cm^2.

Solution:

- According to the beam, we realize that the section having maximum deflection is the section at C.
- Calculate deflection at C by Morh's integration method associated with multiplying Veresaghin's diagram.
- Draw shear force diagram Q_y and bending moment diagram $M_x$at state "m" as in the figure.
- Draw bending moment diagram M_x at state "k" as in the figure.

Multiply diagrams as in the above examples, we get:

$$y_{\max} = y\left(\frac{l}{2}\right) = \frac{Pl^3}{48EJ_x}$$

- According to thecondition of stiffness: $|Y|_{max} \le [\text{f}]$

$$y_{\max} = y\left(\frac{l}{2}\right) = \frac{Pl^3}{48EJ_x} \le [\text{f}]$$

Infer:

$$P \le \frac{48EJ_x[f]}{l^3} = \frac{48.2.10^4.945.1}{(400)^3} \approx 28,1kN$$

(In case of I$N^0$16-section, we haveJ_x = 945cm^4)

Hence, theallowable load is: [P] = 28,1kN

c) The problem of designing

The problem of designing is used to determine the dimensions or minimum sign number of cross-section of beam in order to ensure that the beam has enough stiffness. To solve this problem, we follow the followingprocedure:

- Determinethe position of the section having maximum deflection and maximum slope.
- Calculate maximum deflection, maximum slope by one of the above methods.
- Thanks tothecondition of stiffness, determine the dimensions or sign number of cross-section.

Example 12: Determine thesign number of I-section of beam in the figure 5.28. Know that[f] = 1cm; E =2.104kN/cm^2.

Solution:

- According to the beam, we realize that the section having maximum deflection is the section at B.
- Calculate deflection at B by Morh's integration method associated with multiplying Veresaghin's diagram.
- Draw shear force diagram Q_y and bending moment diagram $M_x$at state "m" as in the figure.
- Draw bending moment diagram M_x at state "k" as in the figure.

Multiply diagrams as in the above examples, we get:

$$y_{\max} = y(l) = \frac{Pl^3}{3EJ_x}$$

- According to thecondition of stiffness: $|Y|_{max} \le [f]$

$$y_{\max} = y(l) = \frac{Pl^3}{3EJ_x} < [f]$$

$$\Rightarrow J_x \ge \frac{Pl^3}{3E[f]} = \frac{10.(200)^3}{3.2.10^4.1} = 1333,3cm^4$$

Consult and choose I $N^0$18-section having J_x = 1440 cm^4.

Conclusion : we choose I $N^0$18-section for the beam.

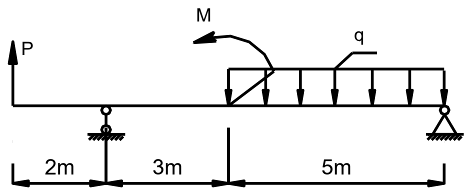
Theoretical questions

1. Raise concept about loading plane, centroidally principal plane of inertia, plane bending and spatial bending.
2. What kind of stress exists on the cross-setion of bended beam? How to calculate and express stress distribution diagram?
3. What is the meaning of section modulus? Raise general formula and formula to calculate in case of simple cross-section. What is the reasonable shape of the cross-section of bended beam?
4. Raise concept aboutthedisplacement of bended beam? Write the differential equation of elastic line.
5. Detailedly raise thecontent of the methods determining deflection and slope.
6. Raise thecondition of stiffness of bended beam and three basic problems.

Numerical problems

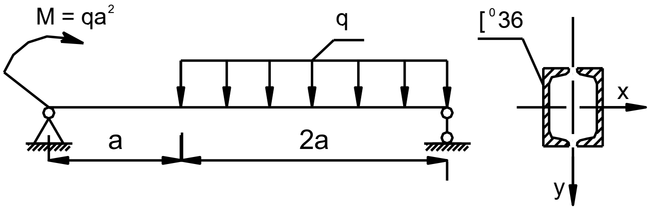
Exercise 1:

Choose the sign number of I-section for beam. Know that: P = 10 kN; M = 40 kNm; q = 20 kN/m; [σ] = 160 MN/m^2.



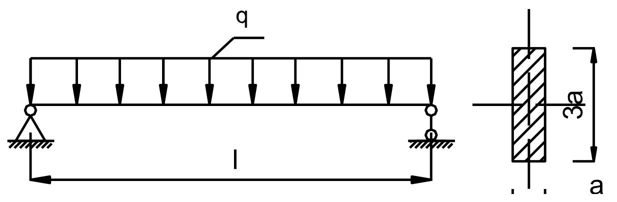
Exercise 2:

Determine the allowable load of the beam. Know that: [σ] = 160 MN/m^2; a = 1m.



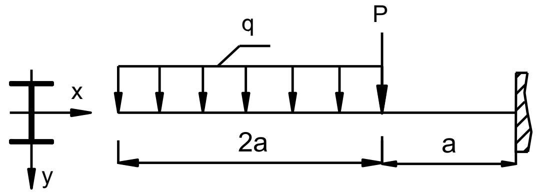
Exercise 3:

Check the strength and stiffness for beam. Know that: q = 50 kN/m; l = 4m; a = 5 cm, [σ] = 160 MN/m^2; $\left[\frac{f}{l}\right] = \frac{1}{400}$; [φ] = 1,6^0, E = 2.10^{11} N/m^2.



Exercise 4:

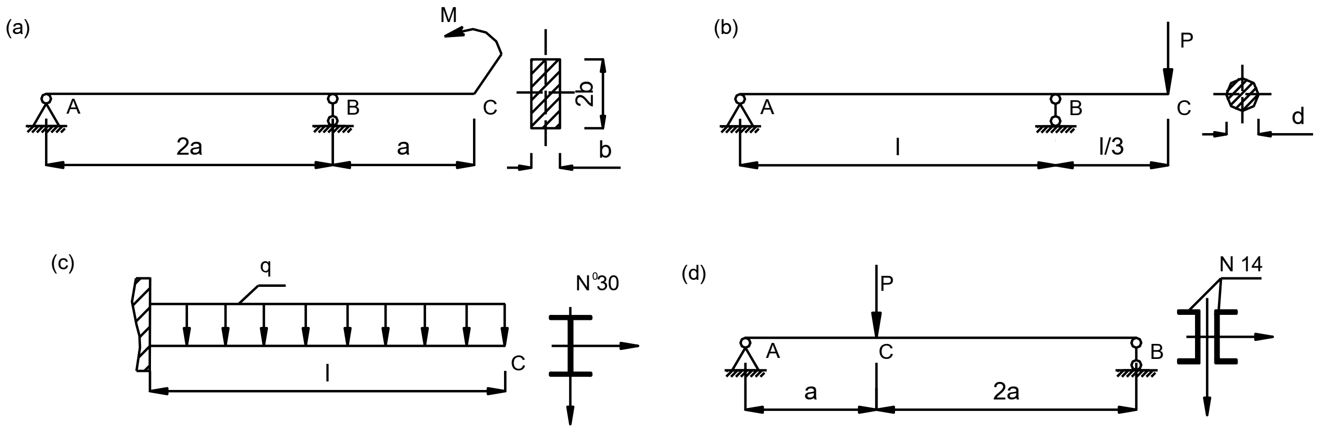
Choose the sign number of I-section for beam. Know that: P = 2qa; q = 40 kN/m; a = 1m; [σ] = 150 MN/m^2; $\left[\frac{f}{l}\right] = \frac{1}{300}$; [φ] = 2,4^0, E = 2.10^5



Exercise 5:

Determine deflection at C, slope at A for the beam as in the figure.

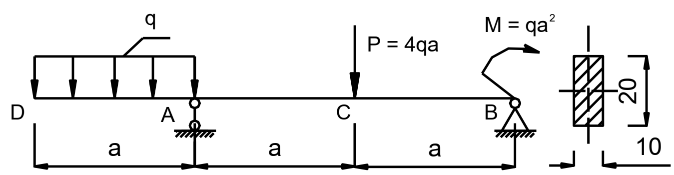
Know that: P = 40 kN; l = 2,4 m; a = 1m; b = 10 cm; E = 2.10^5 MN/m^2; d = 6 cm; M= 20 kNm; q = 20 kN/m.



Exercise 6:

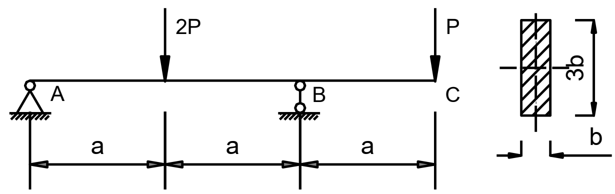
Determine deflection at C and D, slope at A and D for the beam as in the figure. Know that: q = 20 kN/m; a = 1m; E = 2.10^{11} N/m^2.

The cross-section of beam is the rectangle having (bxh) = (10x20) cm^2.



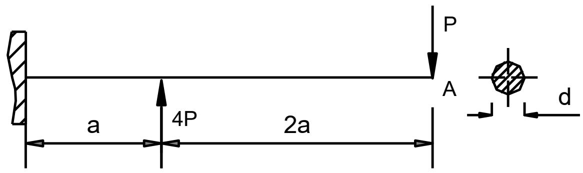
Exercise 7:

Determine the allowable load of the beam. Know that: [σ] = 150 MN/m^2; b = 6 cm; [f_c] = 0,8 cm; [$φ_A$] = 2,4^o; E = 2.10^{4}kN/cm^2.



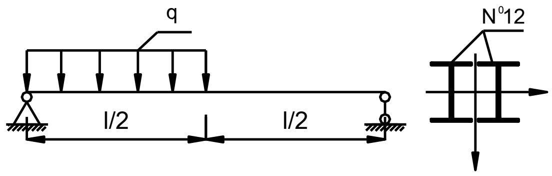
Exercise 8:

Determine the diameter of the beam. Know that: P = 60 kN; a = 1,5 m; [σ] = 150 MN/m^2, [f_A] = 1,2 cm; E = 2.10^{11} N/m^2.



Exercise 9:

Check strength and stiffness for beam. Know that: q = 20 kN/m; l = 4m; [σ] = 150 MN/m^2, [f_B] = 0,6 cm; [φ_A] = 2^0; E = 2.10^5 MN/m^2.

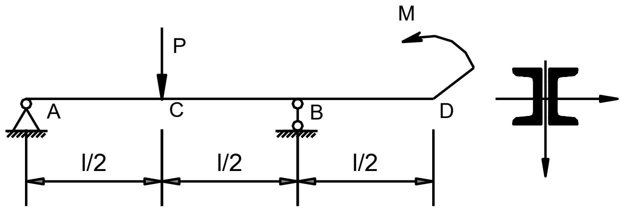


Exercise 10:

The beam having cross-section created by two [-steels is shown as in the figure.

Choose the sign number of I-section for beam. Know that: P = 40 kN; M = 20 kNm; l = 3 m;

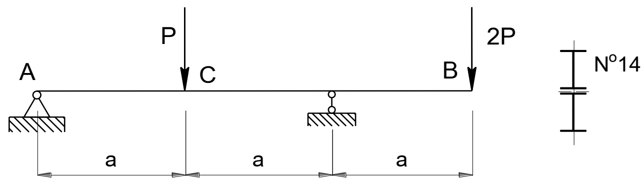
[σ] = 160 MN/m^2; $\left[\frac{f}{l}\right] = \frac{1}{400}$, E = 2.10^{4}kN/cm^2.



Exercise 11:

The beam having cross-section created by two I-steels is shown as in the figure.

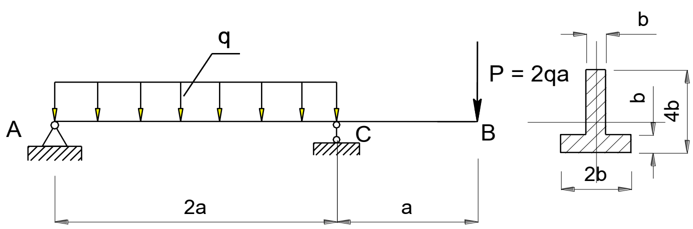
Check strength for beam. Know that: P = 40 kN; a = 1,5 m; [σ] = 160 MN/m^2; $[f_C] = 2\,\text{cm}$ E = 2.10^{4}kN/cm^2.



Exercise 12:

The beam is subjected to loads as in the figue.

Determine the allowable load acting on the beam. Know that: b = 4 cm, a = 1,5 m, [σ] = 160 MN/m^2, $[f_B] = 2\,\text{cm}$, E = 2.10^5 MN/m^2.



APPENDIX 1 –LABORATORY MANUAL

TABLE OF CONTENTS

The number of experiments: …05………… The number of lessons: …05………

Ordinal No.	EXPERIMENT	PLACE	THE NUMBER OF LESSONS	PAGES	NOTE
1	Determine themechanical properties of materials	118 - B5	01		
2	Determine the deformation of round shaft subjected to torsion	118 - B5	01		
3	Determine the deflection of the spring helical, cylindrical, having small pitch	118 - B5	01		

PART I - INTRODUCTION

1.The general target of experiments of the subject

Strength of materials as a basic subject in engineering field is defined as a branch of mechanics of deformable solids that deals with the behaviours of solid bodies subjected to various types of loadings. It provides the future civil engineers with the means of analyzing and designing so that all types of structures operate safely.

The method to study Strength of materials is association between theory and experiments. The study by experiments not only decreases or replaces some complicated calculations but also raises assumptions to establish formulas and checkstheaccuracy of the results found by theory.

Thelaboratory manual of Strength of materials is compiled in order to instruct students the most basic experiments of the subject. Hence, it helps students to get used to research method relying on experiments and understand theimportance of experiments in study.

2.General introduction about equipment in the laboratory

The laboratory of Strength of materials has: versatilely tensile (compressive) machine, tensile (compressive) machine FM1000, torsion testing machine K5, two fatigue testing machines, two impact testing machines, deflection of string measuring machine, torsion testing table, plane bending testing table, axial compression testing table. Besides, there are measuring equipments: calipers, ruler, steel cutting pliers…

3.Progress and time to deploy experiments

After finishing the chapter "Torsion in round shaft", students start experimenting the first three lessons. After finishing the chapter "Buckling of columns", students experiment the last two lessons.

4.The assessment of the experimental results of students

Theexperimental results of students are assessed by answering questions in class, observing students during the process of experiments and checking reports.

5.Preparation of students

Before experimenting, students have to carefully study experiments. The leader of class prepares list of students, divide students into small groups and sends it to lecturer two weeks before experiments.

Experimental curator

Pham Thi Thanh

PART II: DETAILED CONTENT OF EXPERIMENT

LESSON 1 - DETERMINETHE MECHANICAL PROPERTIES OF MATERIALS

1. The purpose of experiment:

Find out relation between force and deformation when pulling or pushing a steel sample, know the way to determine themechanical properties of steel in particular and materials in general. The above mechanical properties of materials are the necessary figures to calculate strength.

2. Theoretical content

We prepare a steel sample as in the figure 1.

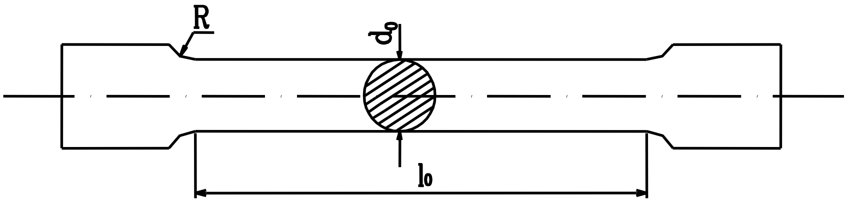


Figure 1

l_o-the initial length of sample

d_o - the initial diameter of sample

Area of cross-section is: $F_0 = \frac{\pi d_0^2}{4}$

Carry out experiment pulling the sample on experimental machine until the sample is cut. We get the graph expressing relationship between P and Δl as shown in the figure 2. According to the graph, we can divide the load-resistant procedure of the sample into three stages:

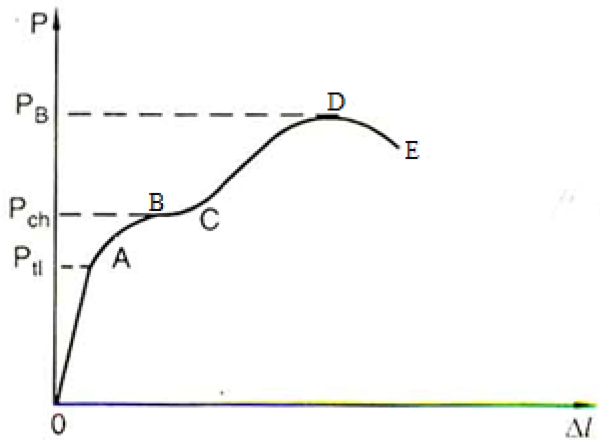


Figure 2

- Proportional stage (elastic stage): The diagram begins with a straight line from the origin O to point A,which means that the relationship between load P and deformation Δl in this initial region is not only linear but also proportional. Beyond point A, the proportionality between stress and strain no longer exists; hence the force at A is called the proportional force P_{pr}. The proportional stress is $\sigma_{pr} = \frac{P_{pr}}{F_0}$

- Yielding stage: With an increase in load beyond the proportional limit, the deformation begins to increase more rapidly for each increment in load. Consequently, the load-deformation curve has a smaller and smaller slope, until, at point B, the curve becomes horizontal. Beginning at this point, theconsiderable elongation of the test specimen occurs with no noticeable increase in the tensile force (from B to C). This phenomenon is known as theyielding stageof the material, and point B is called the yieldingpoint. The maximum load in this stage is signed P_{yiel}. The corresponding stress is known as the yielding stress of the steel $\sigma_{yiel} = \frac{P_{yiel}}{F_0}$. In the region from B to C, the material becomes perfectly plastic, which means that it deforms without an increase in the appliedload.

- Ultimate stage: After undergoing the large deformations that occur during yielding stage in theregion BC, the steel begins to strain harden. During deformation hardening, thematerial undergoes changes in its crystalline structure, resulting in the increased resistance of the material to further deformation. The elongation of the test specimen in this region requires an increase in the tensile load, and therefore the load-deformation diagram has a positive slope from C to D. The load eventually reaches its maximum value, and the correspondingload (at point D) is called the ultimate load P_{ulti}. The corresponding stress is known as the ultimate stress of the steel $\sigma_{ulti} = \frac{P_{ulti}}{F_0}$. Further stretching of the bar is actually accompanied by a reduction in the load, and fracture finally occurs at a point such as E in Fig. 2.

When a test specimen is stretched, lateral contraction occurs. The resulting decrease in cross-sectional area is too small to have a noticeable effect on the calculated thevalues of thestresses up to about point C in Fig 2.2, but beyond that point the reduction in area begins to alter the shape of the curve. In the vicinity of the ultimate stress, the reduction in area of the bar becomes clearly visible and a pronounced necking of the bar occurs (see Figure 3).

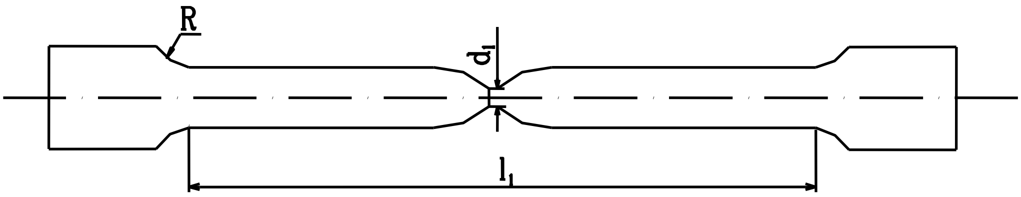


Figure 3

σ_{pr}, σ_{yie}, $\sigma_{ulti}$are characteristics for ductility of materials. Besides, in case of ductile materials when pulled, we can find two characteristics forthe plasticity of materials δ and ψ. They are determined as below:

The extension inthe length of the specimen after fracture to its initial gauge length:

$$\delta = \frac{l_1 - l_0}{l_0}.100\%$$

The percent reduction in area measuring the amount of necking that occurs: $\psi = \frac{F_0 - F_1}{F_0}.100\%$

3. Experimenting machine

Use versatilely tensile (compressive) machine made in Germany. Its model is FM-1000. The maximum tensile force is 1000kG ≈ 10kN and it is driven by hydraulic power. Tensile (compressive) machine has two cantilevers used to keep sample during the process of tension. The upper cantilever is fixed while the lower cantilever can be mobile. Force measuring gauge and graph drawing equipment express relationship between P and Δl.

4. The procedure of experiment

- Measure initial diameter d_o and markthe initial length of sample l_o = 10cm as the figure.

- Fix sample in two cantilevers of the machine, check thework of every part of the machine, control hand to return position “0”.

- Supply electric for the machine and control cantilever to the position marked on the sample.

- Begin pulling the sample, observe the process of pulling until the sample is cut, read thevalues P_{pr}, P_{yiel}, P_{ulti}on force measuring gauge.

- Take the sample out of two cantilevers of the machine, keep the line marked on the sample. Measure dimensions l_1, d_1 again after pulling.

+ Measure l_1: Attach two parts of the sample cut together and measure distance l_1 among two lines marked.

+ Measure d_1: Diameter d_1 is measured at the position which the sample is cut. Hence, we can calculate F_1.

5. Experimental result

After calculating, theexperimental result is written in the following table:

Sample	l_1 (cm)	d_1 (cm)	P_{pr} (kN)	P_{yiel} (kN)	P_{ulti} (kN)	σ_{pr} (kN/cm^2)	σ_{yiel} (kN/cm^2)	σ_{ulti} (kN/cm^2)	δ (%)	ψ (%)
Sample 1										
Sample 2										
Sample 3										

6. Comments

- Compare properties about the strength of the sample with the properties of steel CT3 when pulled.

- Comment the deformation and diameter of the sample after pulling in comparison with the initial sample.

LESSON 2 - DETERMINE THE DEFORMATION OF THE ROUND SHAFT SUBJECTED TO TORSION

1. The purpose of experiment:

Determine relative angle of twist of the round shaft subjected to torsion through experiment. Hence, we can evaluate the accuracy of theoretical formula.

2. Theoretical content

When a round shaft is subjected to torsion, relative angle of twist among two cross-sections of shaft is determined by equation:

* In general case:

Shaft has many segments, M_z, GJ_p vary continuously on each segment.

$$\varphi = \sum_{i=1}^{n} \int_0^{l_i} \left(\frac{M_z}{GJ_p} \right)_i dz$$

In which: $M_z$is internal torque.

G is modulus of rigidity (shear modulus) of material. The sample is made from steel CT3 having $G = 8.10^6$kN/cm^2

$J_p$is polar moment of inertia of cross-section. In case of hollow circular shaft, $J_\rho$is determined: $J_p = \frac{\pi D^4}{32}(1-\eta^4)$

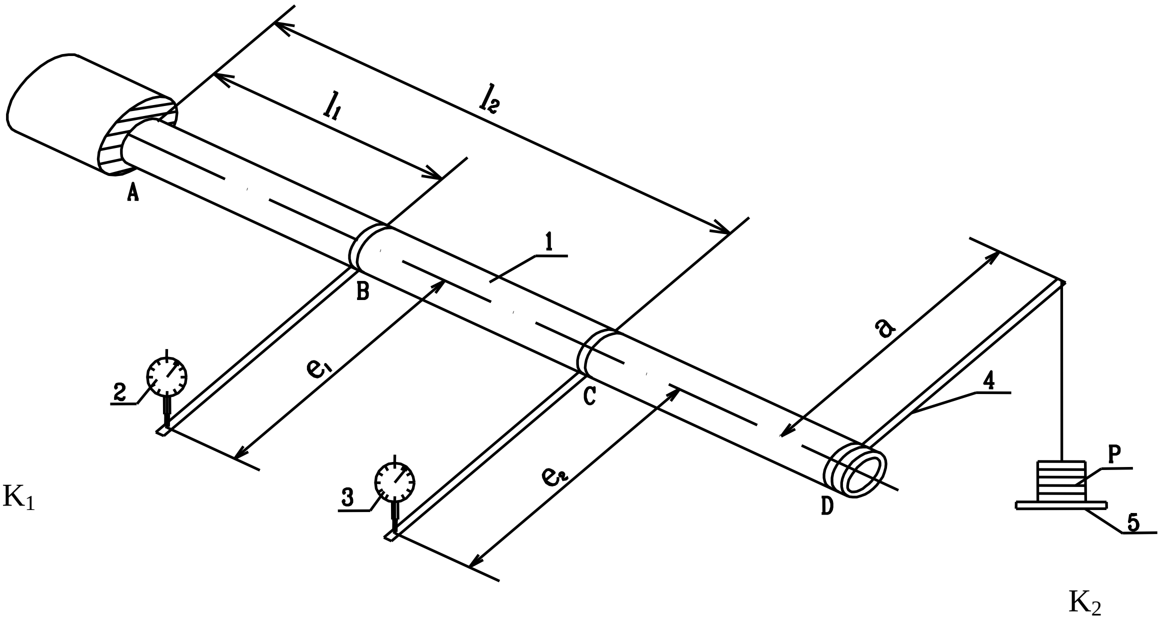
$l_i$is the length of segment i.

* In particular case:

Shaft has many segments, M_z, GJ_p are constant on each segment.

$$\varphi = \sum_{i=1}^{n} \left(\frac{M_z l}{GJ_p} \right)_i$$

4. Experimental layout



1. Hollow circular shaft
2. Gauge K_1
3. Gauge K_2
4. Arm
5. Bracket to put load

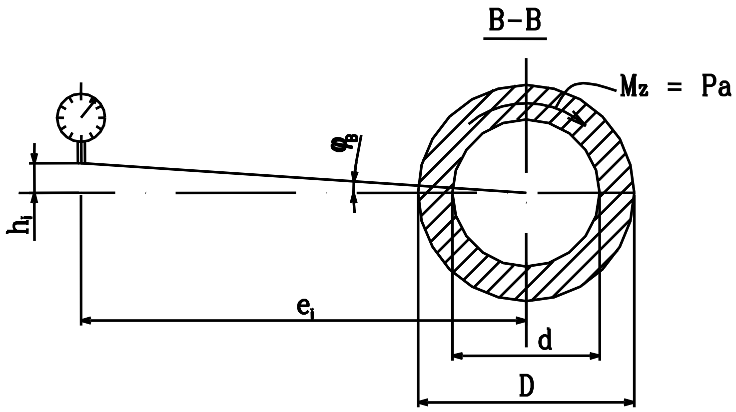


Figure 4

Thanks tothe impact of the load P, shaft is deformed and cross-sections are rotated. At the section B and C, brackets e_1, e_2 are also rotated corresponding angles φ_B và φ_C. Hence, the gauges K_1, K_2 will point corresponding displacements h_1 and h_2. Angles of twist at the section B and C are determined through h_1, h_2, e_1, e_2.

$$\varphi_B = arctg\frac{h_1}{e_1};\ \varphi_C = arctg\frac{h_2}{e_2}$$

4. The procedure of experiment

- Assemble experimental layout.
- Measure the dimensions of shaft: l_1, l_2, d, D, e_1, e_2, a.
- Put loads in turn, read and write indexes on gauges.

5. Experimental result

l_i	P_i (N)	M_{zi} (Nmm)	h_i (mm)	φ_{ith}	φ_{iex}	$\Delta\varphi = \left\|\frac{\varphi_{ith} - \varphi_{iex}}{\varphi_{ith}}\right\| 100\%$
l_1 =	P_1 = 10 ... P_5 = 50					
On	P_1=10 ... P_5= 50					

6. Comments

- Commentthe accuracy of experimental result.
- Analyse reasons.

LESSON3-DETERMINE THE DEFLECTION OF THE SPRING HELICAL, CYLINDRICAL, HAVING SMALL PITCH

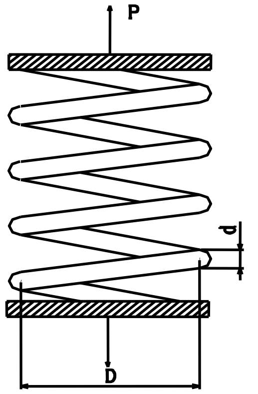
1. The purpose of experiment:

Determine the deflection of the spring helical, cylindrical, having small pitch through experiment. Hence, we can evaluate the accuracy of theoretical formula.

2. Theoretical content

Consider the spring helical, cylindrical, having small pitch, diameter of spring coil D, diameter of spring wire d and number of coils n. If we compress spring by a force P, deflection is determined by equation:

$$\lambda = \frac{8PD^3 n}{Gd^4} \qquad \text{(mm, cm)}$$



3. Experimental layout

When load P cause compressive deformation for spring, gauge K measures corresponding value of deflection λ.

Figure 5

4. The procedure of experiment

- Determine the parameters of spring: D, d, n
- Assemble and control hand to return to the position "0" of the gauge.
- Put load P in turn, read and write the corresponding value of deflection on gauges.

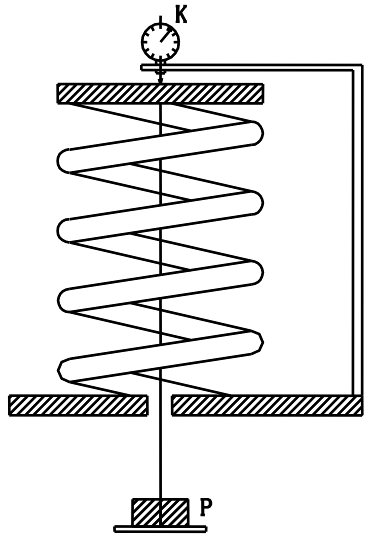


Figure 6

5.Experimental result

The process of calculating values λ is written in the following table:

P_i (N)	λ_i^{th} (mm)	$\sum_{i=1}^{n} \lambda_i^{th}$	λ_i^{ex} (mm)	$\sum_{i=1}^{n} \lambda_i^{ex}$	$\Delta\lambda = \frac{\sum_{i=1}^{n} \lambda_i^{th} - \sum_{i=1}^{n} \lambda_i^{ex}}{\sum_{i=1}^{n} \lambda_i^{th}}.100\%$
$P_1 = 5$ ... $P_8 = 40$					

6. Comments

- Commentthe accuracy of experimental result.
- Analyse reasons.

APPENDIX

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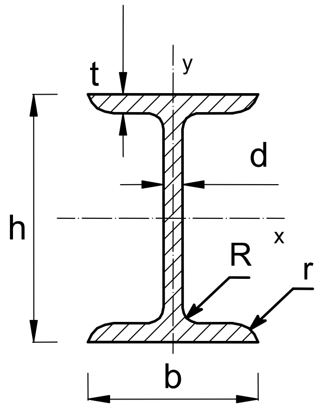
Subject of Strength of materials



PROPERTIES OF SHAPED STEEL

I -Section

ГОСТ 8239-56

The sign number of section	Gravity N/m	Dimensions (mm)						The area of section cm²	The properities of area						
									x-x				y-y		
		h	b	d	t	R	r		J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm
10	111	100	70	4,5	7,2	7,0	3,0	14,2	244	48,8	4,15	28,0	35,3	10	1,58
12	130	120	75	5,0	7,3	7,5	3,0	16,5	403	67,2	4,94	38,5	43,8	11,7	1,63
14	148	140	82	5,0	7,5	8,0	3,0	18,9	632	90,3	5,78	51,5	58,2	14,2	1,75
16	169	160	90	5,0	7,7	8,5	3,5	21,5	945	118	6,63	67,0	77,6	17,2	1,90
18	187	180	95	5,0	8,0	9,0	3,5	23,8	1330	148	4,47	83,7	94,6	19,9	1,99
18a	199	180	102	5,0	8,2	9,0	3,5	25,4	1440	160	5,53	90,1	119	23,3	2,06
20	207	200	100	5,2	8,2	9,5	4,0	26,4	1810	181	8,27	102	112	22,4	2,17
20a	222	200	110	5,2	8,3	9,5	4,0	28,3	1970	197	8,36	111	148	27,0	2,29
22	237	220	110	5,3	8,6	10,0	4,0	30,2	2530	230	9,14	130	155	28,2	2,26
22a	254	220	120	5,3	8,8	10,0	4,0	32,4	2760	251	9,23	141	203	33,8	2,50
24	273	240	115	5,6	9,5	10,5	4,0	34,8	3460	289	9,97	163	198	34,5	2,37
24a	294	240	125	5,6	9,8	10,5	4,0	37,5	3800	317	10,1	178	260	41,6	2,63
27	315	270	125	6,0	9,8	11,0	4,5	40,2	5010	371	11,2	210	260	41,5	2,54
27a	339	270	135	6,0	10,2	11,0	4,5	43,2	5500	407	11,3	229	337	50,0	2,80
30	365	300	135	6,5	10,2	12,0	5,5	46,5	7080	472	12,3	268	337	49,9	2,69
30a	392	300	145	6,5	10,7	12,0	5,5	49,9	7780	518	12,5	292	346	60,1	2,95
33	422	330	140	7,0	11,2	13,0	5,5	53,8	9840	597	13,5	339	419	59,9	2,79
36	486	360	145	7,5	12,3	14,0	6,0	61,9	13380	743	14,7	423	516	71,1	2,89
40	561	400	155	8,0	13,0	15,0	6,0	71,9	18930	974	16,3	540	666	75,9	3,05
45	652	450	160	8,6	14,2	16,0	7,0	83,0	27450	1220	18,2	699	807	101	3,12
50	761	500	170	9,3	15,2	17,0	7,0	96,9	39120	1560	20,1	899	1040	122	3,28
55	886	550	180	10,0	16,5	18,0	7,0	113	54810	1990	20,2	1150	1350	150	3,46
60	1030	600	190	10,8	17,8	20,0	8,0	131	75010	2500	23,9	1440	1720	181	3,62
65	1190	650	200	11,7	19,2	22,0	9,0	151	100840	3100	25,8	1790	2170	217	3,79
70	1370	700	210	12,7	20,8	24,0	10,0	174	133890	3830	27,7	2220	2730	260	3,76
70a	1580	700	210	15,0	24,0	24,0	10,0	202	152700	4360	27,5	2550	3240	309	4,01
70b	1840	700	210	17,5	28,2	24,0	10,0	234	175350	5010	27,4	2940	3910	373	4,09

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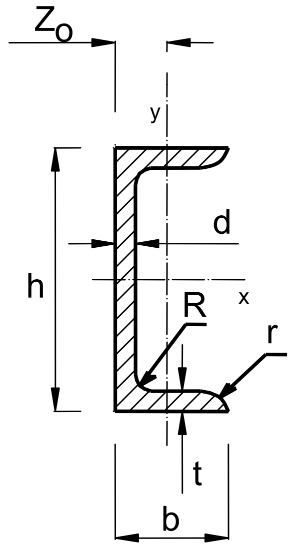
Subject of Strength of materials



PROPERTIES OF SHAPED STEEL

C -Section

ГОСТ 8240-56

The sign number of section	Gravity N/m	Dimensions (mm)						The area of section s cm^2	The properties of area							Z_0
									x-x				y-y			
		h	b	d	t	R	r		J_x cm^4	W_x cm^3	i_x cm	S_x cm^3	J_y cm^4	W_y cm^3	i_y cm	
6	54,2	50	37	4,5	7,0	6,0	2,5	6,90	26,1	10,4	1,94	6,36	8,41	3,59	1,10	1,36
6,5	65,0	65	40	4,5	7,4	6,0	2,5	8,28	54,5	16,8	2,57	10,0	11,9	4,58	1,20	1,40
8	77,8	80	45	4,8	7,4	6,5	2,5	9,91	99,9	25,0	3,17	14,8	17,8	5,89	1,34	1,48
10	92,0	100	50	4,8	7,5	7,0	3,0	11,7	187	37,3	3,99	21,9	25,6	7,42	1,48	1,55
12	108,0	120	54	5,0	7,7	7,5	3,0	13,7	313	52,2	4,78	30,5	34,4	9,01	1,58	1,59
14	123,0	140	58	5,0	8,0	8,0	3,0	15,7	489	69,8	5,59	40,7	45,1	10,9	1,70	1,66
14a	132,0	140	62	5,0	8,5	8,0	3,0	16,9	538	76,8	5,65	44,6	56,6	13,0	1,83	1,84
16	141,0	160	64	5,0	8,3	8,5	3,5	18,0	741	92,6	6,42	53,7	62,6	13,6	1,87	1,79
16a	151,0	160	68	5,0	8,8	8,5	3,5	19,3	811	101	6,48	58,5	77,3	16,0	2,00	1,98
18	161,0	180	70	5,0	8,7	9,0	3,5	20,5	1080	120	7,26	69,4	85,6	16,9	2,04	1,95
18a	172,0	180	74	5,0	9,2	9,0	3,5	21,9	1180	131	7,33	75,2	104	19,7	2,18	2,13
20	184,0	200	76	5,2	9,0	9,5	4,0	23,4	1520	152	8,07	87,8	113	20,5	2,20	2,07
20a	196,0	200	80	5,2	9,9	9,5	4,0	25,0	1660	166	8,15	95,2	137	24,0	2,34	2,57
22	209,0	220	82	5,3	9,9	10,0	4,0	26,7	2120	193	8,91	111	151	25,4	2,38	2,24
22a	225,0	220	87	5,3	10,2	10,0	4,0	28,6	2320	211	9,01	121	186	29,9	2,55	2,47
24	240,0	240	90	5,6	10,0	10,5	4,0	30,6	2900	242	9,73	139	208	31,6	2,60	2,42
24a	258,0	240	95	5,6	10,7	10,5	4.0	32,9	3180	265	9,84	151	254	37,2	3,78	2,67
27	277,0	270	95	6,0	10,5	11	4,5	35,2	4160	308	10,9	178	262	37,3	2,73	2,47
30	318,0	300	100	6,5	11,0	12	5,0	40,5	5810	387	12,0	224	327	43,6	2,84	2,52
33	365,0	330	105	7,0	11,7	13	5,0	46,5	7980	484	13,1	281	410	51,8	2,97	2,59
36	419,0	360	110	7,5	12,6	14	6,0	53,4	10820	601	14,2	350	513	61,7	3,10	2,68
40	483,0	400	115	8,0	13,5	15	6,0	61,5	15220	761	15,7	444	642	73,4	3,23	2,75

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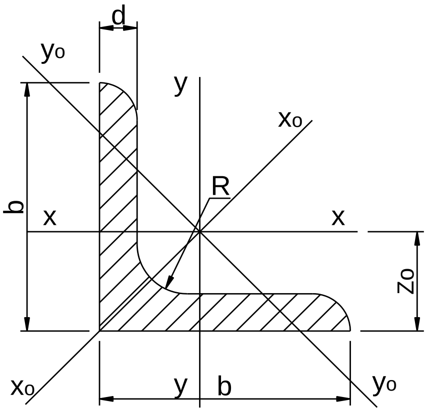

PROPERTIES OF SHAPED STEEL

Equal angle steel

ΓOCT 8240-56

The sign number of section	Dimensions, mm				The area of section cm^2	Gravity per meter N	The properties of area							
							x - x		y - y		y_o - y_o		$x_0 - x_0$	
	b	d	r	R			J_x cm^4	i_x cm	J_{xomax} cm^4	i_{xomax} cm	J_{xomin} cm^4	i_{xomin} cm	J_{xomax} cm^4	Z_o cm
2	20	3	3.5	1.2	1.13	8.9	0.4	0.59	0.63	0.75	0.17	0.39	0.81	0.6
		4			1.46	11.5	0.5	0.58	0.78	0.73	0.22	0.38	1.09	0.64
2.5	25	3	3.5	1.2	1.43	11.2	0.81	0.75	1.29	0.95	0.34	0.49	1.57	0.73
		4			1.86	14.6	1.03	0.74	1.62	0.93	0.44	0.48	2.11	0.76
2.8	28	3	4	1.3	1.62	12.7	1.16	0.85	1.84	1.07	0.48	0.55	2.2	0.8
3.2	32	3	4.5	1.5	1.86	14.6	1.77	0.97	2.8	1.23	0.74	0.63	3.26	0.89
		4			2.43	19.1	2.26	0.96	3.58	1.21	0.94	0.62	4.39	0.94
3.6	36	3	4.5	1.5	2.1	16.5	2.56	1.1	4.06	1.39	1.06	0.71	4.64	0.99
		4			2.75	21.6	3.29	1.09	5.21	1.38	1.36	0.7	6.24	1.04
4	40	3	5	1.7	2.35	18.5	3.55	1.23	5.63	1.55	1.47	0.79	6.35	1.09
		4			3.08	24.2	4.58	1.22	7.26	1.53	1.9	0.78	8.53	1.13
4.5	45	3	5	1.7	2.65	20.8	5.13	1.39	8.13	1.75	2.12	0.89	9.04	1.21
		4			3.48	27.3	6.63	1.38	10.05	1.74	2.74	0.89	12.1	1.26
		5			4.29	33.7	8.03	1.37	12.7	1.72	3.33	0.88	15.3	1.3
5	50	3	5.5	1.8	2.96	23.2	7.11	1.55	11.3	1.95	2.95	1	12.4	1.33
		4			3.89	30.5	9.21	1.54	14.6	1.94	3.8	0.99	16.6	1.38
		5			4.8	37.7	11.2	1.53	17.8	1.92	4.63	0.98	20.9	1.42
5.6	56	3.5	6	2	3.66	30.3	11.6	1.73	18.4	2.18	4.8	1.12	20.3	1.5
		4			4.38	34.4	13.1	1.73	20.8	2.18	5.41	1.11	23.3	1.52
		5			5.41	42.5	16	1.72	25.4	2.16	6.59	1.1	29.2	1.57
6	63	4	7	2.3	4.96	39	18.9	1.95	29.9	2.45	7.81	1.25	33.1	1.69

		5			6.13	48.1	23.1	1.94	36.6	2.44	9.52	1.25	41.5	1.74
		6			7.28	57.2	27.1	1.93	42.9	2.43	11.2	1.24	50	1.78
7	70	4.5	8	2.7	6.2	48.7	29	2.16	46	2.72	12	1.39	51	1.88
		5			6.86	53.8	31.9	2.16	50.7	2.72	13.2	1.39	56.7	1.9
		6			8.15	63.9	37.6	2.15	59.6	2.71	15.5	1.38	68.4	1.94
		7			9.42	73.9	43	2.14	68.2	2.69	17.8	1.37	80.1	1.99
		8			10.7	83.7	48.2	2.13	76.4	2.68	20	1.37	91.9	2.02
7.5	75	5	9	3	7.39	58	39.5	2.31	62.6	2.91	16.4	1.49	69.6	2.02
		6			8.78	68.9	46.6	2.3	73.9	2.9	19.3	1.48	83.9	2.06
		7			10.1	79.6	53.3	2.29	84.6	2.89	22.1	1.48	98.3	2.1
		8			11.5	90.2	59.8	2.28	94.9	2.87	24.8	1.47	113	2.15
		9			12.8	101	66.1	2.27	105	2.86	27.5	1.46	127	2.18
8	80	5.5	9	3	8.63	67.8	52.7	2.47	83.6	3.11	21.8	1.59	93.2	2.17
		6			9.38	73.6	57	2.47	90	3.11	23.5	1.58	102	2.19
		7			10.8	85.1	65.3	2.45	104	3.09	27	1.58	119	2.23
		8			12.3	96.5	73.4	2.44	116	3.08	30.3	1.57	137	2.27
9	90	6	10	3.3	10.6	83.3	82.1	2.78	130	3.5	34	1.79	1.45	2.43
		7			12.3	96.4	94.3	2.77	150	3.49	38.9	1.78	1.69	2.47
		8			13.9	109	106	2.76	168	3.48	43.8	1.77	1.94	2.51
		9			15.6	122	118	2.75	186	3.46	48.6	1.77	2.19	2.55
10	100	6.5	12	4	12.8	101	122	3.09	193	3.88	50.7	1.99	214	2.68
		7			13.8	108	131	3.08	207	3.88	54.2	1.98	231	2.71
		8			15.6	122	147	3.07	233	3.87	60.9	1.98	265	2.75
		10			19.2	151	179	3.05	284	3.84	74.1	1.96	330	2.83
		12			22.8	179	209	3.03	331	3.81	86.9	1.95	402	2.91
		14			26.3	206	237	3	375	3.78	99.3	1.94	472	2.99
		16			29.7	233	264	2.98	416	3.74	112	1.94	542	3.06
11	110	7	12	4	15.2	119	176	3.4	279	4.29	72.7	2.19	308	2.96
		8			17.2	135	198	3.39	315	4.28	81.8	2.18	353	3
12.5	125	8	14	4.6	19.7	155	294	3.87	467	4.87	122	2.49	516	3.36
		9			22	173	327	3.86	520	4.86	135	2.48	582	3.4
		10			24.3	191	360	3.85	571	4.84	149	2.47	649	3.45
		12			28.9	227	422	3.82	670	4.82	174	2.46	782	3.53
		14			33.4	262	482	3.8	764	4.78	211	2.45	916	3.61
		16			37.8	296	539	3.78	853	4.75	224	2.44	1051	3.68
14	140	9	14	4.6	24.7	194	466	4.34	739	5.47	192	2.79	818	3.78
		10			27.3	215	512	4.33	814	5.46	211	2.78	914	3.82
		12			32.5	255	602	4.21	957	5.43	248	2.76	1097	3.9

16	160	10	16	5.3	31.4	247	774	4.96	1229	6.25	319	3.19	1356	4.3
		11			34.4	270	844	4.95	1341	6.24	348	3.18	1494	4.35
		12			37.4	294	913	4.94	1450	6.23	376	3.17	1633	4.39
		14			43.3	340	1046	4.92	1662	6.2	431	3.16	1911	4.47
		16			49.1	385	1175	4.89	1866	6.17	485	3.14	2191	4.55
		18			54.8	430	1299	4.87	2061	6.13	537	3.13	2472	4.63
		20			60.4	474	1419	4.85	2248	6.1	589	3.12	2756	4.7
18	180	11	16	5.3	38.8	305	1216	5.6	1933	7.06	500	3.59	2128	4.85
		12			42.2	331	1317	5.59	2093	7.04	540	3.58	2324	4.89
20	200	12	18	6	47.1	370	1823	6.22	2896	7.84	749	3.99	3182	5.37
		13			50.9	399	1961	6.21	3116	7.83	805	3.98	3452	5.42
		14			54.6	428	2097	6.2	3333	7.81	861	3.97	3722	5.46
		16			62	487	2326	6.17	3755	7.78	970	3.96	4264	5.54
		20			76.5	601	2871	6.12	4560	7.72	1182	3.93	5355	5.7
		25			94.3	740	3466	6.06	5494	7.63	1432	3.91	6733	5.89
		30			111.5	876	4020	6	6351	7.55	1688	3.89	8130	6.07
22	220	14	21	7	60.4	474	2814	6.83	4470	8.6	1159	4.38	4941	5.93
		16			68.6	538	3157	6.81	5045	8.58	1306	4.36	5661	6.02
25	250	16	24	8	78.4	615	4717	7.76	7492	9.78	1942	4.98	8286	6.75
		18			87.7	689	5247	7.73	8337	9.75	2158	4.96	9342	6.83
		20			97	761	5765	7.71	9160	9.72	2370	4.94	10401	6.91
		22			116.1	833	6270	7.69	9961	9.69	2579	4.93	11464	7
		25			119.7	940	7006	7.65	11125	9.64	2887	4.91	13064	7.11
		28			133.1	1045	7717	7.61	12244	9.59	3190	4.89	14674	7.23
		30			142	1114	8117	7.59	12965	9.56	3389	4.89	15753	7.31